Physics 123 section 2 CID ____________

Homework 20

Score ________ / 30

1. (3 pts) The high E string on my guitar has a linear mass density of 3.93×10^{-4}kg/m (I calculated this from information I found at the string manufacturer's web site) and a length of 25.5 inches (64.77 cm). The frequency of the fundamental mode of this string is 330 Hz (this mode is also known as the "first harmonic"). (a) What is the tension in the string? (b) If I pluck the string I excite a whole bunch of modes. If I then lightly touch the string right in the middle of the string, I will damp out all modes except the ones that have a node at that point. After touching the string in the middle, what is the frequency of the lowest frequency mode which is still ringing? (c) Now I pluck the string and then lightly touch it at a point 2/3 of the way from one end to the other. What is the frequency of the lowest frequency mode which is still ringing?

2. (4 pts) (a) Imagine that you stretch your slinky and excite different transverse harmonic modes. Sketch $y(x)$ for the slinky at a moment in time when it is oscillating in its first transverse harmonic mode (also called the fundamental). (b) Sketch $y(x)$ for the slinky at a moment in time when it is oscillating in its second transverse harmonic mode. (c) How should the frequency of the second harmonic compare to the first harmonic? (d) Stretch your slinky to about 7 feet and measure the frequency of the first and second transverse harmonic modes. Do this by measuring about 10 oscillations and dividing the time for the ten oscillations by 10 to get the period for a single oscillation with 10 times greater accuracy. Then use the period to find the frequency.

3. (6 pts) One evening I took a tall glass from my cupboard and measured the frequency of the fundamental mode of the air in the glass. I did this by whacking the bottom of the glass while my wife held a microphone above the glass. The microphone was connected to my computer, and I measured the frequency using "Spectrum Lab," a free program which you can download from the class web page. The inside of the glass is a cylinder which is 7 cm in diameter and 14.3 cm tall. The lowest frequency I measured when I whacked the glass was 570.8 Hz.

(a) If the glass were very narrow (i.e. if the diameter were much smaller than the height) then the waves would propagate almost as if they were one-dimensional. Otherwise you need to use a three-dimensional wave theory to get precise results. Assuming that the waves in the glass are one dimensional, calculate the velocity of sound using the height of the glass and the frequency of the fundamental mode[1].

(b) I then filled the glass with water and found the frequency of the first harmonic to be 3168 Hz. Use this frequency to calculate the speed of sound in water[2]?

(c) Then I put milk into the glass and measured a frequency of 3100 Hz. What is the speed of sound in milk?

(d) I then shook up the milk and found that the frequency of the first harmonic was cut in half. And then over the course of a few seconds the frequency drifted back up. This is because microscopic air bubbles in the milk decrease the density of the milk (air is less dense than milk) and decrease the bulk modulus (air is more compressible than milk). Which changed by a bigger factor, the density or the bulk modulus[3]?

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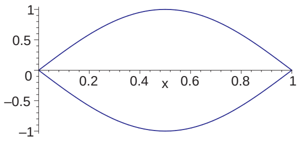
[1] Although this assumption is not really very good in this case, you should still get an answer which is within 5% of the "expected" answer of 343 m/s. This is due to the fact that the wideness of the glass introduces two errors which partially cancel each other. First, the fact that the waves propagate in the glass three-dimensionally means that the wavenumber k is really the sum of three components $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$, resulting in a wavelength for the fundamental mode which is shorter than it would be if the diameter of the glass were smaller. Second, the wide mouth of the glass introduces an "edge effect." Because the mouth is so wide, the oscillating wave pokes out of the cup and the pressure just outside the cup is not fixed at atmospheric pressure. This effectively increases the length of the cup, thereby increasing the wavelength of the fundamental mode.

[2] This answer won't turn out as good. You still have the shorter wavelength due to the three-dimensional nature of the oscillation. But the "edge effect" is gone because there is no water outside of the glass. Because the air above the glass has such a different wave speed than the water in the glass, the oscillation doesn't penetrate out of the glass very far. Still, your answer should be within 25% of the expected value of 1480 m/s.

[3] This is a fun thing that you ought to try. You can do it with water and hear the pitch go down after you shake it. But water releases its air bubbles rather fast. It is easier in milk because the fat in milk increases the viscosity and holds onto the bubbles longer. And it works a lot better if you add ice cream. Make yourself a milk shake in the blender, pour it into a rigid cup (one made of glass works best), and then hit the bottom of the glass. You should hear a very deep, low frequency "thunk."

4. (3 pts) I want to make an organ pipe with a fundamental tone of 440 Hz (middle A). How long should the pipe be, and what is the frequency of the next mode on the pipe above the fundamental if (a) it is open on both ends, (b) it is closed on both ends, or (c) it is open on one end and closed on the other? Assume that v =343 m/s.

5. (5 pts) The minimum and maximum displacement of air molecules along the length of a pipe which is closed at both ends is plotted below. (a) Make a similar plot showing the minimum and maximum *pressure* variation. (b) In this mode at any moment in time all pieces of the air are displaced longitudinally in the same direction. Stretch your slinky to 7 feet, drive this longitudinal mode, and measure its frequency (note, this is different from the previous slinky problem - this time we are driving longitudinal waves). (c) Sketch the minimum and maximum displacement for the second harmonic. (d) If you generate this mode on your slinky, what would you expect the frequency to be? (e) Generate this mode on your slinky (stretched to 7 feet) and measure its frequency.



6. (3 pts) Consider an organ pipe which is closed on the left side and open on the right. Sketch the minimum and maximum of (a) ΔP and (b) s for the two lowest frequency harmonics of the pipe.

7. (3 pts) Imagine a lake on which water waves travel at a velocity v. You and a friend are using magic shoes to hover motionless over the lake. Your friend, some distance away from you, starts dropping pennies into the lake at a frequency f. Each penny makes a ripple which travels toward you, passing you at a frequency f. (a) In terms of v and f, what is the distance L between the ripples? (b) If your friend starts moving toward you with a velocity v_s, how far apart will the ripples be? (c) Now, with your friend still moving across the lake toward you, you start to move toward him at a speed of v_o (relative to the lake). At what frequency will the waves pass you? You've now derived the equation for Doppler shifts.

8. (3 pts) Two trains have identical whistles which sound at 180 Hz. With one train motionless and the other approaching the station, someone waiting at the station hears the whistles beating with a frequency of 2 Hz. (a) What is the velocity of the moving train. (b) What will the beat frequency be when the moving train has passed and is moving away from the station (assuming that it's velocity doesn't change). Assume v =343 m/s.

Extra problems I recommend you work (not to be turned in)

- What would happen if you were to wiggle the slinky side to side horizontally instead of up and down? Will the frequency of the fundamental mode be the same as, higher than, or lower than the fundamental vertical mode? Try it and see! (b) What if you rotate the slinky in circles (the "jump rope" mode)? Will the fundamental "circular" mode be at the same frequency, at a lower frequency, or at a higher frequency? Try it and see!

- Try this if you have a guitar or other stringed instrument: Pluck the highest string and then damp it. Can you still hear the tone faintly? That's because some of the upper strings have harmonics with nearly the same frequency as the fundamental of the string you plucked, allowing them to absorb energy at this frequency and begin to oscillate. Now try this while damping all of the other strings. Now you shouldn't hear the tone after you damp the string. Now damp all of the other strings except one. Pluck the high string and then damp it. See which of the strings can oscillate at the fundamental frequency of the string you plucked.

- Try this if you have access to a piano: Gently push down middle C lightly enough that no sound is made. Keep holding it down. Now strike C an octave above. Let go of the higher note while still holding down middle C. Can you still hear the higher note? That's because the second harmonic of the middle C strings has nearly the same resonance frequency as the fundamental of the C an octave higher. This means that the middle C strings can absorb energy at this frequency and begin to oscillate. Now try holding down middle C and playing different notes to see which ones share harmonics with middle C.

- THE BOOK IS WRONG about vibrating bars! Imagine that you want to make a tuning fork which resonates at 440 Hz. The waves travel down each of the aluminum prongs of the fork at a speed of 5,100 m/s. Each prong is fixed at one end and free at the other, so according to the text, the standing wave ought to be shaped like one quarter of a sine wave. (a) If this were true, how would the prongs need to be to have a fundamental mode at 440

Hz? Does that seem reasonable? (b) The fact is the mode will not be shaped like a sine wave, and the boundary conditions will be different. At the clamped end the end of the bar does not move: $y(0,t) = 0$. The book got that one correct. But the slope at the clamped end is also equal to zero (the bar is stiff, and the clamp rigidly holds the bar such that the angle of the bar at the clamped end is fixed). So they missed one of the boundary conditions at the clamped end. At the other end, the bar is not attached to a massless ring. So the boundary condition there is not that the slope has to equal zero: $dy/dx \neq 0$ at all times at $x = L$. If there is to be no net force on the last infinitesimal piece of the bar at the free end, what should the boundary condition be?