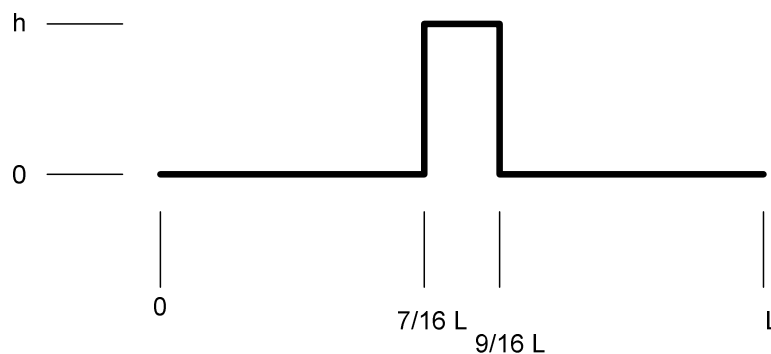


1. (22 pts) A string of length  $L$  is fixed at both ends. At time  $t = 0$  the string is stretched into the shape shown in the figure below. Mathematically, the shape of the string is given by the equation:

$$y(x) = \begin{cases} 0 & \text{if } 0 < x < \frac{7}{16}L \\ h & \text{if } \frac{7}{16}L \leq x \leq \frac{9}{16}L \\ 0 & \text{if } x > \frac{9}{16}L \end{cases} .$$

Using a Fourier transform, we can write  $y(x)$  as a sum of the harmonic modes of the string.

- If we want to perform a Fourier transform to write  $y(x)$  in terms of the harmonic modes of the string, what should we use for  $\lambda_0$  and  $k_0$ ?
- Calculate  $b_0$ .
- Calculate  $a_n$ .
- Calculate  $b_n$ .
- Write  $y(x)$  as a sum of harmonic modes on the string (similar to equation 6.23 but with the parameters you calculated above inserted for  $A_n$ ).



2. (8 pts) One of my colleagues, Justin Peatross, has a laser which produces pulses of light which are only 25 femtoseconds long (one femtosecond =  $10^{-15}$  seconds). What is the minimum bandwidth that the laser gain medium needs to have to produce such pulses? The bandwidth is the maximum frequency of light that it will amplify minus the minimum frequency it will amplify.

**Extra problems I recommend you work (not to be turned in)**

- Find a piano. Push down the sustain pedal and briefly sing a note. What do you hear after you stop singing? The piano is doing a Fourier transform of your voice. Each string only resonates with certain frequencies. Those strings which have a harmonic at the same frequency as one of the frequencies present in the note you sang will absorb sound and begin to oscillate, reproducing your voice.
- Use a computer to calculate what the string in problem 1 will look like at times  $t > 0$  assuming that the speed of waves on the string is 1 m/s and that  $L=1$  m.
- Download the program "Spectrum Lab" from the class web page. Install it and run it. Click on "View/Windows" and then on "Spectrum Lab Components." In the top left-hand corner of the window that opens is a box called "Signal Generator." Make sure that the switch below it points to the right. Now click on "View/Windows" and open the "Test Signal Generator." You can use this new window to make different tones. Turn it on and generate two sines waves (with no AM or FM) at 440 and 441 Hz. Listen to the beats.

- Show that the Fourier transform of a sine wave has just one nonzero coefficient.
- A function that is periodic with a period  $T$  is also periodic with a period  $2T$ . Show that if I use a Fourier transform to write this function as a sum of sines and cosines, I get exactly the same amplitudes with exactly the same sines and cosines whether I use  $T$  or  $2T$  as my fundamental period.