Physics 222 CID ___________

Homework 4

Score ________ / 30

1. (6 pts) An electron and a proton are traveling at the same velocity (i.e. their group velocities are the same). (a) Which one has the longest de Broglie wavelength? (b) Which one has the largest quantum frequency ν? (c) Which one has the larges phase velocity?

2. (6 pts) A beam of neutrons strikes a crystalline piece of material. A particular diffraction peak is at an angle of 10.2 degrees from the incident beam.

- **(a)** What is the angle between the incident beam and the surface of the Bragg plane causing this diffraction peak? I'm asking for the angle from the surface itself, not from the normal.
- **(b)** The spacing between Bragg planes is not just the spacing between atoms in the crystal — it depends on which set of Bragg planes you are considering. Assuming that the Bragg planes which generate this peak are spaced 0.181 nm apart, and assuming that this is a first-order peak ($n = 1$) what is the wavelength of the neutrons?
- **(c)** What is the kinetic energy of the neutrons?

3. (4) Imagine that I have an electron trapped in a thin conducting film on top of a flat insulating surface. As you will learn later, this 3D problem can be decomposed into three 1D problems - so lets just focus on the small dimension of the film. Also, let's assume that the binding energy of the conducting film, ϕ, is so large that we can treat this like an infinite well.

- **(a)** We can't meet the boundary conditions with a traveling wave - we have to have a standing wave. What does this tell us about the velocity of the electron wave function?
- **(b)** If n is a positive integer, write the possible wavelengths that our "1D electron" standing wave can have terms of n.
- **(c)** Use this to find the allowed kinetic energies that our "1D electron" can have.
- **(d)** What total energies can our "1D electron" have?

4. (4 pts) Some times it is difficult, if not impossible, to solve a quantum problem analytically. But often you can use a simpler but similar problem to make estimates. Use the equation for the allowed energies for a particle in a box to estimate the following energies (in eV):

- **(a)** The ground state energy of the electron in a hydrogen atom. Treat the hydrogen atom as box with $L = 10^{-10}$m.
- **(b)** The ground state energy of a proton in a helium nucleus. Use $L = 2 \times 10^{-15}$m.
- **(c)** The energy of the photon that is emitted when a proton in a helium nucleus de-excites from the first excited state ($n = 2$) to the ground state ($n = 1$).

5. (4 pts) Imagine a marble of mass m which is sitting between two large boards spaced a distance L apart. Explain why, even if we used an "ideal" motion detector, we would never measure it's velocity to be lower than $v_{min} = \sqrt{h^2/4m^2L^2}$.

6. (6 pts) If I want to pull an electron out of a large piece of metal, it takes an amount of energy called the "work function." We mentioned this when we discussed the photo-electric effect. But if the piece of metal is very very thin, quantum mechanics places a limit on the lowest kinetic energy the particle can have. Assuming we can treat the thin film as a 1D infinite square well, how thin does a film of aluminum need to be before the lowest energy electrons in the metal have enough kinetic energy to leave the metal? The work function for aluminum is 4.08 eV.

<u>Extra problems I recommend you work (not to be turned in)</u>

- Think about what the wave function of the ground state of a two-dimensional infinite square well would look like (i.e., the particle can move in the x and y directions, and $U = 0$ if $0 < x < L_x$ and $0 < y < L_y$, and $U = \infty$ otherwise). See if you can think of a function which would seem to work.

- Explain why the wave function has to go to zero anywhere $U = \infty$.