Physics 222 CID ____________

Homework 5

Score ________ / 30

1. (6 pts) Although section 3.6 invoked a fair amount of hand-waving, the equation that was derived (Equation 3.18) gives the correct energies for a particle in a one-dimensional infinite square well (particle in a box). But we could also use the uncertainty principle to "guess" the approximate order of magnitude of the lowest allowed energy.

(a) If we let Δx be defined as the size of the well L, what is the smallest that Δp can be?

(b) If we guess that the absolute value of p for the ground state must be on the order of the uncertainty, about how big must the ground-state energy be (using the classical, non-relativistic expressions for E and p).

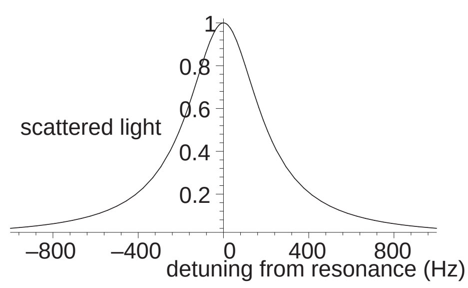
(c) How does this guess compare with the actual ground-state energy?

2. (4 pts) Justin Peatross, a professor in our department, has a laser which can produce pulses of light only 15 femtoseconds long (1 fs = 10^{-15}s).

(a) To make such a laser you need to use a laser amplifying crystal which will amplify a broad range of light frequencies. What is the minimum span of frequencies you need to produce a pulse this small?

(b) Imagine that Justin sends one of these pulses onto a diffraction grating, and then uses a pinhole to catch just a narrow band of wavelengths from the pulse. If he uses an ideal, infinitely fast response detector to detect the light coming through the pinhole, how long will he find that light was present on the detector — exactly 15 fs, more than 15 fs, or less than 15 fs. Assume that the 15 fs pulse has a Gaussian envelope (such that the uncertainty is equal to 1/2, not greater than 1/2)? Explain why.

3. (6 pts) In my lab we are constructing an atomic clock based on an optical frequency transition in calcium atoms. The line width of the transition is 410 Hz. This means that if I had a perfect laser (a pure sine wave) shining on stationary calcium atoms magically suspended in the laser beam, and if I scanned the laser very very slowly through the resonance frequency, the atoms would scatter a fair amount of light as long as the frequency of the light was within a 410 Hz wide region centered on the true transition frequency. If I plot the amount of light scattered out of the laser beam as a function of the laser's frequency, I would get something that looks like the graph below.



Notice that this curve is roughly 410 Hz wide (by eye I'd say that it is somewhat wider, but for this shape of curve, and the way line width is defined, it is exactly 410 Hz wide).

(a) When an atom scatters light, what is really happening is that the atom absorbs a photon, putting an electron in an excited state. Eventually the electron falls down, and light is re-emitted. Given that $\Delta\nu$ =410 Hz, use the energy-time uncertainty relation to find out about how long the electron stays in the excited state before falling back down. Note that electrons in atoms don't usually stay in an excited state for more than a few nanoseconds. This particular excited state lives much longer, however, which is why we are using this particular transition.

(b) Now, imagine that the atoms are not stationary, but are moving at 750 m/s, passing briefly through my laser beam which is 1 cm in diameter. As I scan my laser I get a curve very similar to the curve above, but it will be wider. About how wide will it be?

4. (6 pts) (a) Give three reasons why the planetary model of the atom cannot be correct. (b) Give three reasons that the Bohr model of the atom cannot be a complete description of what really happens in an atom (although it does a pretty good job for most things).

5. (8 pts) In Bohr's theory he assumed that the electron in a hydrogen atom could orbit without emitting radiation if it underwent a circular orbit with an angular momentum equal to an integer times $\hbar$. He chose this quantization condition simply because it fit the empirical data. But his theory can be derived using more fundamental principles of wave mechanics.

(a) Use the Coulomb force law and the assumptions that the electron undergoes a classical circular orbit and that the circumference of a Bohr orbit must be an integer n times the de Broglie wavelength of the electron to derive the allowed orbital radii and electron velocities. (Hint: The acceleration for uniform circular motion plus Coulomb's law gives you an equation with v, r, and known quantities. The requirement that your orbit have an integer number of waves gives you another such equation. Then you have two equations and two unknowns . . .)

(b) Use what you found in (a) to find the kinetic and potential energy of the electron in the n^{th} state. Then add them together to get the total energy.

(c) What does the fact that the total energy is negative mean?

(d) If you double n, what happens to the total energy, velocity, and radius of the electron's orbit?

(e) What is the angular momentum of the n^{th} Bohr orbit?

<u>Extra problems I recommend you work (not to be turned in)</u>

- Use the position-momentum uncertainty relation to find an uncertainty relation for the angle θ and the angular momentum L for a particle confined to move in a circle of radius r.

- If you feel ambitious, find a relation for the angular momentum of elliptical Bohr orbits for which the electron's wave meets up with itself continuously (the same wave condition we applied to circular orbits in problem 2). Remember that velocity, and therefore momentum and wavelength, changes along an elliptical orbit.

- Presumably the earth/sun system obeys quantum mechanics. (a) What is the earth's kinetic energy (as measured in a frame at rest with the sun)? (b) What is the potential energy of the earth (it should be negative). (c) What is the total energy of the earth (kinetic plus potential). It should be negative, because the earth is bound to the sun. (d) Using a Bohr-like model, find the quantum number n of the earth's orbit. (e) What is the minimum amount of additional kinetic energy that the earth can absorb (i.e., how much energy must it absorb to go to the next highest n)? This is an example of the correspondence principle.