Physics 222 CID ____________

Lab 2 - Group and Phase Velocity

Score ________ / 10

In this lab you will study how wave packets propagate in linear media using a computer simulation. You will study both non-dispersive media in which sine-waves of all wavelengths travel at the same speed (like, for example, light traveling in a vacuum) as well as dispersive media (like light traveling through a piece of glass, electron quantum waves traveling through space, and just about every other real system).

The first step is to go to the "labs" page on the class web page and click on Dispersion Lab. This will open the applet in a new window. It may take a few minutes to download and start up, but once it does you should see a screen with two graphs and some text. If you don't have access to a computer or if you have problems running this program, a computer in the walk-in lab area will be running this program running during the week this lab is scheduled. The next step is to click on the red "get help" button in the upper left-hand corner and read the instructions for the software. You may want to play with the program for a bit to make sure that you understand how it works.

Uncertainty First lets explore the uncertainty which is inherent in waves. To do this, first click on "Reset All." In the upper graph you should see a depiction of a Gaussian wave packet (a little "burst" of a sine-wave with a Gaussian-shaped "envelope"). In the lower graph you can see the spectrum of the pulse (the amplitude of each of the sine waves which the computer added together to make the wave packet in the upper graph). On the far right-hand side of the program the computer displays Δx (the standard deviation of the pulse in space), Δk (the standard deviation of the pulse's spectrum), and the product of the two.

We learned in class that in order to make pulses which were very narrow in space, we had to add a wide band of frequencies or wavenumbers together, making it difficult to state with certainty what the frequency of the pulse was. To make a wave packet with a very well defined frequency or wavenumber we had to let the packet extend over a large range in space such that it was difficult to assign a location to the packet with precision. Furthermore, we learned that if we defined uncertainty to be the RMS standard deviation, the uncertainties in x and k follow the uncertainty relation $\Delta x \Delta k \geq \frac{1}{2}$.

Notice that our wave satisfies the above uncertainty relation. Now type in a different value for the pulse width (w). Notice that as the pulse shrinks, its spectrum widens. The uncertainty relation should still hold. Now change the central wavenumber (k) and see what happens.

Now click "Reset All," enter 75 for k, and enter `squarepulse(x/w)` for the "Envelope." Now try different values for the pulse width and fill in the table below. Then answer the question below.

w	Δx	Δk	$\Delta x \Delta k$
0.02			
0.05			
0.08			
0.1			

- Do the values in this table satisfy the uncertainty relation above?

Note that the physical size of the pulse on the screen is about 4 times larger than Δx. This is just due to the fact that we have chosen to define uncertainty as the RMS standard deviation. This is the most commonly used but not always the most useful definition. So, you see, there is uncertainty in our definition of uncertainty. As a result, the uncertainty relation is often written in the less precise form: $\Delta x \Delta k \gtrsim 1$.

Non-dispersive media In this part of the lab we will examine what happens when wave pulses travel in non-dispersive media. In non-dispersive media the angular frequency of a sine wave is simply proportional to the wavenumber of the wave: $\omega(k) = vk$, where v is the velocity that waves travel through the medium. Wait a minute... is that the phase or group velocity? Think about this for one minute, and then answer the following two questions in the space provided.

- The dispersion relation for light traveling through a vacuum is just $\omega(k) = ck$, where c is equal to 2.9979×10^8 m/s. What is the phase velocity for a pulse of light who's central wavelength is 657 nm?

- What is the group velocity for this light pulse?

Now lets use the computer simulation to see what happens to a Gaussian-shaped pulse as it propagates through a non-dispersive medium. First click on the "Reset All" button. There should now be a pretty pulse displayed in the upper graph, with a nice spectrum centered around a wavenumber of 75 m^{-1}in the lower graph. Now click on the "Go!" button to let time run and see what happens. The dispersion relation, shown just below the "Reset All" button, is $\omega(k) = 0.1\mathrm{m/s} \cdot k$. Use this dispersion relation to answer the following questions.

- What is the group velocity for a pulse in this medium centered at 75 m^{-1}?

Now click on the "Stop" button to stop the simulation if it hasn't already stopped, and click on the "Reset t=0" button to set time back to zero. Now plug the group velocity you calculated above into the "x-Axis Velocity" box to make our "view window" move with the pulse. Click on "Go!." If you did your calculation correctly, the pulse should stand still in the window.

Based on what you have seen, answer the following question.

- What happens to the spatial size of a pulse and the spread of frequencies or wavenumbers in a pulse as it travels in a non-dispersive medium?

Dispersive Media Now lets pick a dispersion relation which is a little more interesting: the dispersion relation for a massive particle in free space, $\omega = \hbar k^2/2m$. Click on "Reset All," and then enter the dispersion relation "0.001*k^2". Before you do anything else, use this dispersion relation to calculate the group and phase velocities for a pulse centered around $k = 75\mathrm{m}^{-1}$.

- Group Velocity

- Phase Velocity

Now click on “Go!” and see what happens. Now stop the simulation, set time to $t = -10$, and set the “x-Axis Velocity” equal to the group velocity you calculated above. Press “Go!” again and watch what happens. Now stop the simulation, set time to $t = -2.5$, and set the “x-Axis Velocity” equal to the phase velocity calculated above. Press “Go!” and see what happens (hint: this is the part of the lab where the vertical blue line in the center of the graph is useful). Finally, based on what you saw and in you own words explain what phase and group velocity represent:

- Group velocity is...

- Phase velocity is...

Now, based on what you have seen, answer the following question.

- What happens to the spatial “size” of a pulse when it travels through a dispersive medium?

- What happens to the spectrum of a pulse when it travels through a dispersive medium?

That’s the end of the lab, but I recommend that you take some additional time to play around with this simulation. If you can develop a solid understanding of dispersion, uncertainty, and group and phase velocities, you will be able to better understand how quantum waves evolve, travel through space, and interfere.