

1. (4 pts) Let's derive the ideal gas law. To do this, let's first find the force on a wall due to a single particle. Consider a molecule of gas with mass m and velocity at time $t = 0$ of $v = (v_x, v_y, v_z)$ which is bouncing around in a rectangular box. Two opposite walls of the box have an area A spaced by a distance L . These walls have normal vectors which point in the $+x$ and $-x$ directions, respectively. Assume that all collisions are elastic. (a) What is the change in the momentum of the particle $\Delta\vec{p}$ when the particle bounces off the wall on the right (the $+x$ side of the box)? Note that I'm asking for a vector, so be sure to give a direction. (b) How much momentum is imparted to the wall as a result of this collision? (c) After this collision, how long do we have to wait before the particle collides with the right wall again? (d) We know that $F = ma$, but $a = dv/dt$, so

$$F = ma = m \frac{d}{dt}v = \frac{d}{dt}mv = \frac{dp}{dt}.$$

In other words, force is equal to the change in momentum with time. If I average the force on the right wall due to collisions with this molecule over a long time, what will we get?

2. (4 pts) Now consider N identical particles all going in random directions. The total force on the right wall will be the sum of the forces due to each particle:

$$F = \sum_i F_i.$$

(a) Plug your result from the last problem into this equation and show that the total force is equal to

$$F = \frac{m}{L} N \bar{v}_x^2$$

where \bar{v}_x^2 is the average of v_x^2 for all of the particles. (b) Show that, if the particles are going in random directions, \bar{v}_x^2 is just proportional to \bar{v}^2 , the average of the magnitude of the velocity squared for all of the particles. (c) Use this result to write the force on the wall in terms of the average kinetic energy of the particles. In other words, write the total force as something times \bar{E}_k . (d) Now divide by A to turn F into pressure, and you will have something that looks kind of like the ideal gas law. What does T have to be equal to in order for the equation you derived to be equal to the ideal gas law?

3. (3 pts) Helium atoms have a mass of 6.65×10^{-27} kg. If I have a gas of helium atoms at 25°C , what will the (a) average kinetic energy per atom, and (b) the rms speed of the atoms be? (c) Explain why even though the average velocity of the atoms is zero, the rms speed is not.
4. (3 pts) Carbon dioxide molecules are about 11 times more massive than helium atoms. Pretend that I have a box which contains a mixture of helium gas and carbon dioxide gas. (a) If I measure the kinetic energy of a bunch of particles at random, which ones would I expect to have, on average, a greater kinetic energy: the CO_2 molecules or the He atoms? (b) Which type of particles would tend to have higher velocities? (c) If I had an equal number of CO_2 and He particles, which type would contribute most to the pressure of the gas?
5. (4 pts) Consider a gas of nitrogen molecules (N_2) at a temperature low enough that the vibration modes of the molecule are "frozen out." In other words, let's assume that the molecules have 5 degrees of freedom: 3 translational and 2 rotational. (a) What is the molar specific heat at constant volume? (b) If I put these 3 moles of nitrogen into a rigid container, how much will the temperature of the gas change by if I dump 100 J of heat into the gas? (c) How much will the internal energy of the gas change when I do this? (d) How much work will the gas do in the process of being heated?
6. (4 pts) Consider this same 3 moles of nitrogen gas. (a) What is the molar specific heat at constant pressure? (b) If I put the gas instead into a balloon surrounded by air at atmospheric pressure, how much will the temperature change if I dump 100 J of heat into the gas? (c) How much will the internal energy of the gas change when I do this? (d) How much work will the gas do in the process of being heated?

7. (4 pts) (a) Explain in your own words why the molar specific heat at constant pressure should be higher than the molar specific heat at constant volume. (b) Explain why the change in the internal energy of a gas is always equal to $nC_V\Delta T$, even when it undergoes a process in which the volume changes.
8. (4 pts) The isotopically averaged mass of a copper atom is 1.055×10^{-25} kg. (a) If I have a block of copper that weighs 2 kg, without looking up the heat capacity of copper, calculate how much heat will it take to raise the temperature of the block by 10°C . (b) Explain why the heat capacity at constant pressure and constant volume are essentially the same for solids.

Extra problems I recommend you work (not to be turned in)

- How would the molar heat capacity at constant volume of steam compare to the molar heat capacity of ice?
- How would the molar heat capacity at constant volume of nitrogen gas compare to the molar heat capacity of frozen nitrogen?
- Draw a plot of the constant volume and constant pressure molar heat capacity of gaseous nitrogen as a function of temperature.
- What would the molar heat capacity of a diatomic gas be for an adiabatic process? For an isothermal process? Why aren't these quantities discussed in the book?