Physics 222 CID ____________

Homework 7

Score ________ / 30

You might find the following integrals useful on this assignment. Here n is an integer.

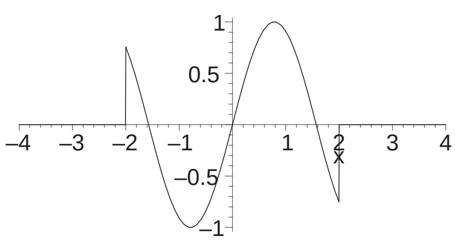
$$\int_0^{n\pi} \sin^2(x)dx = \frac{n\pi}{2}$$

$$\int_0^{n\pi} x\sin^2(x)dx = \frac{n^2\pi^2}{4}$$

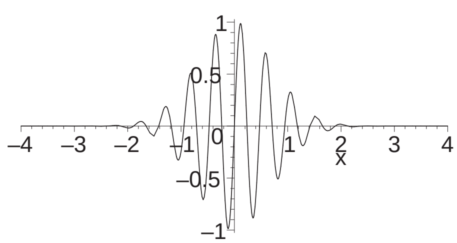
1. (4 pts) If $\Psi = Ae^{i(kx-\omega t)}e^{-x^2/a^2}$ is a properly normalized wave function, what does A equal? Assume that A is real and note that $\int_{-\infty}^{\infty} e^{-x^2}dx = \sqrt{\pi}$.

2. (5 pts) We talked about three properties that a wave function must have in order to be "well-behaved." A wave function which is not "well-behaved" represents a situation which is not physically possible. (a) What "physical impossibility" occurs if the wave function is not single valued? (b) What "physical impossibility" occurs if the second derivative is not finite everywhere. (c) What "physical impossibility" occurs if the first derivative is not continuous? (d) What if the wave function itself is not continuous? (e) What if the wave function doesn't approach 0 as $x \to \pm\infty$?

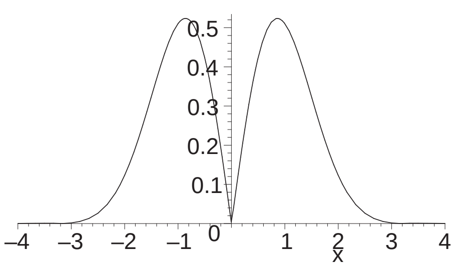
3. (4 pts) For each of the following wave functions, state if it is a "well behaved" wave function. If it is not, explain why it is not.



(a)



(b)



(c)

(d) $\Psi = Ae^{i(kx-\omega t)}$ (Sorry, there's no plot of this one — it is complex, and it goes on forever. But you can imagine what it would look like).

4. (6 pts)

(a) Use the chain rule to show that any function $y(u)$ will solve the classical wave equation with v equal to a constant if $u = kx - \omega t$. Remember, if v is a constant (i.e., it doesn't depend on k) then this represents a non-dispersive medium. (Your book makes the bold statement that all solutions to the classical wave equation are of the form $y(x - vt)$, but that is only true if v is a constant — i.e., if the medium is non-dispersive. It is not true for light traveling through glass, for example.)

(b) Show that $y(x,t) = ax^3 + bt^3$ cannot be a solutions to the classical wave equation in a medium for which v was a constant. (In fact, it can only be the solution for a medium in which v depends on x and t in a very particular way. This would be very strange — in a dispersive medium, v depends on k, and it is possible for it to depend on x (imagine light going through air and then entering glass at some value of x). But a time dependent velocity is unusual (but not impossible). It means that the properties of the medium are changing in time — imagine waves traveling on a rubber band as the rubber band is stretched — or blackbody radiation from the big bang traveling through an expanding space-time. This can have weird effects — like the frequency of a pure sine wave changing in time!)

(c) Show that $\Psi(x,t) = Ae^{i(kx-\omega t)}$ is a solution to Schrodinger's equation in free space (i.e. where U is a constant).

(d) Show that $\Psi(x,t) = A\sin(kx - \omega t)$ does not solve Schrodinger's equation. (What do you expect, it's not complex!) To do this, simply show that if it were a solution it would imply something that cannot be true.

5. (4 pts) Show (by plugging it into Schrodinger's equation) that if Ψ_1 and Ψ_2 are solutions to Schrodinger's equation, then $a\Psi_1 + b\Psi_2$ is also a solution (a and b are constants).

6. (7 pts) Consider a particle in the n^{th} energy state of an infinite square well of size L. Its wave function at time $t = 0$ is given by

$$\psi = \begin{cases} \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} & \text{for} \quad 0 < x < L \\ 0 & \text{otherwise} \end{cases} .$$

(a) At time $t = 0$, what is the expectation value $<$x$>$?

(b) Imagine that I had a million identical particles in one million identical square wells, and each had this same wave function at time $t = 0$. At time $t = 0$ I measure the position x of each particle within its well. Will I measure the same x for every particle?

(c) If I average all of the values I got in part (b), what average position will I get?

(d) If $n = 2$, at what locations is $P(x)$ the largest?

(e) If $n = 2$, at what locations in the range $0 < x < L$ is $P(x)$ equal to zero?

Extra problems I recommend you work (not to be turned in)

- Think about your answer to problem 6(e) and ponder what the expectation value $< x >$ actually tells you about the probability of finding a particle at particular locations.

- Calculate $< x^2 >$ for the wave in problem 6.

- Calculate $< x >$ for a wave function which is equal to the wave function in problem 6 times $e^{i\phi}$ where ϕ is a constant.

- Calculate $< x >$ as a function of time for a particle in the n^{th} state of an infinite square well. The full time-dependent wave function is just the wave at time $t = 0$ times $\exp(-iE_n t/\hbar)$ where E is the energy of the n^{th} state.