1. (6 pts) Consider a wave of the form
   \[ y(x, t) = \frac{y_0}{(kx - \omega t)^4 + 1}. \]
   (a) Is the wave moving in the +x or -x direction?
   (b) What is the velocity of the wave?
   (c) Write an equation for a wave which is identical to this wave but which is moving in the opposite direction.

2. (6 pts) A seismometer detects an earthquake. The faster longitudinal wave (the “P” wave”) is detected 21.2 seconds before the slower transverse wave (the “S” wave). If we know that P waves travel at 7.80 km/s and that S waves travel at 4.50 km/s, how far is the seismometer from the epicenter of the quake?

3. (7 pts) (a) Write down an expression for a sinusoidal wave traveling in the +x direction with an amplitude A, an angular frequency \( \omega \), and a wavenumber \( k \). Assume that \( y(x = 0, t = 0) = 0 \) and that \( dy/dt \) is positive at the position \( x = 0 \) at time \( t = 0 \). (b) Now write the expression for the same wave, but in terms of the amplitude \( A \), the wavelength \( \lambda \), and the frequency \( f \). (c) Find the velocity of the wave in terms of \( \omega \) and \( k \). Then remember this equation and never forget it! (d) Find the period \( T \) of the wave in terms of \( \omega \) and \( k \). (e) Write, in terms of \( A \), \( k \), and \( \omega \), the expression for a wave traveling in the -x direction assuming that \( y(x = 0, t = 0) = A \).

4. (7 pts) I am watching sinusoidal waves travel across a swimming pool. If I look at the water right in front of me, it goes up and down ten times in 11.5 seconds. At the peaks of the wave the water is 4 cm below the edge of the pool. At the lowest points of the wave the water is 6 cm below the edge of the pool. At one particular moment in time you notice that the water height is at a maximum right in front of you, and then drops to a minimum height 0.791 meters away. (a) What is the frequency \( f \) for this wave? (b) What is \( \omega \) for this wave? (c) What is \( \lambda \) for this wave? (d) What is \( k \) for this wave? (e) What is the amplitude \( A \) of this wave? (f) What is the speed of water waves in this pool? Note that the units for \( \omega \) are \text{rad/s}, and the units for \( k \) are \text{rad/m}.

5. (4 pts) A sinusoidal wave travels one wavelength per period, so \( v = \lambda/T \). Use this as a starting point to prove that \( v = \omega/k \). These two equations are really worth remembering!

**Extra problems I recommend you work (not to be turned in)**

- As we will study in a future unit, light is actually a wave. A laser generates a wave which is almost perfectly sinusoidal. The wavelength of light from a helium-neon laser is 633 nm. The speed of light is \( 2.9979 \times 10^8 \text{ m/s} \). What is the wavenumber, frequency, period, and angular frequency of the light from a helium-neon laser?

- Explain what \( x \) and \( y \) physically represent for a transverse wave on a slinky. Explain what they physically represent for a longitudinal wave on a slinky.
1. (3 pts) You are abducted by aliens and placed in a holding cell on an unknown planet. Due to your diligent study of the Starfleet Planetary Guide, you know that if you could determine $g$, the gravitational acceleration on the planet, you would be able to figure out where you are. So you pull a thread from your uniform which is 1.21 meters long and which weighs 0.402 grams. You tie the end to your shoe, which weighs 0.211 kg. You then hold the top of the string with the shoe hanging at the bottom, and you pluck the string near the top. The pulse takes 0.312 seconds to travel down to the shoe. (a) What is $g$? (b) If I hang both shoes, rather than just one shoe from the string, thereby doubling the tension in the string, by what factor with the speed of waves on the string increase?

2. (4 pts) Two triangular shaped pulses are traveling down a string, as shown in the figure below. The figure represents the state of the string at time $t = 0$. The pulse on the left is traveling to the right, and the pulse on the right is traveling to the left, as indicated by the arrows above the pulses in the figure. The speed of waves on the string is 1 m/s. Draw the shape of the string at the following times: $t = 2s$, $t = 2.5s$, $t = 3.5s$, and $t = 5s$.

3. (6 pts) Imagine your slinky stretched to a length $L$ and fixed at both ends.

(a) What is the tension $T$ and the linear mass density $\mu$ of the slinky in terms of its mass $m$, its spring constant $k$, and its length $L$? Assume that the stretched length of the slinky is long enough compared to the length when it is not stretched that the unstretched length is negligible.

(b) What is the wave speed for transverse waves on a slinky (in terms of $m$, $k$, and $L$)?

(c) Have someone hold one end of your slinky (or attach it to something like a doorknob). Take the other end and stretch the slinky until it is about five feet long. Now strike one end of the slinky to make a transverse pulse and watch as the pulse travels to the other end and then reflects back. Time how long it takes for the pulse go out and back 10 times, and use this to calculate the wave speed for transverse waves on the slinky.

(d) Now predict what the wave speed would be if the slinky were stretched to about 10 feet.

(e) Stretch the slinky until it is about 10 feet long and measure the wave speed the same way you did before.

4. (2 pts) A transverse pulse travels down a slinky and reflects off of the end which is being held by a friend of yours. (a) Will the reflected pulse look the same as the incoming pulse, or will it be inverted? Now have someone hold one end of your slinky (or attach it to something like a doorknob). Take the other end and pull it back until the slinky is stretched about 10 feet (don’t stretch it too far or it won’t slink back together again and the slinky will be ruined). Hold your end of the slinky at arms length so that you can see the slinky well. Now quickly strike the top of the slinky with your hand to make a transverse pulse. Watch as the pulse reflects off of the fixed end. (b) Was the reflected pulse inverted?

5. (5 pts) If $a$ and $b$ are real numbers and $\bar{c}$ is a complex number, what is the complex conjugate of (a) $a + b$, (b) $a + ib$, (c) $a + \bar{c}$, (d) $a + i\bar{c}$, and (e) $(a + ib)/[\sin(\bar{c}) + ia]$?

6. (2 pts) Show that $\bar{A} \cdot A = |\bar{A}|^2$. 
7. (5 pts) I want you to know how to derive the rate at which a sine wave transmits power down a string. The basic idea is this — every cycle the sum of the kinetic and potential energy contained in a piece of string of length $\lambda$ passes by any given point on the string. So imagine a long string with a linear mass density $\mu$ under a tension $T$ with a sine wave of the form $y(x, t) = A \sin(kx - \omega t)$ traveling on it. Now consider an infinitesimal piece of the string with an unstretched length (its length when no wave is passing through it) of $dx$. This piece of string will have a mass $dm = \mu dx$. When the wave pass through it, it moves at a velocity $dy/dx$, giving it some kinetic energy. In addition, its length stretches, giving it potential energy. The $x$ component of its length is still $dx$, but it now has a $y$ component of length, such that its total length $ds$ is

$$ds = \sqrt{dx^2 + dy^2} = dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2}.$$ 

We will assume that the amplitude of the wave is small such that $dy \ll dx$. This allows us to use a Taylor series and make the approximation:

$$ds = dx \left(1 + \frac{1}{2} \left(\frac{dy}{dx}\right)^2\right).$$

It turns out that the amount of potential energy is exactly equal to the amount of kinetic energy. So to make your life simpler, I’m going to let you choose to find either the kinetic or potential energy, not both. So choose one version of problem 7 below and work it. Assume that the amplitude of the wave is small enough that $T$ is essentially constant as the wave passes by, and give all of your answers in terms of $A, \mu, \omega$, and $k$. For either one, you will need the following integral:

$$\int_a^{a+\lambda} \cos^2 \left(\frac{2\pi x}{\lambda}\right) dx = \int_a^{a+\lambda} \sin^2 \left(\frac{2\pi x}{\lambda}\right) dx = \frac{\lambda}{2}.$$

**Kinetic:** (a) What is the kinetic energy $dE_k$ that this infinitesimal piece of string contains at time $t = 0$? (b) Now consider a finite piece of the string with an unstretched length one wavelength long. How much kinetic energy does this piece of string have at time $t = 0$?

**Potential:** (a) Remembering that work equals force time distance, what is the potential energy $dU$ contained in an infinitesimal piece of the string at location $x$ at time $t = 0$? Hint - how much work would you have to do to stretch this piece string from its original length $dx$ to its length when the wave is passing through, assuming that the tension in the string doesn’t change significantly as we do this. (b) What is the total potential energy in one wavelength of the string?

8. (3 pts) We know that the the total energy in one wavelength (kinetic plus potential) will pass a point on the string once per period $T$. Use this and the formulas you found above to prove that

$$P = \frac{1}{2} \mu \omega^2 A^2 v.$$

**Extra problems I recommend you work (not to be turned in)**

- Hold one end of your slinky up high and let the other end dangle downward (don’t let it touch the floor). (a) If you whack the end of the slinky to make a transverse pulse, what will happen to the pulse when it reaches the bottom? Will it reflect? Will the reflection be inverted? (b) Try it and see what happens. You may be surprised! (c) Does the dangling end of the slinky act as a free or a fixed end? Why?

- If you hold one end of your slinky up high and let the other end dangle down (without touching the floor), how will the wave speed change as a function of the distance from the bottom of the slinky?

- Use your answers from the above question and from problem 4 to predict the time it would take for a transverse pulse to travel down and back up the slinky 5 times. Now test your prediction.

- Find the other component of the energy in a wave (the one you chose not to work).

- In our derivation for power transmitted by a sine wave we integrated over a wave. As such, what we really found is the “average” power transmitted through the string. It turns out that the power passing by a point goes up and down with time. Find an equation for the instantaneous power passing a point $x$. 

\[ \int_a^{a+\lambda} \cos^2 \left(\frac{2\pi x}{\lambda}\right) dx = \int_a^{a+\lambda} \sin^2 \left(\frac{2\pi x}{\lambda}\right) dx = \frac{\lambda}{2}. \]
1. (3 pts) Imagine a clothesline stretched across your yard. It has a mass of 0.113 kg and a length of 6 m. When you flick the line, the pulse you generate travels down the line at a speed of 19.2 m/s. When the pulse gets to the end, it is completely absorbed without reflection by the flexible pole it is tied to. If you stand near the other end of the line and wiggle it sinusoidally for one minute with an amplitude of 10 cm at a frequency of 3 Hz, how much energy will the flexible pole absorb?

2. (3 pts) The power transmitted by light (and other electro-magnetic waves) has the same dependence on amplitude as the power transmitted along a stretched string. If I measure the light power emitted by a laser and then increase the output of the laser until the power doubles, by what factor has the amplitude of the oscillating electric field in the laser beam increased?

3. (5 pts) (a) Consider the function \( y = A e^{(x-vt)/a^2} \) (where \( A \) and \( a \) are constants, and \( v \) is the speed of waves on the string). Plug this into the linear wave equation and show that it is a solution. (b) Show that \( y = A \sin(bxt) \) is not a solution to the wave equation (where \( A \) and \( b \) are constants). (c) By plugging things into the wave equation, show that if \( y_A(x,t) \) and \( y_B(x,t) \) are solutions to the wave equation, \( y_A + 2.13y_B \) is also a solution.

4. (5 pts) Find the bulk modulus for a gas undergoing an (a) adiabatic, and (b) isothermal process in terms of \( \gamma \) and the mean pressure \( P \).

5. (5 pts) (a) Use Euler’s formula to fill in the real and imaginary parts of the complex numbers in the table below. (b) Plot each of these points in the complex plane on the graph below.

<table>
<thead>
<tr>
<th>( z )</th>
<th>( \Re[z] )</th>
<th>( \Im[z] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5e^{i\theta} )</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>( 5e^{i\pi/8} )</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>( 5e^{i\pi/4} )</td>
<td>0</td>
<td>5</td>
</tr>
<tr>
<td>( 5e^{i3\pi/8} )</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>( 5e^{i\pi/2} )</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>( 5e^{i3\pi/4} )</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>( 5e^{i\pi} )</td>
<td>-5</td>
<td>0</td>
</tr>
<tr>
<td>( 5e^{i5\pi/4} )</td>
<td>0</td>
<td>-5</td>
</tr>
<tr>
<td>( 5e^{i3\pi/2} )</td>
<td>5</td>
<td>0</td>
</tr>
</tbody>
</table>

6. (9 pts) Let’s find the equation that describes the longitudinal vibrations in a rod of solid material. If the waves passing through the rod have a small amplitude, we can imagine that the rod is made up of tiny slices of mass each connected to the pieces on either side of it by tiny massless springs, as shown in the figure below.
The equilibrium length of each of these springs is \( dx \), and each of the thin masses will have a mass of \( m = \rho V = \rho A dx \) where \( A \) is the cross sectional area of our rod. We’ll label each slice of the rod with the location \( x \) of its center when it is in equilibrium (i.e. when no waves are passing through and all of the springs are relaxed - not stretched or compressed).

(a) Find the spring constant of our little springs \( k \) in terms of \( dx \) and the Young’s modulus of the material. Young’s modulus \( E \) is defined by the following equation:

\[
E = \frac{FL}{A \Delta L}
\]

where \( L \) is the length of our rod, \( A \) is the cross section of the rod, and \( \Delta L \) is the amount the length of a rod will shrink when a force \( F \) is applied to each end. (Hint: The number of springs in a rod of length \( L \) is \( L/dx \), and each one will compress a tiny amount when the force is applied.)

(b) If I look at the piece labeled \( x \) while a wave passes through, the spring to its left will be compressed by an amount \( s(x - dx, t) - s(x, t) \), resulting in a force of \( F_{left} = k[s(x - dx, t) - s(x, t)] \). What is the force exerted by the spring on the right of the piece labeled \( x’ \)? (Note that if the spring on the right is compressed it will result in a force in the negative direction - be sure to get your signs right!)

(c) Remember the definition of a derivative:

\[
\frac{\partial f}{\partial x}
\]

\[
\text{where the line and the subscript } x = y \text{ means that we are evaluating the derivative at } y. \text{ If we plug in your equation for } k \text{ from part (a) and apply the definition above, we can write } F_{left} \text{ and } F_{right} \text{ in terms of }
\]

\[
\frac{\partial s}{\partial x}
\]

\[
\text{and } \frac{\partial s}{\partial x} \bigg|_{x=x-dx}.
\]

So, now do it! Find \( F_{left} \) and \( F_{right} \) in terms of these derivatives.

(d) Now add these together to find the total force on the particle, and set it equal to \( ma \). Show that this is just the one-dimensional linear wave equation and determine what \( v \), the speed of longitudinal waves in our rod, is equal to.

**Extra problems I recommend you work (not to be turned in)**

- Get someone to hold the other end of your slinky or hook it on something. Then whack the slinky to send a pulse down it. Right as that pulse is reflecting off of the far end, whack it again to make a second pulse. Watch as the two pulses collide. Is the approximation that your slinky is a linear medium a good one?

- In Physics Phor Phynatics I mentioned that you can write any complex number as a real number times \( e^{i\theta} \): \( z = Ae^{i\theta} \). Find \( A \) and \( \phi \) for the following complex numbers: \( 2 + 3i \), \( -2 + i \), \( (2 + i)/(1 + i) \).

- The solution to the damped harmonic oscillator is of the form \( x = Ae^{-\tau t} \sin(\omega t + \phi) \). Find \( \tau \) and \( \omega \) as a function of \( m \), \( \gamma \), and \( \gamma \).

- Work the same problem, only now use a solution of the form \( x = Ae^{-\tau t}e^{i(\omega t + \phi)} \), realizing that the true solution at any time is just the imaginary part of this.

- The three dimensional wave equation is just

\[
\frac{1}{v^2} \frac{\partial^2 \vec{s}}{\partial t^2} = \frac{\partial^2 \vec{s}}{\partial x^2} + \frac{\partial^2 \vec{s}}{\partial y^2} + \frac{\partial^2 \vec{s}}{\partial z^2}.
\]

Show that any function \( f(\vec{r} - \vec{v}t) \) is a solution to this equation where \( \vec{r} = \hat{i}x + \hat{j}y + \hat{k}z \) and \( \vec{v} \) is a vector in an arbitrary direction with a magnitude equal to the wave velocity \( v \).
1. (5 pts) Imagine that I have a copper wire with a round cross section with diameter \( d = 0.411 \text{mm} \). I splice the end of that wire to another wire, also with diameter \( d = 0.411 \text{mm} \), but which is made of iron. I then pull the joined wires until they are under a tension \( T = 30 \text{ N} \). (a) What is the ratio of the wave velocity on the copper wire to the speed that waves travel on the iron wire (i.e., what is \( v_{\text{copper}}/v_{\text{iron}} \))? (b) What is the ratio of the wave numbers for the two wires \( (k_{\text{copper}}/k_{\text{iron}}) \) for a sine wave with an angular frequency \( \omega \)? (c) If I send a sine wave down the copper wire, what fraction of the power in the incident sine wave is transmitted to the iron wire? Copper has a density of \( 8920 \text{ kg/m}^3 \), and iron has a density of \( 7860 \text{ kg/m}^3 \).

2 (3 pts) If I splice a copper wire with a round cross section with a diameter \( d = 0.411 \text{mm} \) to an iron wire with a different diameter, what should the diameter of the iron wire be if I don’t want waves to reflect at the junction?

3. (5 pts) Imagine two different strings joined together at \( x = 0 \) with an ideal massless knot. Under the following conditions, state whether a wave traveling from one string to the other will transmit with no reflection, transmit with some reflection, not transmit at all, or whether the condition given is not sufficient to determine which will occur. (a) The two strings have the same linear mass density. (b) The two strings have the same diameter. (c) The two strings are made of the same material. (d) The wave velocity is the same on both strings. (e) For any sine wave of a given frequency, the wavenumber will be the same on both strings.

4. (3 pts) A beam of light traveling through air enters a piece of glass at normal incidence (i.e., it is traveling perpendicular to the face of the glass). Light travels at \( 2.9979 \times 10^8 \text{ m/s} \) in air. It travels at \( 1.9594 \times 10^8 \text{ m/s} \) in the glass. What fraction of the incident light power is reflected off of the surface of the glass?

5. (4 pts) Use Euler’s formula to prove that \( \cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b) \) and that \( \sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b) \). Hint: first note that \( e^{i(a+b)} = e^{ia}e^{ib} \). Then apply Euler’s formula to each of the exponentials. Finally, note that the real part of the stuff on the left side of the equals sign must be equal to the real stuff on the right side, and that the imaginary stuff on the left must equal the imaginary stuff on the right. This lets you separate your equation into two equations which will lead to the two equations you are trying to prove.

6. (4 pts) (a) If \( \bar{z} = a + ib \), where \( a \) and \( b \) are real numbers, show that the real part of \( \bar{z} \) is equal to \( (\bar{z} + \bar{z}^*)/2 \). (b) Find a similar equation which will give you the imaginary part of \( \bar{z} \). (c) Use these equations, along with Euler’s formula, to show that \( \cos(\theta) = (e^{i\theta} + e^{-i\theta})/2 \). (d) Find a similar relation for \( \sin \theta \).

7. (6 pts) (a) Write the complex number \( \bar{z} = 5 - 14i \) as a real number times the exponential of an imaginary number. In other words, if I write \( \bar{z} = Ae^{i\phi} \), what are the real numbers \( A \) and \( \phi \)? (b) If \( \bar{z} = 25e^{i(0.35)} \), write \( \bar{z} \) as a real number plus an imaginary number. In other words, if I write \( \bar{z} = a + ib \), what are the real numbers \( a \) and \( b \)?

Extra problems I recommend you work (not to be turned in)

- Show that in the limit as \( \mu_2 \to 0 \) or \( \mu_2 \to \infty \), our equation for the transmitted and reflected amplitudes and powers are consistent with what we deduced earlier for a string with a fixed or a free end.

- Electronic transmission lines (like the cables that you use to connect your computer to the Internet) have finite bandwidth. In other words, they will allow sine waves to propagate through them only if the frequency of the sine wave is below some given frequency. Above this frequency, the sine waves will not propagate well. If a given cable can carry sine waves with frequencies from 0 Hz to 750 MHz, what is the shortest (in time) pulse that we can send down the cable?

- If you are really ambitious, consider what happens when I have a short piece of string of length \( L \) and linear mass density \( \mu_s \) connected to two long ropes with mass density \( \mu_r \). I pull the system to a tension \( T \), and then send a sine wave from one end with a frequency \( \omega \). How much of the power is transmitted through the string to the second rope? This problem involves three regions with boundary conditions at two different locations.
1. (4 pts) (a) Explain in your own words what phase velocity is. (b) Explain in your own words what group velocity is.

2. (6 pts) Imagine that I have a string which I can send waves down. I make various measurements of the speed of sine waves and determine that they travel at a velocity given by $v_{\text{sine}} = (0.637 \text{m} \cdot \text{s}) \omega^2$. (a) Is this velocity the group velocity, the phase velocity, or neither? (b) Find the dispersion relation for the string. If I have a pulse with an average $k$ of 1.42 radians per meter, what is the pulse’s (c) phase velocity and (d) group velocity. (e) At what speed would I expect the center of mass of the pulse to travel down the string?

3. (4 pts) You’ve all heard about ultrasonic imaging, a method used to see inside of a patient. It is frequently used to check on the condition of unborn babies. Take a look at the ultrasound below of my daughter Elena at the age of 5 months. (a) Note that the frequency of sound waves (highlighted in the image with reverse text) is 4.0 MHz, or 4 million cycles per second. Why does it have to be so high? (b) Note that the power is listed at 0 dB and the gain is listed as -20 dB. By what factor is the power reduced by when the gain is -20 dB? (In other words, what do we have to divide the power by in order to reduce the power by 20 dB?)

4. (2 pts) Liquid mercury has a bulk modulus of $2.80 \times 10^{10} \text{N/m}^2$ and a density of $1.36 \times 10^3 \text{kg/m}^3$. What is the speed of sound in mercury?

5. (3 pts) Remember that the bulk modulus tells us how the volume of a piece of some material changes when the pressure surrounding it changes by an amount $\Delta P$. If the initial volume of the object is $V$ and the change in volume is $\Delta V$, the bulk modulus is given by the equation

$$B = -\frac{\Delta P}{\Delta V/V}.$$ 

(a) The speed of sound in water is 1480 m/s. The density of water is 1000 kg/m$^3$. What is the bulk modulus for water? (b) The bulk modulus for a gas depends on how the change in pressure occurs — i.e. whether the gas is compressed adiabatically, isothermally, etc. Imagine that I compress some air in a piston to measure the bulk modulus of air. Which process will result in the largest measured bulk modulus, an adiabatic compression or an isothermal one? (c) Which bulk modulus should I use for sound waves, adiabatic, isothermal, or neither?

6. (6 pts) Imagine a slinky which has a total mass $m$, a cross sectional area $A$, and a spring constant $k$. In order to be able to talk about sound waves on a slinky in terms of the equations and ideas we have developed for
3-dimensional sound waves, we can let \( P = F/A \) and \( \rho = m/AL \). Also, \( V = AL \) and \( \Delta V = A \Delta L \). When we plug this into the equation for the bulk modulus, we get

\[
B = \frac{\Delta F/A}{\Delta L/L} = \frac{L}{A} \frac{\Delta F}{\Delta L}.
\]

For small \( \Delta L \), the last part is just the derivative of \( F \) with \( L \). And we know that since our slinky’s uncomprssed length is tiny, \( F \approx -kL \). (a) Find the bulk modulus for a slinky which is stretched to a length \( L \) in terms of \( m \), \( A \), \( k \), and \( L \). (b) Find the speed of “sound waves” on your slinky in terms of \( m \), \( A \), \( k \), and \( L \). (c) How does the speed of compression waves change with \( L \)? (d) Stretch your slinky until it is 5 feet long (hook it to something or have a friend hold the other end), and then whacking the end to make a compression wave. Measure the time that it takes for the pulse to travel back and forth 10 times and calculate the speed of compression waves on your slinky. (e) Now do the same thing with the slinky stretched to 10 feet. Does the speed of compression waves vary with length as you predicted?

7. (5 pts) (a) How many decibels does the sound level change by if the intensity of the sound wave goes up by a factor of 2, 4, or 8? Notice a pattern? (b) How many decibels does the sound level change by if the intensity of the sound wave goes up by a factor of 10, 100, 1000? Notice a pattern? (c) By what factor does the intensity change if the sound level changes from 131 dB to 147 dB? (d) How many decibels does the sound level change by if the amplitude (i.e. the maximum displacement or the maximum pressure difference) goes up by a factor of 2?

**Extra problems I recommend you work (not to be turned in)**

- How does the speed of longitudinal waves on a slinky compare to the speed of transverse waves that you found in problem 4 of homework #14?

- Consider the following function. Note that this is just the sum of sinusoidal waves with different amplitudes and different frequencies, but which all travel at the same velocity (\( v = 1 \)).

\[
y(x, t) = \sum_{n=1}^{100} e^{-n^2/1000} \cos(2\pi n[x - t])
\]

(a) Estimate the phase velocity of this wave. (b) Estimate the group velocity of this wave. If you have a computer with Maple, MatLab, some programming language you know how to use, or something similar, plot the function from \( x = -0.1 \) to \( x = 0.1 \) at times \( t = 0 \), 0.02, 0.04, 0.06, 0.08, and 0.1.

- Now consider this slightly different function:

\[
y(x, t) = \sum_{n=1}^{10} e^{-n^2/1000} \cos(2\pi n[x - n^{1/4}t]).
\]

This function is also a superposition of sinusoidal waves, but now the velocity of each component depends on its frequency. (c) Estimate the phase velocity of this wave? (d) Estimate the group velocity of this wave. Try plotting this function from \( x = -0.1 \) to \( x = 0.1 \) for \( t = 0 \), 0.01, 0.02, 0.03 and 0.04. Do you see the wave disperse?

If you have access to the physics department computer lab, those computers have Maple and MatLab installed. If you have a computer but don’t have MatLab, you might want to get Octave, a free program which is similar to MatLab. You can find more about it at my web page:

http://www.physics.byu.edu/faculty/durfee/.
1. (3 pts) The array of speakers in the public address system at the BYU football stadium produces waves which, near to the speakers, approximate plane waves. Imagine that the gain is turned up too high, and the PA system begins to whine, emitting sine waves with a frequency $f$ and an intensity of $I$ traveling at the speed of sound $v$. The pressure variations in the wave are given by $\Delta P = \Delta P_{\text{max}} \sin(kx - \omega t)$. What are $\Delta P_{\text{max}}$, $k$, and $\omega$ in terms of $\rho$, $f$, $I$, and $v$?

2. (6 pts) Two metal-heads place their radios, tuned to different stations, next to each other to see which is the loudest. A sound detector is placed 10 meters away from the radios. It reads 95 dB when the first radio is on, and 93 dB when the second one is on. (a) What is the sound intensity, in Watts per square meter, at the detector when only the first radio is turned on? (b) What is the sound intensity at the detector when only the second radio is on? (c) What will the meter read (in dB) when both radios are on? (d) What will it read (in dB) when both radios are on if we move the meter to a distance of 25 meters away from the radios? Assume that the combined size of the two radios is much smaller than 10 meters.

3. (3 pts) A drop of water falls into a perfectly calm pond generating ripples that travel out in circular rings. Prove that as each ring expands, the amplitude of the ring drops off as $1/\sqrt{r}$ where $r$ is the radius of the ring. Assume that the waves propagate non-dispersively (i.e., the radial thickness of a ring doesn’t change as the ring expands).

4. (3 pts) A firework explodes 77.2 meters above you. You record the explosion with a microphone and you find that the average intensity of the sound was 71.3 dB and that the sound lasted for 2.32 ms. How much was the total sound energy released by the explosion (in Joules)? Assume that the sound waves were spherical.

5. (5 pts) Imagine that I am standing 20 meters from an object which is emitting sound. A meter I am holding indicates that the sound level is 75.3 dB. Now I move toward the object until I am just 15 meters away. (a) What will my meter read now (in dB) if the object is a very small speaker which is emitting nearly perfectly spherical waves? (b) What will my meter read now if the object is a huge array of speakers emitting a nearly perfect plane wave?

6. (7 pts) A friend of mine gets into his car and honks the horn. I measure its frequency to be 421 Hz. Then he drives away, turns around, and drives toward me. He honks the horn again, and now I measure a frequency of 428.2 Hz. (a) How fast is my friend going? (b) What frequency will I hear right as he is passing me? (c) What frequency will I hear after he has passed me and is driving away? Assume that the speed of sound is 343 m/s.

7. (3 pts) You are in the passenger seat of a car which is moving at 20.2 m/s on a road which runs right along a train track. You have your window open and your arm is out the window doing the “airplane wing” thing with your hand. A train approaches you from behind travelling at 47.3 m/s blowing its horn. You hear the train’s horn at a frequency of 648 Hz. What frequency will you hear after the train has passed you? Assume that the speed of sound is 343 m/s.

Extra problems I recommend you work (not to be turned in)

- Why is it important that the two radios in problem 2 be tuned to different stations?

- In the first problem you explored how the amplitude of 2-dimensional “spherical waves” waves drop of. Now imagine that you are on a starship and the transporter malfunctions, sending you to a parallel universe. Everything seems strange. You notice that if you double your distance from a tiny object which is emitting sound, the sound intensity goes down by a factor of 16. How many spatial dimensions are there in this universe?

- If a 100 Watt audio amplifier plugged into a speaker could really put out 100 Watts of audio power (they don’t), what would be the intensity (in dB) of sound 1 meter from a small speaker driven by a 100 Watt amplifier running at its maximum amplitude?
- In my lab we have a device called an acousto-optic modulator. If you send light through it in just the right way, you get a beam out which is just like the beam coming in except the phase changes linearly in time. Imagine I send light in who's electric field is described by a sine wave: \( E = E_0 \sin(kx - \omega t) \). After passing through the modulator, the field is shifted in phase: \( E = E_0 \sin(kx - \omega t + \phi) \). The phase \( \phi \) is not a constant, but changes linearly in time as \( \phi = At \). Show that the wave exiting the modulator is equivalent to a sine wave without a \( \phi \) term but which has a higher frequency \( \omega' = \omega + A \).