

- (4 pts) Deriving the time-dilation equation: Imagine that your good friend Albert is flying past you in a spaceship at velocity v (relative to you) while you are floating out in deep space. Albert has a mirror glued to the roof of the bridge of his spaceship, a distance L above him. He pulls out a laser pistol and shoots a burst of light upward at the mirror, it reflects back, and burns a bald spot in his hair. (a) How long does it take the light to travel from the pistol to Albert's hair? We'll call that time Δt_p . (b) In your reference frame, the mirror is moving at a velocity v . If you observe that the light took a time Δt to go up and back to Albert, how far did the mirror move in this time? (c) Your answer to part (b) implies that the light traveled a path which looks like two sides of a triangle. How long is that path (in terms of L , v , and c)? (d) Knowing that the speed of light is constant, find the time it takes to travel this path, Δt , in terms of L , v , and c . (e) How does Δt_p compare to Δt ? In other words, find the time dilation equation!
- (4 pts) Imagine that you lived in a universe where the speed of light was 10 m/s. You hold one end of a meter stick and put your eye right at the 10 cm mark. A friend named Jane holds up the other end and puts her eye right at the 90 cm mark. You and Jane both have wrist watches that are synchronized to each other. A third friend Bill throws a second meter stick like a spear right past the meter stick you are holding. At exactly 10:38 and 1.0200212 seconds you note that the very back of the thrown meter stick passes just in front of your eye. At precisely the same time Jane notes that the very front of the thrown meter stick passes just in front of her eye. How fast was the thrown meter stick moving?
- (4 pts) Bill likes to throw things (living in a universe in which $c = 10$ m/s tends to make people cranky). While you and Jane are still standing at the 10 and 90 cm marks of a meter stick, the two of you synchronize your infinite accuracy atomic wrist watches. Then Bill picks up your battery powered travel alarm clock and throws it past you. Like your wrist watch, the alarm clock is very precise and never loses or gains a second. It is not, however, synchronized to your and Jane's wrist watches. As it passes your eye you note that the time on the clock flying past is 10:40 and 14.8894 seconds. At that moment your wrist watch reads 10:40 and 10.4533 seconds. When the clock passes Jane's eye, her wrist watch reads 10.40 and 10.5922 seconds. What time does the clock flying past Jane read as it passes her eye? Give your answer to 1/100 of a second.
- (4 pts) Now for something a little more realistic. A Λ^0 particle is an unstable particle that can be created in a particle collider. If you make a bunch of Λ^0 particles at rest, they will decay with an average decay time of 0.260 ns. (a) If I make a stream of Λ^0 particles which are traveling at a speed of 0.955 c , what average particle decay time will I measure? (b) If they travel very close to the speed of light we could measure an average decay time of 1 hour. How far from the speed of light are they when this is true (i.e. what is $c - v$ when we measure a decay time of 1 hour)? You will need to use the approximation $(1 + \epsilon)^x \simeq 1 + x\epsilon$ for small ϵ unless you have a calculator with 30 digits of precision.
- (4 pts) Martha travels on a spaceship at a constant velocity from the Earth to Mars. Jim stays at home on Earth. Both of them note the time and location of the two events: Martha passing the Moon and Martha reaching Mars. (a) Which of the two will measure the proper length between the two events? (b) Which of the two will measure the proper time between the two events? Assume that the Moon and Mars don't move appreciably with respect to the Earth during the time of Martha's journey.
- (6 pts) A space deep space probe is launched from the Earth and passes a deep space station on its way into the unknown. It reaches its final speed of 0.811 c relative to the station just before it passes the space station. After passing the station it travels in a straight line at a constant speed. The probe has an on-board atomic clock and is programmed to send a microwave signal back to the station every year (as measured by the clock on board the probe). It sends its first signal right as it passes the station. Right as the probe's clock tells it to send its second signal back to the station it is passing a comet. (a) From the reference frame of someone on the space station, how much time elapses from the time that the probe sends its first pulse to the time that it sends its second pulse (give the time that it *sends* the pulse, in years, not the time that the pulse is received back at the

station). (b) From the reference frame of the probe, how far away is the station when it sends its second pulse? (c) From the reference frame of someone on the station, how far away is the comet? (d) If someone on the space station measures the time between the first pulse arriving at the station and the second pulse arriving at the station, what length of time will they measure (in years)?

7. (4 pts) Ever since the big bang different parts of the universe have been flying away from each other. Astronomers can figure out how fast a star is moving away from us by looking at atomic emission lines and measuring how much the lines have been Doppler shifted from the wavelength of the lines which we measure in experiments on Earth. They often use a parameter known as the “red-shift.” This parameter is usually represented as z and is defined to be the wavelength they measure for the light coming from the star, λ_m , minus the wavelength measured in an experiment in which the atoms emitting the light are at rest, λ_0 , all divided by λ_0 . In other words $z \equiv (\lambda_m - \lambda_0)/\lambda_0$. Imagine that astronomers look at a particular emission line from hydrogen atoms in a particular star and find that the red-shift of the line is 0.7. How fast is that star moving relative to the Earth?

Extra problems I recommend you work (not to be turned in)

- Read the book, “Mr. Tompkins in Wonderland” by George Gamow (also sold under the name “Mr. Tompkins in Paperback”). You will enjoy it, and you will better understand relativity (and quantum mechanics)! The HBLL library has several copies under the following call numbers: **QC 71 .S775 1999, QC 71 .G25 1965, QC 173.5 .G36x, and QC 6 .G23 1940.**
- Why isn't the “twin paradox” a problem? How would you solve the problem correctly?
- Just because I can't travel faster than the speed of light, it doesn't mean that I can't travel any further than about 100 light years from Earth before I die (where 100 years is about the longest someone might live). Why not?