Example Exam for Physics 222

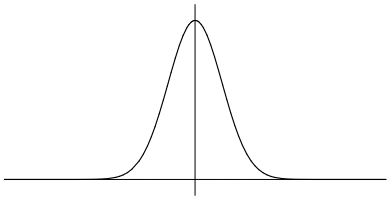
Remember that these questions are only intended to give you a feel for what my tests are like. They do not cover all of the material you will be expected to know on the real exam.

Useful Integrals:
If a is a real constant greater than zero, then...

$$\int_{-\infty}^{\infty} e^{-a^2x^2}dx = \frac{\sqrt{\pi}}{a} \qquad \int_{-\infty}^{\infty} x^2 e^{-a^2x^2}dx = \frac{1}{2}\frac{\sqrt{\pi}}{a^3}$$

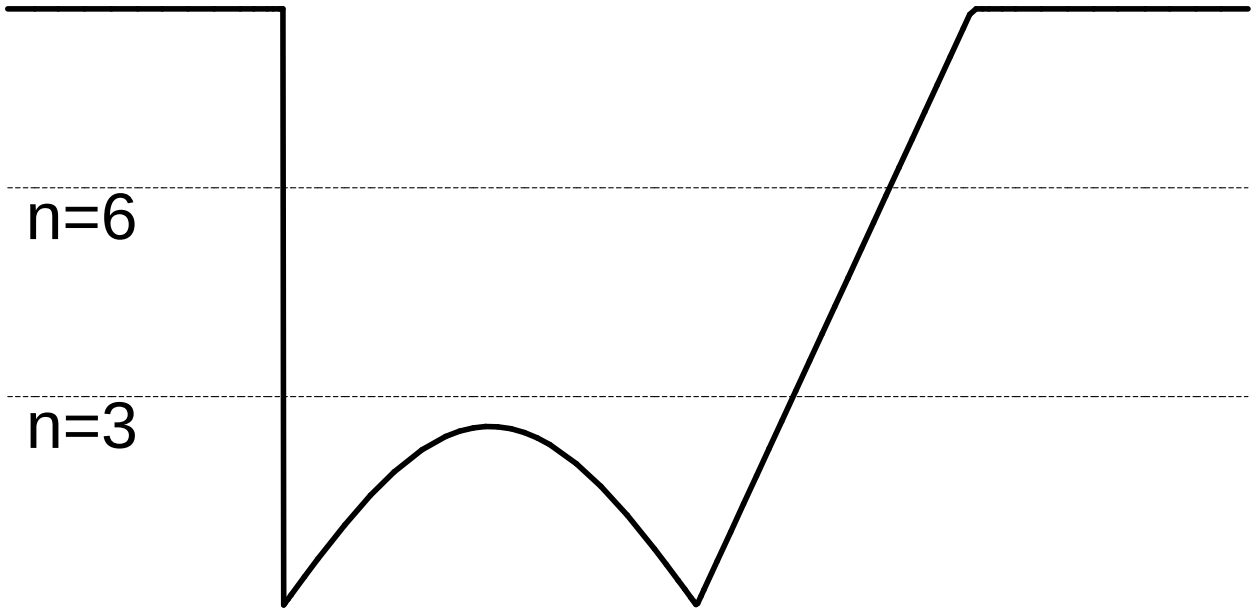
$$\int_{-\infty}^{\infty} xe^{-a^2x^2}dx = 0 \qquad \int_{-\infty}^{\infty} x^3 e^{-a^2x^2}dx = 0$$

1. An electron in free space has a wave function at time $t = 0$ which is given by $\Psi(x, t = 0) = Ae^{-x^2/a^2}$ where a is a constant. The wave function at time zero is plotted below



(a) Assuming that A is real, what is A?
(b) At time $t = 0$ what is $< p >$?
(c) At time $t = 0$ what is $< x >$?
(d) Is this function an eigenfunction of momentum?

2. The dark line below represents the potential $U(x)$. The energies of the $n = 3$ and $n = 6$ energy eigenstates are shown with dashed lines. Make a sketch of what the stationary state wave functions will look like on the dashed lines. Assume that the $n = 1$ state is the ground state.



3. A lithium atom ($Z = 3$) has been stripped of two of its electrons, such that only one electron remains.

(a) What is the energy of the ground state of the ion?

(b) What is the shortest wavelength of light that the ion can emit?

(c) What is the lowest final state n_f that can be involved in the emission of a visible photon? (Remember that visible light includes wavelengths between 380 and 760 nm).

4. The dispersion relation for deep water waves (i.e. waves for which $h \gg \lambda$) is given by the relation $\omega = \sqrt{gk}$ where $g = 9.8$ m/s^2 is the acceleration due to gravity. Imagine that you are sitting in a row boat in the middle of Utah lake on a calm day. You grab a paddle, dip it into the water, and swish it quickly back and forth 15 times in the space of 2 seconds, generating a water wave packet which travels towards the shore.

(a) Find an algebraic expression in terms of g and the average wavenumber k for the speed at which the individual ripples within the wave packet travel.

(b) Find an algebraic expression in terms of g and the average wavenumber k for the speed at which the packet as a whole travels towards the shore.

(c) Find an algebraic expression for k as a function of the average frequency f and the phase speed v_p.

(d) Plug the relation found in (c) into the equations found in (a) and (b) and solve the equations to get a numerical answer for the two velocities found in (a) and (b).

5. A particle in an infinite square well of length L has a wave function at time $t = 0$ which is given by

$$\Psi(x, t = 0) = \sqrt{\frac{1}{3L}}\left[\sqrt{2}\sin\left(\frac{3\pi x}{L}\right) + 2e^{i\pi/6}\sin\left(\frac{2\pi x}{L}\right)\right]$$

(a) What is the wave function at an arbitrary time t?

(b) If you measured the energy of the particle, what is the probability of finding it in the ground state?

(c) What is $< E >$ at an arbitrary time t?

6.

(a) What is does the term "zero-point" energy mean?

(b) Describe the difference between classical and quantum uncertainty.

(c) What is the correspondence principle?

(d) Explain how the photoelectric effect led to Einstein's conclusion that light was quantized.

(e) What are two of the properties that a wave function must have in order to be "well behaved."