

1. (5 pts) An electron, a proton, and a photon all have a de Broglie wavelength of 100 nm. (a) What is the momentum of each of the three particles (in kg m/s)? (b) What is the velocity of the three particles (in m/s)? (c) What is the kinetic energy of the three particles (in eV)?
2. (5 pts) An electron, a proton, and a photon all have a kinetic energy of 1.21 MeV. (a) What is the velocity of the three particles? (b) What is the momentum of the three particles? (c) What is the de Broglie wavelength of the three particles? Give all quantities in SI units.
3. (2 pts) A nitrogen molecule has a mass of about 4.65×10^{-26} kg. In thermal equilibrium, the molecules in a gas have an average kinetic energy of $(3/2)kT$. What is the de Broglie wavelength of a nitrogen molecule with this energy, assuming that the gas is at a temperature of 25°C ? (Note that what you are calculating is close to but not equal to the average de Broglie wavelength for nitrogen molecules in the gas - think carefully about it. Also, note how much smaller this is than the wavelength of a photon of visible light - this is partly why atom interferometers show so much promise.)
4. (4 pts) (a) What is the minimum potential difference (in Volts) that I would need to accelerate an electron through in order to produce x-rays with a wavelength of 0.1 nm? Assume that the electron is initially at rest. (hint: $hc = 1239.8 \text{ eV nm}$). (b) What is the de Broglie wavelength (in nm) of an electron accelerated through this potential difference?

In the book it states that the probability of finding something at a particular point is proportional to $|\Psi|^2$. This is technically true, but I need to clarify something. The probability of finding a particle at an exact location is usually zero — for any normal wave function there are an infinite number of points where the particle could be found and the probability of finding it at any given point is zero. For example, if the wave function for a particle in one dimension is non-zero from $x = 0$ to $x = 1$ m at some particular time, the probability of finding the particle at $x = 0.4$ m is zero. If you think you found one at 0.4 m, you should look a little closer, and you'll find that it is really a little off — say maybe 0.4000000001 m or 0.399999999 m. If you think you've found one precisely at 0.4 m, just measure with more precision, get more decimal points, and eventually you will find that it wasn't *really* at 0.4 m. However, that said, not all zeros are alike, and it is possible to discuss the *most probable* location to find the particle. That is the location where $|\Psi|^2$ is the largest. Even if the probability of finding it there is zero, it is still a “bigger” zero than anywhere else. In other words, what I really mean is that the probability of finding the particle within an infinitesimal range about that point is bigger than anywhere else.

Usually what we ask is not the probability of finding the particle at a particular place, but the probability of finding it in a range of locations. The probability of finding a particle in a particular region at a given time is just the integral of $|\Psi|^2$ over the volume of the region at that time: $\int |\Psi|^2 dV$. If the wave function describes a single particle, the integral over all of space should be equal to 1, since the probability of finding the particle *somewhere* is 100%.

5. (4 pts) Imagine that a particle in a one-dimensional universe has a wave function at time $t = 0$ which is equal to $\Psi(t = 0) = -Ae^{-bx^2}$ where A is a positive real number. (a) What is the “most probable” location to find the particle if we measure its location at time $t = 0$. Use the definition of “most probable” that I gave above. (b) If the probability of finding the particle *somewhere* is 1, what is A equal to? The integral you need is $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.
6. (6 pts) The dispersion relation for a medium ($\omega(k)$) tells us the angular frequency of sine waves in the medium as a function of the wavenumber of the sine wave.
 - (a) In a non-dispersive medium, sine waves of all wavelengths and frequencies travel at the same velocity: $v_p = C$ where C is a constant. I have labeled this velocity v_p for phase velocity since only phase velocity has meaning for a pure sine wave. Because the velocity of a sine wave is given by $v_p = \omega/k$, the dispersion relation for

a non-dispersive medium is just $\omega(k) = Ck$. Imagine a pulse (consisting of an infinite sum of sine waves, with an average wavenumber of k_0) traveling through this non-dispersive medium. At what velocity will the “center of mass” of the pulse travel?

- (b) Now imagine a dispersive medium in which the velocity of a sine wave with a wavenumber k is given by: $v_p = Ak^3$, where A is a constant. What is the dispersion relation $\omega(k)$ for this medium?
- (c) If the pulse from part (a) were travelling in the medium from part (b), what would its phase and group velocities be?

7. (4 pts) In the text they show that the phase velocity for a free massive particle is equal to the speed of light squared divided by the group velocity (see equation 3.3). They found this using the relativistic energy $E = \gamma mc^2$. (a) Find the phase velocity (as a function of the group velocity) for non-relativistic systems (i.e., using $E = mv^2/2$). (b) Why doesn't equation 3.3 in the text become your answer to part (a) in the limit as $v \rightarrow 0$? (c) Why doesn't this matter?

Extra problems I recommend you work (not to be turned in)

- Find a dispersion relation for which the phase velocity is greater than the group velocity.
- Find a dispersion relation for which the phase velocity is less than the group velocity.
- Find a dispersion relation which gives a phase and group velocity which move in the opposite direction.
- Imagine that a particle in one dimension has a wave function $\Psi = Ae^{-i(kx-\omega t)}$. (a) What is $|\Psi|^2$? Note that $|Q|^2 = Q^*Q$. (b) In terms of the constants A , k , and ω , what is the probability of finding the particle somewhere between $x = 0$ and $x = x_0$ at time $t = 0$? (c) If the probability of finding the particle *somewhere* at a given time is equal to 1, what must A be equal to?