Physics 123 section 2 CID ____________

Homework 35

Score ________ / 30

1. (15 pts) Consider two plane waves, both with a wavelength λ and amplitude A, which are illuminating a wall. Assume that the wall defines the x-y plane (such that every point on the wall has a z coordinate of zero). The two plane waves are traveling in the x-z plane. One of the wave vectors makes an angle with respect to the z axis of $\theta/2$ and the other makes an angle of $-\theta/2$, such that the angle between the two wave vectors is θ, as shown in the figure below

(a) Write down the x, y, and z components of $\vec{k}$ for the two waves in terms of λ and θ.

(b) Equation 8.4 in Physics phor Phynatics shows that we can represent a plane wave with an equation of the form

$$\tilde{y} = \tilde{A}e^{i(\vec{k}\cdot\vec{r}-\omega t)}.$$

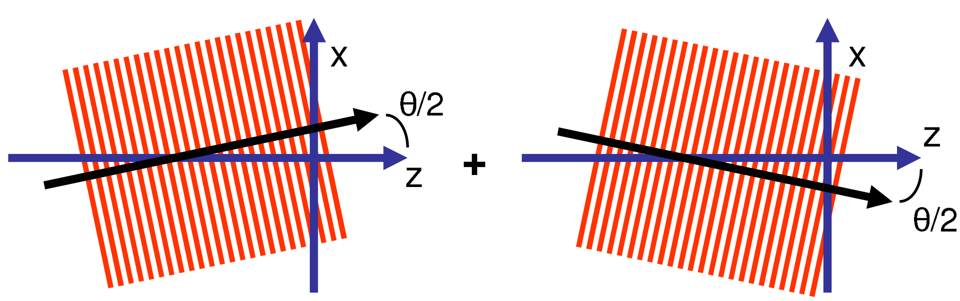
Both of the plane waves have the same amplitude, but I haven't told you their overall phase. So let's write the complex amplitude of the first wave as $\tilde{A}_1 = Ae^{i\phi_1}$ and the complex amplitude of the second wave as $\tilde{A}_2 = Ae^{i\phi_2}$. Now write two equations of the form above to represent the two plane waves in terms of λ, ω, t, θ, A, x, y, z, ϕ_1, and ϕ_2.

(c) When you add the two waves together, since both of the waves have the same amplitude, you should be able to write the total wave as

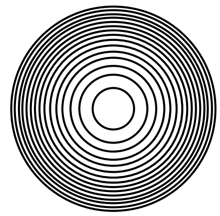
$$\tilde{y}_{total} = Ae^{i(kz\cos(\theta/2)-\omega t)}\left[e^{iG} + e^{iH}\right]$$

where $k = 2\pi/\lambda$ is the wavenumber (the magnitude of the wave vector), and G and H are functions of x, θ, λ, ϕ_1, and ϕ_2. Find these two functions.

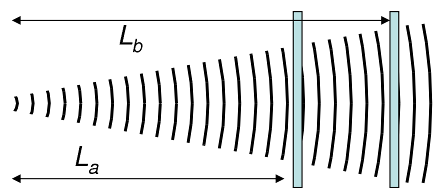
(d) The two interfering plane waves will make an interference pattern of lines on the wall. You get bright fringes when $G - H = 2\pi m$, where m is an integer. Use this fact to find the distance between adjacent bright fringes in terms of θ and λ. (Hint — you can check your answer by taking two limits in θ. In the limit as $\theta = 0$, the two plane waves are parallel, and you shouldn't get fringes. In this limit, the spacing between fringes should go to ∞. In the limit as $\theta = 180°$, we have two counter-propagating waves — a standing wave. In this case we expect fringes to be spaced by $\lambda/2$.)



2. (15 pts) When you worked your walk-in-lab with the Michelson interferometer, you probably noticed that what you saw was somewhat different than what we discussed in class. Instead of a bright blob of light that flashed on and off as the path lengths changed, you saw rings, like the ones pictured below. This is because in class we assumed that the waves going through the interferometer were plane waves. In fact, the light in a laser beam usually expands like a piece of a spherical wave (just like speakers make sound waves which don't travel equally in all directions, but which never-the-less curve and drop off in amplitude like a spherical wave).



In a Michelson interferometer we use mirrors and a beamsplitter to cause one part of the expanding light wave to interfere with another part. The mirrors and beamsplitters fold the path of the light, but if we unfold it in our mind, it would look like the figure below. The light traveling one path travels a longitudinal distance L_a before hitting the screen, and the light traveling the other path travels a distance L_b.



To figure out what the fringe pattern would look like, consider a point on the screen a distance s from the center. The contribution from path a has traveled a distance of $\sqrt{L_a^2+s^2}$ from the point source, and the contribution from path b has traveled a distance of $\sqrt{L_b^2+s^2}$ from the point source to reach this point on the screen. If the difference in those two paths is equal to an integer times λ, we will get constructive interference — a bright fringe. So the equation which determines where the bright maxima occur is

$$\sqrt{L_b^2+s^2}-\sqrt{L_a^2+s^2}=m\lambda.$$

This hand-waving approach will tell us where the bright fringes are, but not HOW bright they are (or how close the dark fringes really get to zero intensity). There are lots of things we've neglected in the above analysis, including the fact that the wave that has traveled the longest distance has also spread out more — so its amplitude won't be the same as the other wave. So now let's do a more complete analysis, and see if we can extract the same equation above from it.

(a) Use equation 8.6 from Physics phor Phynatics to write down a complex representation for a wave at point s on the screen which has traveled a longitudinal distance L_a to reach the screen. Assume that the wave has a complex amplitude of $\tilde{B}$ at a distance r_0 from the source, and write the equation in terms of $\tilde{B}$, r_0, λ, ω, t, s, and L_a.

(b) Now do the same thing for an identical wave, but which has traveled a longitudinal distance L_b.

(c) Note that the amplitudes of the two waves in (a) and (b) are different unless $L_a = L_b$. Draw a phasor diagram showing two complex numbers with different magnitudes and phases added together, and note that no matter what the two phases are, the two waves can never completely cancel each other out.

(d) Note from your figure that you get the biggest sum when the phases of the two phasors you drew in (c) differ by $2\pi m$ where m is an integer. So to find where the bright fringes occur, write down an equation which states that the difference in phase for the wave in (a) and the wave in (b) is equal to $2\pi m$, and show that this is equivalent to the equation I gave you above.

Extra problems I recommend you work (not to be turned in)

- (a) If $L_a = 10$ cm and $L_b = 20$ cm and $\lambda = 650$ nm, what is the radius of the first two bright fringes in problem 2 above? (b) What is m for the first bright fringe? (c) What is the minimum intensity reached in the first dark fringe (it is not zero) relative to the intensity of a beam with amplitude $\tilde{B}$?

- (a) Calculate the form of the interference pattern created when I illuminate a foil with two small holes in it with a laser. (b) Show that the locations of the interference maxima are where we predicted them to be when we did a more simple treatment of a two-slit pattern. (c) Show that the dark fringes don't go all the way to zero.