

0. (0 pts) Have a friend hold one end of your slinky or hook it over a door handle. Then start waving your end back and forth. Look at the far end of the slinky, where the wave is reflecting off of the fixed end. Do you see a standing wave?
1. (5 pts) Two senile old men are sitting on a park bench. One of them whistles for his dog (which has been dead for years) at a frequency of 857 Hz. A long way from the bench, the sound waves from his whistling can be described by the equation $\Delta P_1 = \Delta P_{max} \sin(kx - \omega t + \phi_1)$ where $\Delta P_{max} = 0.0142 \text{ Pa}$ and $\phi_1 = 2.41 \text{ rad}$. (a) What is ω ? (b) Assuming that the speed of sound is 343 m/s, what is k ? (c) The second old man joins in, whistling at the same frequency and with the same amplitude, but with a different phase $\phi_2 = -0.45 \text{ rad}$. What is the amplitude of the resultant wave when both are whistling? (d) What is the frequency of the resultant wave?
2. (7 pts) The trig identity that the book gives you in section 18.1 should have made the last problem very simple. But things get uglier if the two waves don't have the same amplitude, or if there are more than two sources. Imagine that a speaker is placed some distance north of you. Two other speakers are then placed at different points along the line between you and the first speaker. All of the speakers are driven with the same sine wave with an angular frequency $\omega = 2764 \text{ rad/s}$. When just the first speaker is on, you measure pressure oscillations at your location described by the equation $\Delta P_1 = A_1 \sin(\omega t + \phi_1)$ where $A_1 = 2.45 \text{ Pa}$ and $\phi_1 = 0.254 \text{ rad}$. When just the second speaker is on, you measure pressure oscillations at your location described by a similar equation, but with $A_2 = 1.89 \text{ Pa}$ and $\phi_2 = -1.12 \text{ rad}$. When just the third speaker is on you get a similar equation with $A_3 = 1.05 \text{ Pa}$ and $\phi_3 = 0.0123 \text{ rad}$. When all three speakers are on, you will get pressure oscillations at your location which are described by the equation $\Delta P_{total} = A_{total} \sin(\omega t + \phi_{total})$.
- (a) Think for a moment about how you would find A_{total} and ϕ_{total} using different trig identities, etc. Then for the answer to this part of the question, write down "I've thought about this, and it looks like it is really hard."
- It turns out that this problem *isn't* too hard if we just use complex exponentials. To do we will write the three waves as complex exponentials: $\Delta \tilde{P}_n = A_n e^{i(\omega t + \phi_n)} = \tilde{A}_n e^{i\omega t}$ where $n = 1, 2, \text{ or } 3$ indicating which speaker, and $\tilde{A}_n = A_n e^{i\phi_n}$. Since $e^{i\theta} = \cos\theta + i\sin\theta$, the imaginary part of each $\Delta \tilde{P}_n$ equation will be exactly equal to the real ΔP_n equation you were given above. So the pressure oscillations we will measure when all of the speakers are on will just be the imaginary part of the sum of these three complex equations: $\Delta P_{total} = \Im[\Delta \tilde{P}_1 + \Delta \tilde{P}_2 + \Delta \tilde{P}_3]$.
- (b) The sum of our three complex waves can be written as $\Delta \tilde{P}_{total} = \Delta \tilde{P}_1 + \Delta \tilde{P}_2 + \Delta \tilde{P}_3 = \tilde{A}_{total} e^{i\omega t}$. Find the real and imaginary parts of \tilde{A}_{total} .
- (c) Use the \tilde{A}_{total} that you just found to find A_{total} and ϕ_{total} .
3. (4 pts) Two speakers are fixed to the side of a building. One is fixed to the wall at the same height as your head. The other is 5 meters directly above the first one. They are both being driven by the same amplifier at a frequency of 220 Hz. Starting a long distance away from the building you begin walking toward the lower speaker. As you approach the speaker you notice that sometimes the tone gets louder and sometimes it gets quieter. Each time the sound intensity reaches a minimum and starts to increase again you write down your distance from the lower speaker. What are all of the distances at which you will hear a minimum? Assume that the speed of sound is 343 m/s. Hint: If you get an equation with a square root in it that seems to keep you from solving it, just take everything which is not under the square root to the other side of the equation and square both sides.
4. (5 pts) Two traveling waves traveling in opposite directions down a string interfere to form a standing wave. The two waves have the following form: $y_1(x, t) = A \sin(kx - \omega t + \phi_1)$ and $y_2(x, t) = A \sin(kx + \omega t + \phi_2)$, where $k = 0.341 \text{ rad m}^{-1}$ and the velocity of the traveling waves is 12.3 m/s. The phases ϕ_1 and ϕ_2 are both positive numbers. (a) If ϕ_1 is fixed but ϕ_2 slowly increases in time, which way will the nodes of the standing wave move, in the $+x$ or $-x$ direction? (b) If ϕ_2 increases by $\pi/2$ radians, how far do the nodes in the standing wave move?

5. (4 pts) Two sine waves traveling down a string interfere to create a standing wave. The displacement of the string is given by $y(x,t) = 3.21\text{cm} \cdot \sin(0.342\text{rad m}^{-1} \cdot x) \cos(32.2\text{rad s}^{-1} \cdot t)$. (a) What is the amplitude of each of the two interfering waves? (b) What is the wavelength of each of the two interfering waves? (c) What is the speed at which the two interfering waves are traveling?
6. (5 pts) I clap my hands and record the sound with a microphone attached to my computer. When I analyze the sound I find that it lasts for 2.97 ms. I note that when I clap my hands next to my guitar one of the strings start to ring. I tune the string around and find that a string will only vibrate audibly if it is tuned such that its resonance frequency is above 325 Hz. According to the uncertainty relation, I should be able to tune my string up to higher frequencies and still have it ring. (a) Given this lowest string frequency which I can excite with my clap and the duration of the pulse, what is the highest frequency at which the uncertainty relation guarantees the string will ring? (b) Is it possible that I would hear the string ring if it was tuned above the frequency you found in (a), and why or why not?

Extra problems I recommend you work (not to be turned in)

- What would happen if the two senile old men in the problem above whistled at different frequencies?
- Tap one fingernail on a hard surface over and over again. Listen to the sound. Now tap two fingers at once over and over again and listen to the sound. If you listen carefully, you will note that the tone changes from strike to strike when you tap two fingernails at once. Why?
- If you shine laser light at just the right frequency onto a cloud of really cold atoms, the atoms will be attracted and trapped in the focus of the beam where the light intensity is highest. If you reflect the laser beam back on itself, it forms a standing wave, and the atoms will collect in the anti-nodes. A clever atomic physicist figured out that if you shift the frequency of the beam before reflecting it back you can make a standing wave that slowly moves, pulling the atoms along with it like an atomic conveyor belt! If I focus a laser beam with a wavelength of 657 nm onto some atoms, and then shift the frequency of the light by 10 MHz and reflect it back to make a moving standing wave, what will the velocity of the conveyed atoms be?