Physics 222 CID ____________

Homework 1

Score ________ / 30

This assignment is designed to help you find out what math you need to review for this course. You can get help during the review session or during office hours. You should have this information in textbooks from previous courses, and I think everything you need to know can actually be found online at sources such as Wikipedia.

1. (6 pts) **Differentiation and Integration**. Solve the following integrals and derivatives. Assume that a is a real constant and that n is an integer. Assume that u is a function of x.

- **(a)** Write down the solutions to the following derivatives. Look them up if you need to. The last one should $\frac{\partial}{\partial x} ax^n$, $\frac{\partial}{\partial x}\sin(ax)$, $\frac{\partial}{\partial x}\cos(ax)$, $\frac{\partial}{\partial x}e^{ax}$, $\frac{\partial}{\partial x}f(u)$
- **(b)** Without using a table or a computer, solve the following derivatives. $\frac{\partial}{\partial x}(1+\sin(ax))^n$, $\frac{\partial}{\partial x}\sin(\pi e^{ax})$, $\frac{\partial}{\partial x}\left(\frac{3}{x}\right)$
- **(c)** Write down the solutions to the following integrals. Look them up if you need to.$\int x^n dx$, $\int \sin(ax)dx$, $\int \cos(ax)dx$, $\int e^{ax}dx$

2. (6 pts) **Complex Numbers.** The imaginary number i is just equal to $\sqrt{-1}$, and that $i^2 = -1$. An asterisk in the superscript means complex conjugation. Taking the complex conjugate of a number or an expression means turning each i in the expression into a $-i$. For example, if $a = (i + exp(2i))/(1 + i)$, then $a^* = (-i + exp(-2i))/(1 - i)$. Assume that b and c are constants.

- **(a)** If $a = b + ic$, evaluate the following: a^*, ab, $a + a^*$, $a - a^*$, a^*a
- **(b)** If $a = e^{b+ic}$, evaluate the following: a^*, ab, a^*a

3. (9 pts) **Euler's Equation and Trig Identities.** This one you might not have seen before, but it is really handy. Euler's equation states that $e^{i\theta} = \cos(\theta) + i\sin(\theta)$.

You can find all kinds of trig identities using this formula. You can also often use it to replace sines and cosines with exponentials, which can make solving equations easier.

- **(a)** Note that $e^{i(a+b)} = e^{ia}e^{ib}$. This means that $\cos(a + b) + i\sin(a + b) = [\cos(a) + i\sin(a)][\cos(b) + i\sin(b)]$. Note that the real part of the left-hand side must equal the real part of the right-hand side, and that the imaginary part of the left-hand side must equal the imaginary part of the right-hand side. Use this fact to separate the equation into two equations and find a trig identity for $\cos(a + b)$ and $\sin(a + b)$.
- **(b)** Find a way to write $\sin(\theta)$ and $\cos(\theta)$ in terms of complex exponentials.
- **(c)** If I add together two sine waves with the same frequency but different phases and amplitudes, the result is a third sine wave with the same frequency: $A\sin(\omega t + \phi_A) + B\sin(\omega t + \phi_B) = C\sin(\omega t + \phi_C)$. Unfortunately, finding C and ϕ_C can be messy. This problem is trivial when we deal with complex exponentials, since $e^{i(\omega t+\phi)} = e^{i\phi}e^{i(\omega t)}$. What do you get when you add $Ae^{i(\omega t+\phi_A)} + Be^{i(\omega t+\phi_B)}$?
- **(d)** Use your Euler's equation or your answer to part (b) to prove that $\sin(a) + \sin(b) = 2\cos([a - b]/2)\sin([a + b]/2)$.

4. (9 pts) **Integrating Products of Sines.** Do these integrals without using a table or a computer. Instead, draw a sketch of the two sine/cosine waves and their product, and ask yourself what the area under the curve is. Remember that the area under a curve is just the average y value of the curve times the distance in x that you integrate over.

- **(a)** Solve $\int_0^{2\pi}\cos(x)\cos(x)dx$, $\int_0^{2\pi}\cos(3x)\cos(3x)dx$
- **(b)** Solve $\int_0^{2\pi}\cos(x)\sin(x)dx$, $\int_0^{2\pi}\cos(x)\cos(3x)dx$