Physics 123 section 2 CID ____________

Homework 16

Score ________ / 30

1. (5 pts) Imagine that I have a copper wire with a round cross section with diameter $d = 0.411\text{mm}$. I splice the end of that wire to another wire, also with diameter $d = 0.411\text{mm}$, but which is made of iron. I then pull the joined wires until they are under a tension $T = 30$ N. (a) What is the ratio of the wave velocity on the copper wire to the speed that waves travel on the iron wire (i.e., what is v_{copper}/v_{iron})? (b) What is the ratio of the wave numbers for the two wires (k_{copper}/k_{iron}) for a sine wave with an angular frequency ω? (c) If I send a sine wave down the copper wire, what fraction of the power in the incident sine wave is transmitted to the iron wire? Copper has a density of 8920 kg/m^3, and iron has a density of 7860 kg/m^3.

2 (3 pts) If I splice a copper wire with a round cross section with a diameter $d = 0.411\text{mm}$ to an iron wire with a different diameter, what should the diameter of the iron wire be if I don't want waves to reflect at the junction?

3. (5 pts) Imagine two different strings joined together at $x = 0$ with an ideal massless knot. Under the following conditions, state whether a wave traveling from one string to the other will transmit with no reflection, transmit with some reflection, not transmit at all, or whether the condition given is not sufficient to determine which will occur. (a) The two strings have the same linear mass density. (b) The two strings have the same diameter. (c) The two strings are made of the same material. (d) The wave velocity is the same on both strings. (e) For any sine wave of a given frequency, the wavenumber will be the same on both strings.

4. (3 pts) A beam of light traveling through air enters a piece of glass at normal incidence (i.e. it is traveling perpendicular to the face of the glass). Light travels at 2.9979×10^8 m/s in air. It travels at 1.9594×10^8 m/s in the glass. What fraction of the incident light power is reflected off of the surface of the glass?

5. (4 pts) Use Euler's formula to prove that $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$ and that $\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$. Hint: first note that $e^{i(a+b)} = e^{ia}e^{ib}$. Then apply Euler's formula to each of the exponentials. Finally, note that the real part of the stuff on the left side of the equals sign must be equal to the real stuff on the right side, and that the imaginary stuff on the left must equal the imaginary stuff on the right. This lets you separate your equation into two equations which will lead to the two equations you are trying to prove.

6. (4 pts) (a) If $\tilde{z} = a + ib$, where a and b are real numbers, show that the real part of $\tilde{z}$ is equal to $(\tilde{z} + \tilde{z}^*)/2$. (b) Find a similar equation which will give you the imaginary part of $\tilde{z}$. (c) Use these equations, along with Euler's formula, to show that $\cos(\theta) = \left(e^{i\theta} + e^{-i\theta}\right)/2$. (d) Find a similar relation for $\sin\theta$.

7. (6 pts) (a) Write the complex number $\tilde{z} = 5 - 14i$ as a real number times the exponential of an imaginary number. In other words, if I write $\tilde{z}$ as $Ae^{i\phi}$, what are the real numbers A and ϕ? (b) If $\tilde{z} = 25e^{i(0.35)}$, write $\tilde{z}$ as a real number plus an imaginary number. In other words, if I write $\tilde{z}$ as $a + ib$, what are the real numbers a and b?

Extra problems I recommend you work (not to be turned in)

- Show that in the limit as $\mu_2 \to 0$ or $\mu_2 - \to \infty$, our equation for the transmitted and reflected amplitudes and powers are consistent with what we deduced earlier for a string with a fixed or a free end.

- Electronic transmission lines (like the cables that you use to connect your computer to the Internet) have finite bandwidth. In other words, they will allow sine waves to propagate through them only if the frequency of the sine wave is below some given frequency. Above this frequency, the sine waves will not propagate well. If a given cable can carry sine waves with frequencies from 0 Hz to 750 MHz, what is the shortest (in time) pulse that we can send down the cable?

- If you are really ambitious, consider what happens when I have a short piece of string of length L and linear mass density μ_s connected to two long ropes with mass density μ_r. I pull the system to a tension T, and then send a sine wave from one end with a frequency ω. How much of the power is transmitted through the string to the second rope? This problem involves three regions with boundary conditions at two different locations.