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Homework 37

Score ________ / 30

1. (6 pts) A car of mass $m_1 = 1000$ kg is sliding without friction on ice at a velocity $u_1 = 20$ m/s when it strikes another car of mass $m_2 = 1200$ kg which was standing still. The two cars lock together and slide together without friction after the collision. (a) Use the principle of momentum conservation to find the velocity of the two cars after the collision. (b) An observer riding on a bicycle in the same direction as the cars watches the collision while traveling at a velocity $v = 10$ m/s. Find the initial and final velocities of the two cars as measured in an inertial reference frame which is at rest with the bicycle rider using the Galilean transformations. (c) Show that the velocities that you found in (b) satisfy momentum conservation in the bicycle rider's reference frame.

2. (5 pts) Now consider the same car crash as viewed by a bicycle rider who is moving at a speed $v = 10$ m/s at a direction which is orthogonal to the direction that the cars are moving. Let the bike rider be moving in the x direction and the cars moving in the y direction. (a) Use the Galilean transformations to find the x and y components of the cars' velocities be before and after the collision. (b) Show that momentum is conserved in the reference frame of the bike rider.

3. (5 pts) If we assume that Newton's second law ($\vec{F} = m\vec{a}$) is true in a particular inertial reference frame, show that it is also true in the frame of an observer which is moving at a velocity v in the x direction.

4. (5 pts) If we assume that Newton's second law ($\vec{F} = m\vec{a}$) is true in a particular inertial reference frame, show that it is *not* true in the frame of an observer which is accelerating at a constant rate a_0 in the x direction.

5. (3 pts) An observer in a train and an observer standing next to the tracks see different objects moving around (a dog running, a bicycle riding past, etc.) and measure each velocity in their reference frame. At the end of the day they compare notes and find that in most cases they disagree in the magnitude of the velocities that they measured. Which two will the observers always agree on?

6. (3 pts) There are two postulates from which all of special relativity is derived. Write these postulates down in your own words.

7. (3 pts) (a) What important fact did the results of the Michelson-Morley experiment suggest? (b) How did this help lead to Einstein's special theory of relativity?

<u>Extra problems I recommend you work (not to be turned in)</u>

- An observer at rest with respect to the Earth finds that objects falling under gravity accelerate at a constant rate of 9.8 m/s. According to Galilean relativity, will an observer on a train moving at 10 m/s see that objects falling under gravity accelerate at a constant rate in their frame? If so, what is that rate?

- An observer at rest with respect to the Earth finds that objects falling under gravity accelerate at a constant rate of 9.8 m/s. According to Galilean relativity, will an observer on a rocket moving vertically away from the surface of the earth at 10 m/s see that objects falling under gravity accelerate at a constant rate in their frame? If so, what is that rate?

- An observer at rest with respect to the Earth finds that objects falling under gravity accelerate at a constant rate of 9.8 m/s. According to Galilean relativity, will an observer on a rocket accelerating vertically away from the surface of the earth at 10 m/s^2 see that objects falling under gravity accelerate at a constant rate in their frame? If so, what is that rate?

- I throw a ball into the air. An observer at rest with respect to the Earth sees the ball fly in a parabolic path. By Galilean relativity, will an observer on a train see the ball move in a parabolic path?

- If I measure the momentum of a ball in two different inertial reference frames, will I get the same measurement in both frames? Why or why not?

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Homework 38

Score ________ / 30

1. (4 pts) Deriving the time-dilation equation: Imagine that your good friend Albert is flying past you in a spaceship at velocity v (relative to you) while you are floating out in deep space. Albert has a mirror glued to the roof of the bridge of his spaceship, a distance L above him. He pulls out a laser pistol and shoots a burst of light upward at the mirror, it reflects back, and burns a bald spot in his hair. (a) How long does it take the light to travel from the pistol to Albert's hair? We'll call that time Δt_p. (b) In your reference frame, the mirror is moving at a velocity v. If you observe that the light took a time Δt to go up and back to Albert, how far did the mirror move in this time? (c) Your answer to part (b) implies that the light traveled a path which looks like two sides of a triangle. How long is that path (in terms of Δt, L, v, and c)? (d) Knowing that the speed of light is constant, find the time it takes to travel this path, Δt, in terms of L, v, and c. (e) How does Δt_p compare to Δt? In other words, find the time dilation equation!

2. (4 pts) Imagine that you lived in a universe where the speed of light was 10 m/s. You hold one end of a meter stick and put your eye right at the 10 cm mark. A friend named Jane holds up the other end and puts her eye right at the 90 cm mark. You and Jane both have wrist watches that are synchronized to each other. A third friend Bill throws a second meter stick like a spear right past the meter stick you are holding. At exactly 10:38 and 1.0200212 seconds you note that the very back of the thrown meter stick passes just in front of your eye. At precisely the same time Jane notes that the very front of the thrown meter stick passes just in front of her eye. How fast was the thrown meter stick moving?

3. (4 pts) Bill likes to throw things (living in a universe in which $c = 10$ m/s tends to make people cranky). While you and Jane are still standing at the 10 and 90 cm marks of a meter stick, the two of you synchronize your infinite accuracy atomic wrist watches. Then Bill picks up your battery powered travel alarm clock and throws it past you. Like your wrist watch, the alarm clock is very precise and never looses or gains a second. It is not, however, synchronized to your and Jane's wrist watches. As it passes your eye you note that the time on the clock flying past is 10:40 and 14.8894 seconds. At that moment your wrist watch reads 10:40 and 10.4533 seconds. When the clock passes Jane's eye, her wrist watch reads 10.40 and 10.5922 seconds. What time does the clock flying past Jane read as it passes her eye? Give your answer to 1/100 of a second.

4. (4 pts) Now for something a little more realistic. A Λ^0 particle is an unstable particle that can be created in a particle collider. If you make a bunch of Λ^0 particles at rest, they will decay with an average decay time of 0.260 ns. (a) If I make a stream of Λ^0 particles which are traveling at a speed of 0.955 c, what average particle decay time will I measure? (b) If they travel very close to the speed of light we could measure an average decay time of 1 hour. How far from the speed of light are they when this is true (i.e. what is $c - v$ when we measure a decay time of 1 hour)? You will need to use the approximation $(1 + \epsilon)^x \simeq 1 + x\epsilon$ for small ϵ unless you have a calculator with 30 digits of precision.

5. (4 pts) Martha travels on a spaceship at a constant velocity from the Earth to mars. Jim stays at home on Earth. Both of them note the time and location of the two events: Martha passing the Moon and Martha reaching Mars. (a) Which of the two will measure the proper length between the two events? (b) Which of the two will measure the proper time between the two events? Assume that the Moon and Mars don't move appreciably with respect to the Earth during the time of Martha's journey.

6. (6 pts) A deep space probe is launched from the Earth and passes a deep space station on its way into the unknown. It reaches it's final speed of 0.811 c relative to the station just before it passes the space station. After passing the station it travels in a straight line at a constant speed. The probe has an on-board atomic clock and is programmed to send a microwave signal back to the station every year (as measured by the clock on board the probe). It sends it's first signal right as it passes the station. Right as the probe's clock tells it to send its second signal back to the station it is passing a comet. (a) From the reference frame of someone on the space station, how much time elapses from the time that the probe sends its first pulse to the time that it sends its second pulse (give the time that it *sends* the pulse, in years, not the time that the pulse is received back at the station).

(b) From the reference frame of the probe, how far away is the station when it sends its second pulse? (c) From the reference frame of someone on the station, how far away is the comet? (d) If someone on the space station measures the time between the first pulse arriving at the station and the second pulse arriving at the station, what length of time will they measure (in years)?

7. (4 pts) Ever since the big bang different parts of the universe have been flying away from each other. Astronomers can figure out how fast a star is moving away from us by looking at atomic emission lines and measuring how much the lines have been Doppler shifted from the wavelength of the lines which we measure in experiments on Earth. They often use a parameter known as the "red-shift." This parameter is usually represented as z and is defined to be the wavelength they measure for the light coming from the star, λ_m, minus the wavelength measured in an experiment in which the atoms emitting the light are at rest, λ_0, all divided by λ_0. In other words $z \equiv (\lambda_m - \lambda_0)/\lambda_0$. Imagine that astronomers look at a particular emission line from hydrogen atoms in a particular star and find that the red-shift of the line is 0.7. How fast is that star moving relative to the Earth?

Extra problems I recommend you work (not to be turned in)

- Read the book, "Mr. Tompkins in Wonderland" by George Gamow (also sold under the name "Mr. Tompkins in Paperback"). You will enjoy it, and you will better understand relativity (and quantum mechanics)! The HBLL library has several copies under the following call numbers: **QC 71 .S775 1999**, **QC 71 .G25 1965**, **QC 173.5 .G36x**, and **QC 6 .G23 1940.**
- Why isn't the "twin paradox" a problem? How would you solve the problem correctly?
- Just because I can't travel faster than the speed of light, it doesn't mean that I can't travel any further than about 100 light years from Earth before I die (where 100 years is about the longest someone might live). Why not?

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Homework 39

Score ________ / 30

1. (5 pts) Two lights attached to a space ship moving at half the speed of light (relative to the earth) flash on momentarily. An observer on the space ship notes that the flashes occur simultaneously. What conditions must be met for the flashes to occur simultaneously according to an observer on the earth?

2. (2 pts) Bill is on a rocket ship traveling at a speed of $0.91c$ relative to the Earth. Jane is on another ship traveling at $0.95c$ relative to the Earth but in the opposite direction. How fast does Bill measure Jane moving relative to him?

3. (8 pts) Jimmy Neutron is Chasing Finbar Calamitous in a rocket ship. Carl, a friend of Jimmy who is watching from Earth, measures the velocity of Finbar's ship as 0.851c and the velocity of Jimmy's ship as 0.899c. At some moment in time (lets call it $t = 0$) Carl uses an ACME distometer to correctly measure that the two ships are 1.89×10^9 meters apart.

(a) According to Carl, how long will it take for Jimmy to catch Finbar?

(b) How fast is Finbar traveling as measured by Jimmy?

(c) According to Jimmy, how much time will pass between Carl making his measurement and him overtaking Finbar? Assume that Jimmy is just passing the Earth when Carl makes his measurement. (Hint: the distance between Jimmy and Finbar depends on the reference frame — it is not the same as measured by Jimmy and Carl. And you will probably make a mistake if you try to use length contraction, because there is no frame in which both Jimmy and Finbar are stationary. You can either use the Lorentz transformations to transform the two space-time events — Jimmy passing Earth and Jimmy catching up with Finbar — and find the time between them, or you can note that both events happen at the same place in Jimmy's frame and use time dilation.)

(d) If we now assume that Jimmy's pursuit has him traveling nearly directly toward the Earth but he is still 9.10×10^9 meters from the Earth when Carl makes his measurement, how much time will Jimmy measure between the time Carl makes his measurement and the time that Jimmy overtakes Finbar. (Hint - time dilation won't work on this one, so you'd better use Lorentz transformations).

4. (8 pts) Jimmy Neutron is returning from a trip to the center of the galaxy, traveling at a speed of 0.871c relative to the Earth. Right as Jimmy flies past the Earth he verifies that his watch is synchronize with Carl's. Carl is standing still on the surface of the Earth. In both Carl's and Jimmy's frames the Earth is at a location $x = 0$, $y = 0$, and $z = 0$ at time $t' = t = 0$. Carl sets up his coordinate system such that Jimmy is moving in the $+x$ direction, and Jimmy sets his up such that his y and z directions are the same as Carl's and such that Carl is moving in the $-x$ direction. After Jimmy has passed Earth, Carl observes a supernova looking through a telescope. Taking into account the time that it took for light to reach him he determines that the supernova occurred at a time $t = -1.45 \times 10^9$ seconds at a location of $x = 3.29 \times 10^{17}$m, $y = 1.53 \times 10^{17}$m, and $z = 1.69 \times 10^{17}$m. (a) As measured in Carl's reference frame, how far is Jimmy from the supernova when it occurs. (b) As measured in Jimmy's reference frame, how far is Jimmy from the supernova when it occurs?

5. (7 pts) Imagine that you are on a rocket ship bound for mars. Before it launches, you measure the distance from you to mars to be 7.8×10^{10} m (78 million kilometers). Then, in a period of 1 nanosecond you accelerate up to a speed of 0.995 c going straight toward mars. (a) How far does mars appear to be from you now? (b) About how fast did mars "appear" to be moving toward you as you accelerated? (c) Why doesn't this violate the rule that nothing can travel faster than the speed of light?

<u>Extra problems I recommend you work (not to be turned in)</u>

- A meter stick is placed at an angle of 20° from the x axis. A rocket flies by at a speed of 0.9c in the $+x$ direction. How long, and at what angle to the x axis will the meter stick be as measured by someone in the rocket ship?

- If you were traveling in a rocket at 0.9c relative to the Earth, what would you see if you held up a mirror in front of your face?

- You run a red light. You are pulled over. You explain to the traffic officer that you didn't know that the light was red — because you were moving the red light (at 660 nm) was Doppler shifted to appear green (at 520 nm). How fast were you going?

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Homework 40

Score ________ / 30

1. (5 pts) Explain why communication or travel faster than the speed of light violates causality.

2. (5 pts) At what speed is a particle's momentum twice that given by the classical momentum relation?

3. (5 Pts) A particle with a rest mass of m, traveling at a speed of 0.98 c has a head-on collision with a particle of mass $3m$ which is initially at rest. If the larger mass moves away with a velocity of 0.89 c, what is the final velocity of the smaller mass?

4. (5 pts) Two grams of hydrogen burn, combining with 15.9 grams of oxygen to produce water. In the process, 5.72×10^5 Joules of energy are released. How much mass is lost in this process (i.e. how much less mass does the water have compared to the sum of the mass of the hydrogen and the oxygen we started with)?

5. (5 pts) The largest power reactor in the world is the RBMK-1500 reactor at the Ignalina nuclear power plant in Italy. The nuclear reactions in the fuel rods generates 4.80 GW (4.80×10^9 Watts) of heat. This heat creates steam which drives turbines to generate 1.50 GW of electricity. If a new set of fuel rods is placed in the reactor, and then the reactor runs at full power for a year, how much less will the fuel rods weigh at the end of the year?

6. (5 pts) A 1 kg block of copper is heated from a temperature of 25°C to a temperature of 150°C. How much does the mass of the copper change?

Extra problems I recommend you work (not to be turned in)

- At what speed u does the classical momentum equation ($p = mu$) differ from the relativistic form ($p = \gamma mu$) by less than 0.1 percent?
- At what speed u does the classical kinetic energy equation differ from the relativistic form by less than 0.1 percent?
- If an object with a rest mass of 10 kg is accelerated with a constant force of 10 N, how long will it take before the object reaches (a) half the speed of light, (b) the speed of light?
- An electron has a mass of $9.1093897 \times 10^{-31}$ kg. A proton has a mass of $1.67262171 \times 10^{-27}$ kg. A hydrogen atom is made of an electron bound to a proton. It takes 2.1790×10^{-18} J of energy to separate the electron and the proton of a hydrogen atom. To 9 decimal places, how much does a hydrogen atom weigh?
- A photon has no rest mass. Can a photon have momentum? Why or why not?
- A 1 gram meteor strikes the moon and makes a huge crater. From the size of the crater you determine that the energy released in the collision was equivalent to detonating 1 million tons of TNT. How fast was the meteor going before it hit the moon?

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Homework 41

Score ________ / 30

1. (7 pts) An object with mass $m = 1$ kg is moving with a velocity which has an x component $u_x = -0.4$ c, a y component $u_y = 0.5$ c and a z component $u_z = 0$. (a) What are the three components of the object's momentum? (b) What is the kinetic energy of the particle? (c) What is the total energy of the object (including its rest energy)? Give all answers in S.I. units.

2. (8 pts) Now consider the same object as seen by an observer in a reference frame moving at a velocity $v = 0.7$ c in the x direction relative to the frame used in problem 1. (a) Find the three components of the object's momentum and the total energy (including the rest energy) in the new frame by first transforming the three components of the particle's velocity and then plugging these transformed values into the same equations you used above. (b) Now use the momentum/energy transformation equations below and show that you get the same values:

$$\begin{aligned} p'_x &= \gamma(p_x - vE/c^2) \\ p'_y &= p_y \\ p'_z &= p_z \\ E' &= \gamma(E - vp_x) \end{aligned}$$

3. (8 pts) I place a perfect clock which never looses or gains time and has a display which reads out to 1/100,000,000 of a second on top of a 100 meter tall building. I place an identical clock on the ground. Both clocks are synchronized at time t=0. (a) After ten years, a physicist named Liz standing next to the lower clock compares the times on the two clocks. If she assumes that the lower clock is correct, will she conclude that the upper clock is running fast or running slow? (b) By how much will she perceive the upper clock's time to differ from the lower clock's time? (c) If a physicist named Bill is standing next to the upper clock, will he conclude that the lower clock is running fast or slow? (d) By how much will Bill perceive the lower clock's time to differ from the upper clock's time? Assume that the acceleration due to gravity near the Earth is 9.80 m/s^2 and that the observers take into account and subtract the time that it takes for light from the faces of the clocks to reach their eyes when they are measuring the time. *Remember, relativity is NOT about things seeming to be delayed because it takes light time to reach your eye. This is a separate effect. Remember my illustration of a whole team of people making measurements directly in front of them...*

4. (7 pts) (a) If I compress a 1 kg brick into a non-rotating black hole, what will be the radius of its event horizon (also known as the Schwarzschild radius)? (b) Classically an object undergoing a circular orbit of radius r with a velocity v undergoes an inward radial acceleration equal to $a = v^2/r$. Since light travels at the speed of light ($v = c$), what does r need to be for a photon of light to make a circular orbit around an infinitesimally big speck with a mass of 1 kg? The gravitational constant $G = 6.6726 \times 10^{-11}$N m^{2}kg^{-2}, and the force from a point mass of mass M on a second point of mass m a distance r away is given by $F = mMG/r^2$.

Extra problems I recommend you work (not to be turned in)

- In class we derived an equation for the gravitational redshift assuming the acceleration due to gravity is constant. This works well when we are comparing two clocks near the surface of the Earth, but not for clocks far away from the Earth. Derive an equation for the fractional shift in the frequency of two clocks ($\Delta f/f$) if the two clocks are a distance r_1 and r_2 from the center of a spherically symmetric planet with a mass m. Assume both clocks are above the surface of the planet.

- Imagine a photon is created a distance r away from the 1 kg infinitesimal speck and travels radially away from the speck. As it travels away it gets red-shifted. How big must r be in order for the photon to not get red-shifted down to zero frequency before it gets infinitely far away from the speck?

- Use the velocity transformation equations to derive the energy/momentum transform equations.

- (a) What makes up the missing mass of the universe? (b) What is the fundamental mechanism which causes the collapse of a quantum wave function? (c) What general method can be used to combine relativity with quantum mechanics? (d) Why is the universe made of matter instead of antimatter? (e) What is the analytical form of the nuclear weak force? (f) Can CPT be violated? ... Aaaahhh! So many unanswered questions! This may be the last homework, but there are still so many problems to solve! Why are you still reading this... get busy! There are great discoveries to be made!!!