1. (15 points) Show that you can separate Laplace’s equation in spherical coordinates into an angular part which is solved by the spherical harmonics, and a radial part which has the form of Euler’s equation. Note that, although Farlow calls the polar angle $\theta$, let’s use the more standard notation where the polar angle is $\phi$ and the azimuthal angle is $\theta$, so that our equations look like the Wikipedia page on spherical harmonics.

2. (10 points) Now let’s solve a problem where we can assume that our solution to Laplace’s equation has no angular dependence (only dependence on $r$). Assume that I have a spherical metal shell of radius $r = a$ which is held at a voltage $V = V_0$. Assuming that the potential at $r = \infty$ is zero, what is the potential inside and outside of the sphere?

3. (15 points) Now let’s assume that I have a spherical surface of radius $a$ whose potential depends on the azimuthal angle (what Farlow calls $\phi$, but is usually called $\theta$) according to the equation

$$V(a, \phi) = V_0 + V_1 \cos(\theta).$$

Find $V(r, \theta)$ everywhere outside of the sphere. To solve the $\theta$ part, let $x = \cos(\theta)$ and turn the ODE into one we’ve seen before. To solve the $r$ part, just accept that the solution to this equation (which is a form of Euler’s differential equation) is $Cr^\gamma$ where $C$ and $\gamma$ are constants.