

- (4 pts) (a) Explain in your own words what phase velocity is. (b) Explain in your own words what group velocity is.
- (6 pts) Imagine that I have a string which I can send waves down. I make various measurements of the speed of sine waves and determine that they travel at a velocity given by  $v_{sine} = (0.637\text{m} \cdot \text{s})\omega^2$ . (a) Is this velocity the group velocity, the phase velocity, or neither? (b) Find the dispersion relation for the string. If I have a pulse with an average  $k$  of 1.42 radians per meter, what is the pulse's (c) phase velocity and (d) group velocity. (e) At what speed would I expect the center of mass of the pulse to travel down the string?
- (4 pts) You've all heard about ultrasonic imaging, a method used to see inside of a patient. It is frequently used to check on the condition of unborn babies. Take a look at the ultrasound below of my daughter Elena at the age of -5 months. (a) Note that the frequency of sound waves (highlighted in the image with reverse text) is 4.0 MHz, or 4 million cycles per second. Why does it have to be so high? (b) Note that the power is listed at 0 dB and the gain is listed as -20 dB. By what factor is the power reduced by when the gain is -20 dB? (In other words, what do we have to divide the power by in order to reduce the power by 20 dB?)



- (2 pts) Liquid mercury has a bulk modulus of  $2.80 \times 10^{10} \text{N/m}^2$  and a density of  $1.36 \times 10^4 \text{kg/m}^3$ . What is the speed of sound in mercury?
- (3 pts) Remember that the bulk modulus tells us how the volume of a piece of some material changes when the pressure surrounding it changes by an amount  $\Delta P$ . If the initial volume of the object is  $V$  and the change in volume is  $\Delta V$ , the bulk modulus is given by the equation

$$B = -\frac{\Delta P}{\Delta V/V}.$$

- The speed of sound in water is 1480 m/s. The density of water is  $1000 \text{kg/m}^3$ . What is the bulk modulus for water? (b) The bulk modulus for a gas depends on how the change in pressure occurs — i.e. whether the gas is compressed adiabatically, isothermally, etc. Imagine that I compress some air in a piston to measure the bulk modulus of air. Which process will result in the largest measured bulk modulus, an adiabatic compression or an isothermal one? (c) Which bulk modulus should I use for sound waves, adiabatic, isothermal, or neither?
- (6 pts) Imagine a slinky which has a total mass  $m$ , a cross sectional area  $A$ , and a spring constant  $k$ . In order to be able to talk about sound waves on a slinky in terms of the equations and ideas we have developed for

3-dimensional sound waves, we can let  $P = F/A$  and  $\rho = m/AL$ . Also,  $V = AL$  and  $\Delta V = A\Delta L$ . When we plug this into the equation for the bulk modulus, we get

$$B = -\frac{\Delta F/A}{\Delta L/L} = -\frac{L}{A} \frac{\Delta F}{\Delta L}.$$

For small  $\Delta L$ , the last part is just the derivative of  $F$  with  $L$ . And we know that since our slinky's uncompressed length is tiny,  $F \approx -kL$ . (a) Find the bulk modulus for a slinky which is stretched to a length  $L$  in terms of  $m$ ,  $A$ ,  $k$ , and  $L$ . (b) Find the speed of "sound waves" on your slinky in terms of  $m$ ,  $A$ ,  $k$ , and  $L$ . (c) How does the speed of compression waves change with  $L$ ? (d) Stretch your slinky until it is 5 feet long (hook it to something or have a friend hold the other end), and then whacking the end to make a compression wave. Measure the time that it takes for the pulse to travel back and forth 10 times and calculate the speed of compression waves on your slinky. (e) Now do the same thing with the slinky stretched to 10 feet. Does the speed of compression waves vary with length as you predicted?

7. (5 pts) (a) How many decibels does the sound level change by if the intensity of the sound wave goes up by a factor of 2, 4, or 8? Notice a pattern? (b) How many decibels does the sound level change by if the intensity of the sound wave goes up by a factor of 10, 100, 1000? Notice a pattern? (c) By what factor does the intensity change if the sound level changes from 131 dB to 147 dB? (d) How many decibels does the sound level change by if the amplitude (i.e. the maximum displacement or the maximum pressure difference) goes up by a factor of 2?

**Extra problems I recommend you work (not to be turned in)**

- How does the speed of longitudinal waves on a slinky compare to the speed of transverse waves that you found in problem 4 of homework #14?
- Consider the following function. Note that this is just the sum of sinusoidal waves with different amplitudes and different frequencies, but which all travel at the same velocity ( $v = 1$ ).

$$y(x, t) = \sum_{n=1}^{100} e^{-n^2/1000} \cos(2\pi n[x - t])$$

(a) Estimate the phase velocity of this wave. (b) Estimate the group velocity of this wave. If you have a computer with Maple, MatLab, some programming language you know how to use, or something similar, plot the function from  $x = -0.1$  to  $x = 0.1$  at times  $t = 0, 0.02, 0.04, 0.06, 0.08$ , and  $0.1$ .

- Now consider this slightly different function:

$$y(x, t) = \sum_{n=1}^{10} e^{-n^2/1000} \cos(2\pi n[x - n^{1/4}t]).$$

This function is also a superposition of sinusoidal waves, but now the velocity of each component depends on its frequency. (c) Estimate the phase velocity of this wave? (d) Estimate the group velocity of this wave. Try plotting this function from  $x = -0.1$  to  $x = 0.1$  for  $t = 0, 0.01, 0.02, 0.03$  and  $0.04$ . Do you see the wave disperse?

**If you have access to the physics department computer lab, those computers have Maple and MatLab installed. If you have a computer but don't have MatLab, you might want to get Octave, a free program which is similar to MatLab. You can find more about it at my web page:**

<http://www.physics.byu.edu/faculty/durfee/>.