Physics 222 CID ___________

Lab 4 - Stationary States

Score ________ / 10

In this lab you will develop the ability to predict what the stationary states of arbitrary 1-dimensional potentials are like. You will do this using the same computer program used in the "square well" lab.

The simple harmonic oscillator (sho)

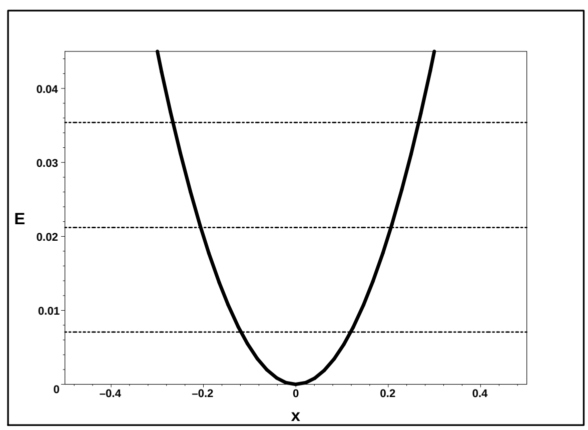
Before we get into unfamiliar teritory, lets review the simple harmonic oscillator. First answer the following question:

How do the energies of the stationary states of a sho scale with n? $E \propto$ ______________

Now calculate the energy of the three lowest-energy states for a particle with a mass such that $m/\hbar^2 = 5000$ S.I. units (i.e., $m = 5.55 \times 10^{-65}$ kg) in a harmonic potential with a frequency such that $m\omega^2 = 1$ in S.I. units (i.e. $\omega = 1.34 \times 10^{32}$ rad/s) and record them below.

Now sketch below what stationary states will look like on the dotted lines on the graph below. Remember the four rules. First, if we label the ground state as $n = 0$, then the number of zero crossings is just equal to n. Second, the wavefunction decays exponentially in regions where $E < V$, decaying more rapidly in x when V is large. Third, the wavefunction oscillates in regions where $E > V$. Since the kinetic energy (which is equal to $E-V$) is proportional to the curvature of the wavefunction, the oscillations have a shorter period where $E - V$ is large (i.e. where V is small). Fourth, the amplitude of the wave is related to the probably of finding the particle near a particular location. Classically, the probability of finding a particle is greater in regions where the classical velocity is lower. Similarly, the wavefunction has a larger amplitude where the kinetic energy ($E - V$) is smaller.

$n =$	0	1	2
E_n			

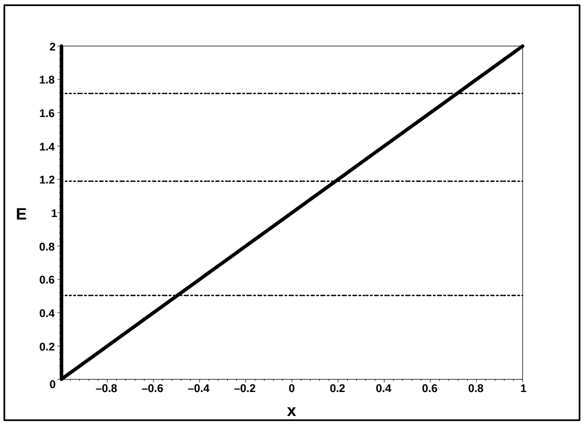


Now enter a simple harmonic oscillator potential into the simulation by entering "`0.5*x^2`" for "V(x)" and set "mass/hbar^2" to 5000.0 and n to 0. Now click on "Recalculate," and you should see the ground state wavefunction of a harmonic oscillator. Now look at different values of n and see if the simulation verifies the energies you calculated and the waveforms you sketched. Also look at the plot of E vs. n and make sure your answer to the first question is correct.

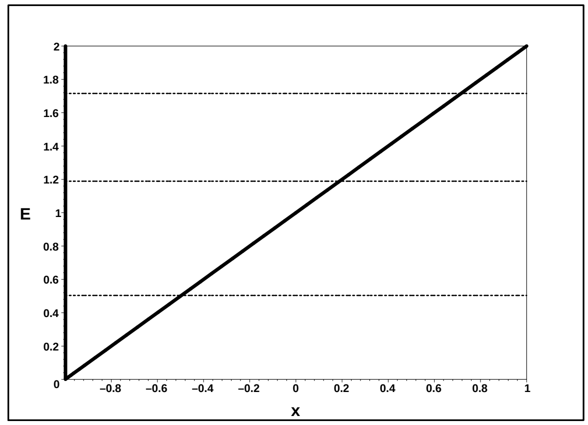
Funky potential "A"

Now consider a potential which is equal to $x + 1$ from $x = -1$ to $x = 1$ but which jumps up to infinity at $x = -1$, as shown in the two figures below. The dotted lines represent the energies of the $n = 0$, $n = 2$, and $n = 4$ stationary states. Using our rules, sketch what you think the wavefunctions of these states will look like on the leftmost graph below. Then enter the function **"x+1"** for "V(x)" and set "mass/hbar^2" to 50.0 and click on Recalculate. Look at the wavefunctions as you change n and see how good your guesses were. Now sketch the wavefunctions calculated by the simulation in the rightmost graph below.

Predicted



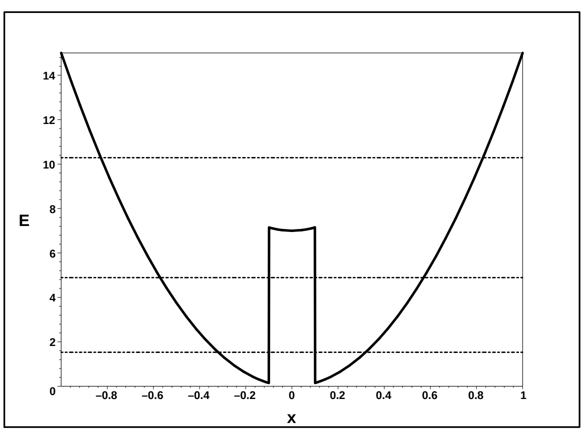
From Simulation



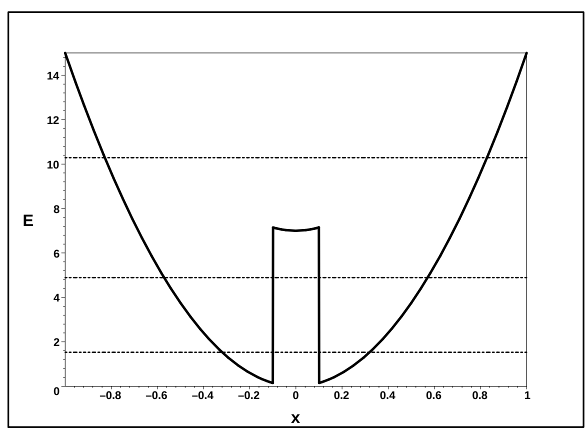
Funky potential "B"

Now consider the potential below. The dotted lines represent the energies of the $n = 0$, $n = 5$, and $n = 12$ stationary states. Using our rules, sketch what you think the wavefunctions of these states will look like on the leftmost graph below. Then enter the function **"15*x^2 + 7*squarepulse(x*10)"** for "V(x)" and set "mass/hbar^2" to 50.0 and click on Recalculate. Look at the wavefunctions as you change n and see how good your guesses were. Now sketch the wavefunctions calculated by the simulation in the rightmost graph below.

Predicted



From Simulation



You should note a few interesting things. Since this potential is symmetric about the origin, the stationary state wavefunctions are all either symmetric or anti-semmetric (in other words they are either purely even or purely odd functions of x). Also, take a look at the plot of the stationary state energies. The states with energies below the top of the spike in the middle of the potential show up in pairs; the energies of the $n = 0$ and $n = 1$ states are nearly the same, as are the energies of 2 and 3, 4 and 5, and 6 and 7. It is common for this to happen whenever the potential has a "double well" shape in which there are two separate classically allowed regions in which we could find the particle. When the energy of the state is above the bump, the two classically allowed regions are no longer divided, and we don't get energies in pairs anymore. This principle will appear again when we study covalent bonding and the "band" theory of solids.