Physics 123 section 2 CID ____________

Homework 15

Score ________ / 30

1. (3 pts) Imagine a clothesline stretched across your yard. It has a mass of 0.113 kg and a length of 6 m. When you flick the line, the pule you generate travels down the line at a speed of 19.2 m/s. When the pulse gets to the end, it is completely absorbed without reflection by the flexible pole it is tied to. If you stand near the other end of the line and wiggle it sinusoidally for one minute with an amplitude of 10 cm at a frequency of 3 Hz, how much energy will the flexible pole absorb?

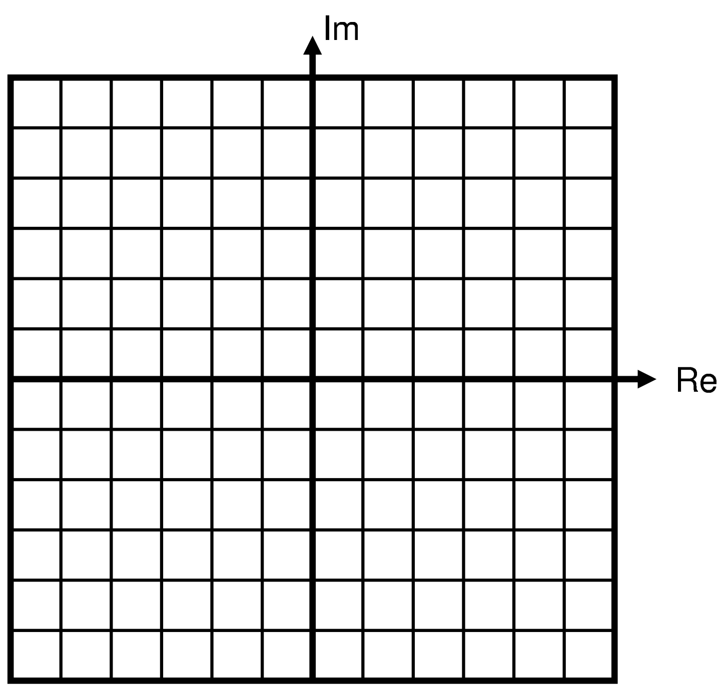
2. (3 pts) The power transmitted by light (and other electro-magnetic waves) has the same dependence on amplitude as the power transmitted along a stretched string. If I measure the light power emitted by a laser and then increase the output of the laser until the power doubles, by what factor has the amplitude of the oscillating electric field in the laser beam increased?

3. (5 pts) (a) Consider the function $y = Ae^{(x-vt)^2/a^2}$ (where A, and a are constants, and v is the speed of waves on the string). Plug this into the linear wave equation and show that it is a solution. (b) Show that $y = A\sin(bxt)$ is not a solution to the wave equation (where A and b are constants). (c) By plugging things into the wave equation, show that if $y_A(x,t)$ and $y_B(x,t)$ are solutions to the wave equation, $y_A + 2.13y_B$ is also a solution.

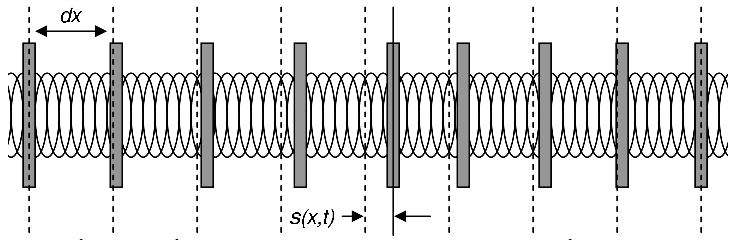
4. (5 pts) Find the bulk modulus for a gas undergoing an (a) adiabatic, and (b) isothermal process in terms of γ and the mean pressure P.

5. (5 pts) (a) Use Euler's formula to fill in the real and imaginary parts of the complex numbers in the table below. (b) Plot each of these points in the complex plane on the graph below.

$\tilde{z}$	$\Re[\tilde{z}]$	$\Im[\tilde{z}]$
$5e^0$		
$5e^{i\pi/8}$		
$5e^{i\pi/4}$		
$5e^{i3\pi/8}$		
$5e^{i\pi/2}$		
$5e^{i3\pi/4}$		
$5e^{i\pi}$		
$5e^{i5\pi/4}$		
$5e^{i3\pi/2}$		



6. (9 pts) Let's find the equation that describes the longitudinal vibrations in a rod of solid material. If the waves passing through the rod have a small amplitude, we can imagine that the rod is made up of tiny slices of mass each connected to the pieces on either side of it by tiny massless springs, as shown in the figure below.



The equilibrium length of each of these springs is dx, and each of the thin masses will have a mass of $m = \rho V = \rho A dx$ where A is the cross sectional area of our rod. We'll label each slice of the rod with the location x of its center when it is in equilibrium (i.e. when no waves are passing through and all of the springs are relaxed - not stretched or compressed).

(a) Find the spring constant of our little springs k in terms of dx and the Young's modulus of the material. Young's modulus E is defined by the following equation:

$$E = \frac{FL}{A\Delta L}$$

where L is the length of our rod, A is the cross section of the rod, and ΔL is the amount the length of a rod will shrink when a force F is applied to each end. (Hint:The number of springs in a rod of length L is L/dx, and each one will compress a tiny amount when the force is applied.)

(b) If I look at the piece labeled x while a wave passes through, the spring to its left will be compressed by an amount $s(x-dx,t) - s(x,t)$, resulting in a force of $F_{left} = k[s(x-dx,t) - s(x,t)]$. What is the force exerted by the spring on the right of the piece labeled x? (Note that if the spring on the right is compressed it will result in a force in the *negative* direction - be sure to get your signs right!)

(c) Remember the definition of a derivative:

$$\left.\frac{\partial f}{\partial x}\right|_{x=y} = \frac{f(y+dx) - f(y)}{dx}$$

where the line and the subscript $x = y$ means that we are evaluating the derivative at y. If we plug in your equation for k from part (a) and apply the definition above, we can write F_{left} and F_{right} in terms of

$$\left.\frac{\partial s}{\partial x}\right|_{x=x} \quad \text{and} \quad \left.\frac{\partial s}{\partial x}\right|_{x=x-dx}.$$

So, now do it! Find F_{left} and F_{right} in terms of these derivatives.

(d) Now add these together to find the total force on the particle, and set it equal to ma. Show that this is just the one-dimensional linear wave equation and determine what v, the speed of longitudinal waves in our rod, is equal to.

Extra problems I recommend you work (not to be turned in)

- Get someone to hold the other end of your slinky or hook it on something. Then whack the slinky to send a pulse down it. Right as that pulse is reflecting off of the far end, whack it again to make a second pulse. Watch as the two pulses collide. Is the approximation that your slinky is a linear medium a good one?

- In Physics Phor Phynatics I mentioned that you can write any complex number as a real number times $e^{i\phi}$: $\tilde{z} = Ae^{i\phi}$. Find A and ϕ for the following complex numbers: $2 + 3i$, $-2 + i$, $(2 + i)/(1 + i)$.

- The solution to the damped harmonic oscillator is of the form $x = Ae^{-t/\tau}\sin(\omega t + \phi)$. Find τ and ω as a function of m, k, and γ.

- Work the same problem, only now use a solution of the form $x = Ae^{-t/\tau}e^{i(\omega t + \phi)}$, realizing that the true solution at any time is just the imaginary part of this.

- The three dimensional wave equation is just

$$\frac{1}{v^2}\frac{\partial^2 \vec{s}}{\partial t^2} = \frac{\partial^2 \vec{s}}{\partial x^2} + \frac{\partial^2 \vec{s}}{\partial y^2} + \frac{\partial^2 \vec{s}}{\partial z^2}.$$

Show that any function $f(\vec{r} - \vec{v}t)$ is a solution to this equation where $\vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$ and $\vec{v}$ is a vector in an arbitrary direction with a magnitude equal to the wave velocity v.