

For the problems in this assignment use a coordinate system with an origin at the center of our film. Assume that the film is in the  $x - y$  plane, and that a vector which is normal to the film will only have a  $z$  component. Also assume that objects are placed on the  $-z$  side of the film and viewed by placing your eye on the  $+z$  side.

1. (4 pts) Assume that I make a hologram with a reference beam which is a plane wave traveling in the  $+z$  direction with a wavenumber  $k$ . At the origin, the electric field from the plane wave is given by

$$E_{ref}(x = y = z = 0) = R_0 \sin(\omega t).$$

This wave can be represented as a complex exponential of the form

$$\tilde{E}_{ref}(x, y, z) = \tilde{R}(x, y, z)e^{i\omega t}.$$

(a) What is  $\tilde{R}(x, y, z)$ ? (b) Evaluate this at  $z = 0$  to find  $\tilde{R}$  in the plane of the film.

2. (5 pts) Assume that the object we want to make a hologram of is a tiny point at a location  $(0, 0, -L)$ . It scatters light which makes an electric field at the origin equal to

$$E_{obj}(x = y = z = 0) = A_0 \sin(\omega t).$$

This wave can be represented with the complex wave

$$\tilde{E}_{obj}(x, y, z) = \tilde{A}(x, y, z)e^{i\omega t}.$$

(a) Remembering that the intensity of light from a point source falls off as  $1/r^2$ , what is  $\tilde{A}(x, y, z)$ ? (b) Evaluate this at  $z = 0$  to find  $\tilde{A}$  in the plane of the film.

3. (3 pts) Now assume that we put a lens in the beam right before it enters the film which focuses the light from the point source to the location  $(0, 0, L)$ . (a) What will  $\tilde{A}(x, y, z)$  for any point between the film and the point where the light comes to a focus? (b) Evaluate this at  $z = 0$  to find  $\tilde{A}$  in the plane of the film and show that it is just equal to the complex conjugate of what we found in the last problem.
4. (3 pts) (a) Show that if we use the reference beam from problem one to make a hologram of the point in problem 2, the second of the three terms in Equation 9.12 in *Physics phor Phynatics* will generate a real image of a point in front of the film. (b) How far from the film will the image form?
5. (5 pts) Now consider a reference beam which is a plane wave traveling at an angle  $\theta$  from the normal of the film. The wave is traveling in the  $y - z$  plane, and it's direction of propagation has a negative  $z$  and a negative  $y$  component. Its electric field at the origin is again equal to

$$E_{ref}(x = y = z = 0) = R_0 \sin(\omega t).$$

Now what is  $\tilde{R}(x, y)$  in the plane of the film?

6. (4 pts) Now reverse the propagation direction for the reference beam in problem 5, such that it has a  $+z$  and a  $+y$  component. Find  $\tilde{R}(x, y)$  for this beam and show that it is the complex conjugate of what we found in problem 5.
7. (6 pts) I make a hologram of a point source at infinity. Because my point source is at infinity, the waves striking the film are actually plane waves. The waves from the object hit the film at an angle  $\theta$  and are identical to the waves you worked with in problem 5 except we will replace  $R_0$  with  $A_0$ . I use the reference beam in problem 1 to make the hologram. Calculate the intensity  $I(x, y)$  that the film is exposed to. Assume that  $I(x, y) = \gamma|\tilde{E}(x, y)|^2$ . Write this as a real function.

### Extra problems I recommend you work (not to be turned in)

- Imagine that you make a hologram of a point at a location  $(0, 0, -L)$  using a reference beam which is not a plane wave, but light coming from a point at location  $(0, 0, -L_r)$ . When you go to look at the hologram you don't set things up exactly the same way, and the reference beam ends up coming from a point at a location  $(0, 0, -(L_r + \delta))$  where  $\delta$  is small. What is the location of the virtual image that you will see?
- If I make a hologram of a point source using a reference beam which is a plane wave but which is not at normal incidence to the film, the real image I get is not exactly a point. Show that this is true.
- Note that when you illuminate the pattern in problem 7 to view the hologram, the three terms in Equation 12 will generate something like the reference beam (i.e. a plane wave traveling in the  $+z$  direction), a real image at  $\infty$  (i.e. a plane wave traveling in the  $+y, +z$  direction), and a virtual image at  $-\infty$  (i.e. a plane wave traveling in the  $-y, -z$  direction). Take the Fourier transform of the pattern in 7 and show that it indeed has only 2 non-zero terms  $k = 0$ , and  $k = k_0$  (the  $k = 0$  term represents a wave which is traveling in the  $z$  direction, and the  $k = k_0$  term represents two waves interfering at an angle to make a standing wave in the  $x - y$  plane). Find  $k_0$ . Compare this pattern to a diffraction grating — what is similar and what is different?
- Skim chapter 10 in *Physics phor Phynatics* and answer the following questions
  1. It is possible to filter the light from a light bulb to achieve the same temporal coherence as a laser. A typical laser used of atomic physics has a linewidth on the order of 10 MHz. In other words, the oscillating electric field in the laser beam is not a pure sine wave, but is an infinite sum of sine waves. If the lowest frequency term in the sum which has a significant amplitude is  $f_{low}$ , then the highest frequency term which has a significant amplitude would be  $f_{high} = f_{low} + 10$  MHz. A light bulb, on the other hand, generates a significant amount of its power at all visible wavelengths. If our light bulb contains significant power from 400 to 650 nm, about what fraction of the light will be need to throw away to get the same temporal coherence as a 10 MHz linewidth laser? Give your answer to 3 digits past the last 9 (for example, 0.999213 or 0.9999222, etc.)
  2. To get the same spatial coherence as a typical laser, we should mask off our light bulb and only use light which comes from a spot which is about one wavelength in diameter. If my light bulb has a filament which is made of a round wire which is 0.3 mm in diameter and 10 mm long, about what fraction of the light must we throw away in order to get the same spatial coherence as a typical 633 nm HeNe laser? Give your answer to 3 digits past the last 9.
  3. I send a laser beam polarized vertically through a half-wave plate with it's fast axis rotated  $15^\circ$  from vertical.
    - (a) At what angle relative to the vertical will the laser beam be polarized after traveling through the wave plate?
    - (b) If I reflect the light coming out of the wave plate right back through it again, at what angle relative to the vertical will the laser beam be polarized after traveling back through the wave plate a second time?Imagine that I use an EOM to modulate the phase of a beam of light such that it's electric field at a given point is equal to

$$E = E_0 \sin((\omega t + a \sin(\omega_{rf} t)),$$

where  $\omega$  is the frequency of the light before it is modulated,  $\omega_{rf}$  is the modulation frequency, and  $a$  is the modulation depth (the maximum phase shift that the light gets at any given time). Show that this can be written as a sum of terms of the form  $A \sin((\omega + m \cdot \omega_{rf})t)$ .