Physics 123 section 2

Homework 32

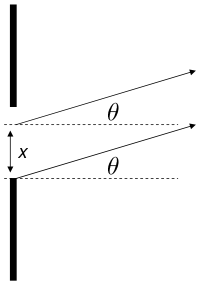
CID ____________

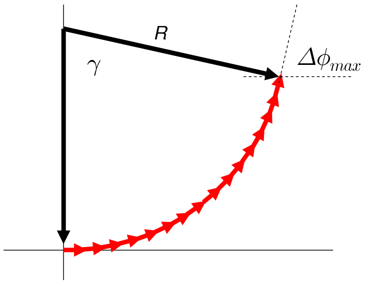
Score ________ / 30

1. (4 pts) You are standing inside your room facing the door, which is open just a crack. You can't see your roommates (because they are on the opposite side of the door). But you can still hear them talking. Not until you fully close the door does the sound of their voice vanish. If light is a wave, and sound is a wave, and neither sound or light waves can penetrate your door, then why can you still hear them talking when you can't see them?

2. (10 pts) Lets derive the intensity pattern for a single slit diffraction. Give all answers in terms of λ (the wavelength of light), a (the width of the slit), θ, and R (the radius of curvature of the circular arc in the figure below).

- **(a)** Two rays of light, one coming from a point at the bottom of the slit and one from a point a distance x from the bottom, travel at an angle θ relative to the normal of the slit, as shown on the left side of the figure below. When the two nearly parallel rays meet at a screen a long distance away, what is the difference in the lengths of the paths that the two rays have traveled? Hint: Draw a line from each of the two points on the slit to a common point on the screen. Then draw a short line between two rays to make an isosceles triangle. In the limit as the screen is very far away, this short line will be perpendicular to the two lines.
- **(b)** What will $\Delta\phi$ (the difference in the phase of the two beams at the screen) be?
- **(c)** In your mind, divide up the slit into tiny pieces of equal length. Each of those pieces will contribute a tiny piece to the total oscillating electric field at the point on the screen. Each of these contributions is represented by a vector in the complex plane. All of these vectors will have the same length, but will have different phase angles. As you add them in the complex plane, you will trace a piece of a circle, as shown on the right side of the figure below. If the phase of the contribution from the bottom of the slit as 0, what is the phase of the contribution from the top of the slit? (It is labeled $\Delta\phi_{max}$ on the figure.)
- **(d)** What is the angle γ in the figure? Hint: In your mind, rotate the dotted lines in the figure by 90°.
- **(e)** The amplitude of the oscillating electric field is equal to the distance from the origin to the end of the last red arrow in the diagram. What is that length? Hint: Draw a line from one end of the red arc to the other. Then draw a line from the center of the circle which cuts that line in half to make two right triangles.
- **(f)** The maximum electric field amplitude in the pattern is where θ is zero. In this case all of the little red vectors would have the same phase, and would add together to form a line. This line would have the same length as the arc length of the red arrows in the figure. Knowing that a circular arc has a length $R\gamma$, what is the ratio of the amplitude of the oscillating field at θ to the amplitude at $\theta = 0$.
- **(g)** Knowing that intensity is proportional to amplitude squared, find the single-slit intensity pattern.



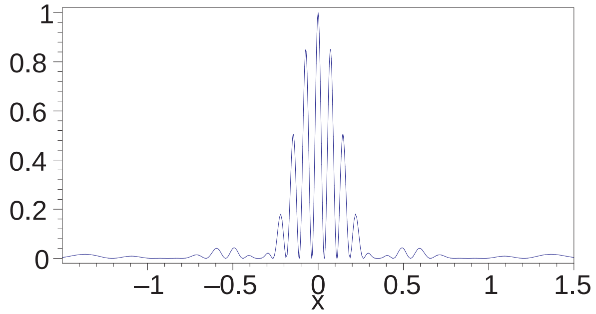


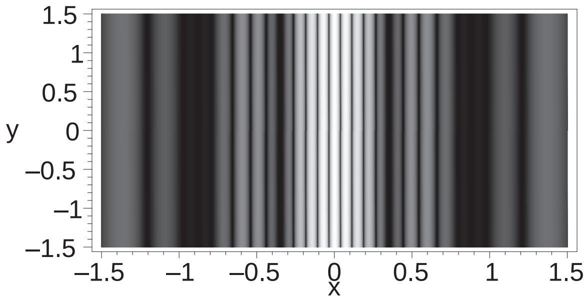
3. (4 pts) What happens to the width of the central bright fringe of a single-slit interference pattern if (a) I make the slit thinner, or (b) I make the wavelength of the light shorter?

4. (6 pts) You are working in a forensics lab, and you have a human hair who's diameter you need to measure. So you shine light from a HeNe laser with a wavelength of 633 nm at the hair and look at the diffraction pattern.

The hair acts like an "inverse" slit, letting all of the light through except for a narrow line of light absorbed by the hair. The light makes a diffraction pattern on a screen which is 1 meter away from the hair. You realize that the resulting light field is just the field made by the laser beam with no hair blocking it minus the field produced by a single slit the same width as the hair: $E = E_{laser\,beam} - E_{single-slit}$. But since the interference pattern is much bigger than the beam of laser light, $E_{laser\,beam}$ is equal to zero and $E = -E_{single-slit}$ over most of the diffraction pattern. Of course your eye can't see the sign of the electric field (you only see the intensity), so you see essentially the same pattern you would see if it were a slit rather than a hair. The width of the central peak on the screen turns out to be 5.73 mm. What is the diameter of the hair?

5. (6 pts) Below is a plot of the calculated intensity in the interference pattern and a calculation of an image of what interference pattern would look like if two slits which are 2 μm wide, whose centers are spaced 9 μm apart, were illuminated by a laser with a wavelength of 660 nm. As you can see, there are 9 thin fringes in the central "clump" of fringes, and 4 in the clump to either side of the central clump. How many fringes would there be in the central clump if the slits were spaced 15 μm apart?





Extra problems I recommend you work (not to be turned in)

- The book gives you an equation to find the angles at which complete destructive interference will occur in a single-slit diffraction pattern. In class we discussed a simple (but complete) explanation as to why this is the correct equation. (a) Can you come up with an equation for the angles at which complete constructive interference will occur? Can you verify it with a simple explanation? (b) If you take the equation from the book for the intensity in a single slit pattern and use that to find where the maximums occur, does it agree with the equation you came up with in (a)? It turns out that it is a little more complicated to find the location of maximums than minimums in a single-slit diffraction pattern. (c) Can you come up with a relation for the position of the intensity maximums using a phasor diagram?

- Derive the intensity formula for a single-slit interference pattern by integration: Divide up the slit into tiny pieces of size dx. Each of these tiny pieces will produce a field $dE = A\sin(\omega t + \phi)dx$ at a point θ on the screen. (The amplitude depends on the amount of light passing through the slit, which depends on the size of dx. Therefore, the amplitude of the sine wave depends on dx.) Integrate this equation from $x = -a/2$ to $x = a/2$ to find the total electric field.

- (a) If the light striking a slit of width a does not strike the slit perpendicularly to the slit, but at some angle ϕ from perpendicular, show that the angles at which dark fringes occur is given by $\sin(\theta - \phi) = m\lambda/a$. (b) Imagine that you shine a laser perpendicular to a slit and look at the diffraction pattern. What will happen to the center part of the diffraction pattern (where θ is small) if you rotate the slit a tiny bit?

- I can take a piece of glass and print a pattern on it. When I shine a laser through the glass I will get a diffraction pattern. For example, if I print black everywhere except for a narrow line, I will get a single-slit diffraction pattern. I can write down a function $T(x)$ which represents the transmission of light as a function of the horizontal position on the glass x. For example, for the single-slit pattern $T(x) = 0$ for $x < -a/2$ and for $x > a/2$, and $T(x) = 1$ for $-a/2 \leq x \leq a/2$, where a is the thickness of the slit. Explain why the diffraction pattern for some arbitrary transmission pattern $T(x)$ can be found from the Fourier transform of $T(x)$.

- Hold a penny a few inches above a table and illuminate it with a light a few feet above it. Notice that the edges of the shadow are fuzzy. Is this primarily due to diffraction or something else? To find out, move the light source further away and see what happens.