1. (10 points) In the last homework, we derived the equation for heat transferred through the lead of a transistor. We came up with

\[ k \frac{\partial^2 T}{\partial x^2} = \rho c \frac{\partial T}{\partial t}, \]

where \( k \) is the thermal conductivity of the lead, \( \rho \) is the mass density of the lead, and \( c \) is the specific heat of the lead.

(a) Separate this PDE into two ODEs.

(b) Use the separated equations to find a general set of solutions to this equation. (Hint, one of these two equations could be solved with sine with an arbitrary phase shift, an sine plus a cosine or with exponentials. Since we know they are really the same, all should work. But applying boundary conditions will be easier if we use a sine plus a cosine.)

2. (6 points) We want to solder the lead to our circuit, but we don’t want to destroy the transistor. Let’s call the length of the lead \( L \) and assume that it is initially at room temperature \( T_0 \) everywhere. Let’s also assume that no heat comes in or out at the \( x = 0 \) end, and that the \( x = L \) end is kept at a temperature \( T_H \) by a soldering iron. Write down equations for the initial condition and boundary conditions, and apply them to your general solution to get three equations (which will could be used, in theory, to determine the unknown constants). (Hint, the equations should involve infinite sums.)

3. (10 points) OK, let’s not go any further with this set of boundary/initial conditions. Let’s do an easier set of conditions. Assume that the lead is completely insulated from the world (i.e. no heat enters or leaves) and at time \( t = 0 \) has a temperature distribution

\[ T(x,0) = T_A + T_B \cos \left( \frac{\pi x}{L} \right). \]

Find \( T(x,t) \) for all times greater than \( t = 0 \).

4. (14 points) In Physics 222 you should have met the time-independent Schrödinger’s equation - the equation we use to find the allowed energies of a system and the quantum wavefunctions that go along with those energies. The equation in 2D Cartesian coordinates looks like this:

\[ -\frac{\hbar^2}{2m} \left( \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + U \psi = E \psi, \]

where \( \hbar, m, E, \) and \( 2 \) are constants, and \( U \) is the potential (which is usually a function of position).

(a) Let’s look at this equation in zero potential. Set \( U = 0 \) and then separate this PDE into two ODEs. Then find a general set of solutions to this equation. As in the last problem, use the sine plus cosine forms in your solutions.

(b) Now let’s assume that the particle that this equation describes is in a 2D box, such that it’s wave equation has to be zero outside of the box. Assume that one corner of the box is at \( (0,0) \), and that it extends a distance \( W \) in the \( x \) direction and \( H \) in the \( y \) direction. The boundary conditions in this problem, then, are that \( \psi \) must be zero along each of the four edges of the box. Using these boundary conditions, you can make two sets of constants in your arbitrary solution \( \psi \) to zero, and turn two of them into something known times an integer. Use these boundary conditions to find the allowed values for \( E \) which give a solution to the PDE, and find the wavefunctions \( \psi_E \) that go with those allowed energies. Note that your wavefunctions will still have an unknown constant - there’s one more boundary condition in quantum that we haven’t used (the fact that the wave must be normalized). Don’t worry about it for now.

Wow, we just solved Schrödinger’s equation. How cool is that? How are we ever going to top that?