Physics 123 section 2 CID ___________

Homework 12

Score ________ / 30

1. (4 pts) I have two containers of gas. They are both 0.5 m^3 in volume and are both at 300 K. They are both filled with gas at atmospheric pressure. They are connected together through a small tube with a valve. I open the valve and allow the gasses to mix together. After a long time, such that the gasses are thoroughly mixed, how much has the entropy of the system increased from what it was when I opened the valve if (a) both containers are initially filled with nitrogen molecules, and (b) if one is initially filled with nitrogen molecules and the other with helium atoms?

2. (4 pts) Imagine that I toss four coins in the air and count how many come up “heads.” Make a table. Label the left-most column “Macrostate: Number of heads,” and write down the numbers 0 through 4 in a column below this heading. Label the middle column “Possible Microstates,” and in this column list all of the possible microstates which result in the macrostate in the left column (for example, HHTH, TTHH, etc.). Label the right-most column “Total number of microstates,” and in this column write down the number of microstates for each macrostate. See table 22.1 in the text for reference.

3. (4 pts) For this problem, refer to the table you made for the above problem. (a) Using your table, if you tossed four coins and counted how many of them came up “heads,” what is the number you are most likely to get? (b) Use your table to calculate the probability of getting all heads? (c) Which macrostate has the highest entropy? (d) Using the formula $s = k_B \ln g$, where s is the entropy, k_B is Boltzmann’s constant, and g is the number of microstates for a given macrostate, calculate the entropy for each macrostate in your table.

- If you want you can imagine that “heads” represents a particle in a higher energy state than “tails.” At a temperature of absolute zero there is no thermal energy, so all of the coins are “tails” and the entropy is zero. As you add heat, the entropy goes up. In this “toy model” the entropy actually reaches a maximum and then goes to zero again when we have all “heads.” In most real systems, however, the effective “coins” have an infinite number of “sides” — there are an infinite number of energy states that a particle can have. As such, you never get to the point where entropy reaches a maximum and starts to drop.

4. (4 pts) My mother-in-law told me about an interesting “fact” she had heard. Imagine that you had an egg in each hand and you smashed them together, causing one of the eggs to break. She heard that if you held one hand still and smashed the eggs together by moving the other one, the moving egg would always break and the stationary one would not. My wife and I objected. According to relativity, the odds should be equal for either one to break. So we decided to prove her wrong. We got some eggs and smashed them together. The first two times we did it, guess what? The moving egg broke both times! At this point my mother-in-law was certain that we had just proved her right (she has since taken a class in statistics). So we did it four more times, breaking a total of 6 eggs. In the end, three times the moving one broke, and three times the stationary one did, in agreement with Galilean relativity. (a) If I did the experiment twice, what are the odds that the moving egg would break both times? (b) If I did the experiment six times, what are the odds that the moving egg would break all six times? (c) If I wanted to be 99% sure that my mother-in-law was right, how many times would I have to do the experiment and see the moving egg break each time?

5. (6 pts) You are welcome to work this problem in groups — just make sure that you are taking part and that you understand everything that’s going on.

(a) If I toss a coin many times, about what fraction of the time should I get heads?

(b) Toss a coin 100 times. Each time write down a 1 if you get heads and 0 if you get tails. Calculate the fraction of tosses which resulted in heads after 1 toss, 2 tosses, 9 tosses, 16 tosses, 36 tosses, 81 tosses, and 100 tosses. Plot on a graph the fraction of heads results as a function of N, the number of tosses.

(c) Now calculate the difference between the fractions you calculated in part (b) and the expected value from part (a). Plot this on another graph as a function of N. On this same graph plot the functions $f_{max}(N) = 1/\sqrt{N}$ and $f_{min}(N) = -1/\sqrt{N}$. It can be shown (you can derive it, or ask me during office hours if you are interested), that most of the time the absolute difference from the expected value will be less than $1/\sqrt{N}$.

6. (4 pts) Imagine that I have a spring pressure gauge, like the one we discussed in class, with a moving plate area of 1 cm^2. If it is placed in a container filled with air at room temperature and atmospheric pressure, on average about 10^{24} molecules will collide with this surface every second (by the way, you should know how to calculate that from the last chapter). Like any real gauge, our gauge wont respond to pressure changes instantaneously. Using the expected $1/\sqrt{N}$ fluctuation discussed above, calculate by what fraction of the average pressure the readout of our gauge would fluctuate by if the gauge response time is one second, one μs, or one fs? (A fs or femtosecond is 10^{-15} seconds, about the time it takes light to travel 0.3 μm.)

7. (4 pts) Imagine that you are doing exit polls to determine the winner of an election between two candidates. It is a very close election, with both receiving very close to 1/2 of the votes. Imagine that you are very careful to poll a balanced cross-section of voters. (a) If you poll 100 people, how close can the election be (in percentage points) if you want to be reasonably certain that you predict the right winner from your poll? (b) What if you poll 10,000 people?

Extra problems I recommend you work (not to be turned in)

- Calculate how many molecules strike a 1 cm^2 plate per second in a room temp, atmospheric pressure gas.

- How can you use the microscopic picture of entropy to explain why heat flows from high temperature to low temperature?

- Derive the $1/\sqrt{N}$ law for coin tosses (hint — consider the table you made, and ask yourself how many ways are there to get $\frac{1}{2}$ heads if you toss a coin N times how many ways to get $\frac{3}{4}$ heads, etc.)