Physics 123 section 2, Fall 2003 *Practice Exam # 1 With Solutions*

This is the actual exam that I gave last year. Below are the constants, etc., which I gave the students on the exam. Hopefully this gives you an idea of what you need to do to prepare for your exam.

Constants and material properties you may find useful:

Universal Gas Constant	$R = 8.31451 \frac{\mathrm{J}}{\mathrm{K\,mol}}$
Boltzmann's Constant	$k_B = 1.38065 \times 10^{-23} \frac{\mathrm{J}}{\mathrm{K}}$
Avogadro's Number	$N_A = 6.02214 \times 10^{23}$
Stephan-Boltzman Constant for Blackbody Radiation	$\sigma = 5.6696 \times 10^{-8} frac\,\mathrm{W\,m^2\,K^4}$
Acceleration of Gravity Near the Earth	$g = 9.80 \frac{\mathrm{m}}{\mathrm{s^2}}$
Absolute Zero	$T_0 = -273.15\,^\circ\mathrm{C} = 0\,\mathrm{K}$
Density of Water	$\rho_{water} = 1000 \frac{\mathrm{kg}}{\mathrm{m^3}}$
Density of Type 316 Stainless Steel	$\rho_{316} = 0.29 \frac{\mathrm{lb}}{\mathrm{in^3}}$
Density of Aluminum	$\rho_{Al} = 2.70 \times 10^3 \frac{\mathrm{kg}}{\mathrm{m^3}}$
Absolute Zero	$T_0 = -273.15\,^\circ\mathrm{C} = 0\,\mathrm{K}$
Specific Heat of Water	$c_{water} = 4186 \frac{\mathrm{J}}{\mathrm{kg\,^\circ C}}$
Specific Heat of Ice	$c_{ice} = 2090 \frac{\mathrm{J}}{\mathrm{kg} \cdot {}^\circ\mathrm{C}}$
Specific Heat of Aluminum	$c_{Al} = 900 \frac{\mathrm{J}}{\mathrm{kg} \cdot {}^\circ\mathrm{C}}$
Latent Heat of Fusion for Water	$L_{water} = 3.33 \times 10^5 \frac{\mathrm{J}}{\mathrm{kg}}$
Mass of a Helium Atom	$m_{He} = 6.6465 \times 10^{-27}\,\mathrm{kg}$
Radius of a Helium Atom	$r_{He} = 0.032 \times 10^{-9}\,\mathrm{m}$

Equations you may find useful:

Surface Area of a Sphere	$A_{sphere} = 4\pi r^2$
Volume of a Sphere	$V_{sphere} = \frac{4}{3}\pi r^3$
Relating Boltzmann's Constant and the Gas Constant	$R = k_B \cdot N_A$

Useful conversion factors:

$$1\,\mathrm{oz} = 2.835 \times 10^{-2}\,\mathrm{kg}$$
$$1\,\mathrm{lb} = 0.4536\,\mathrm{kg}$$
$$1\,\mathrm{in} = 2.54\,\mathrm{cm}$$
$$1\,\mathrm{m} = 100\,\mathrm{cm}$$
$$1\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{Pa}$$

1. Imagine that we make a submarine in the shape of a cube, 5 meters on a side. Let's make the walls out of type 316 stainless steel, and make them 1 centemeter thick. Just for simplicity, let's assume that the inside of the cube is a perfect vacuum.

(a) If we put our submarine into the water, how deep will the bottom of the submarine sink into the water?

(b) What volume of the vacuum inside the submarine will we have to replace with water to achieve neutral buoyancy? (Neutral buoyancy occurs when the buoyant force exactly cancles gravity.)

(c) If our submarine can only withstand pressures of 10 atmospheres (gauge pressure), how deep can it dive before it is crushed like a tin can?

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$L = 5\,\mathrm{m}, t = 0.01\,\mathrm{m}$
$\rho_{water} = 1000\,\mathrm{kg/\,m^3}, \rho_{steel} = 0.29\,\mathrm{lb/\,in^3} = 8027\,\mathrm{kg/\,m^3}$

$V_{sub} = L^3$
$V_{steel} = 6 \cdot L^2 \cdot t = 1.5\,\mathrm{m^3}$
... or, depending on how you interpreted the problem,
$V_{steel} = L^3 - (L - 2t)^3 = 1.494\,\mathrm{m^3}$
$m_{sub} = V_{steel} \cdot \rho_{steel} = 12041\,\mathrm{kg}$ (or $11992\,\mathrm{kg}$, depending on the volume you used)

(a) We simply balance gravity with the buoyant force

$$m_{sub}g = V_{displaced} \cdot \rho_{water}g = L^2 y \rho_{water} g$$

$$y = \frac{m_{sub}}{L^2 \rho_water} = \frac{12041\,\mathrm{kg}}{(5\,\mathrm{m})^2 1000\,\mathrm{kg/\,m^3}}$$

$\boxed{y = 0.482\,\mathrm{m}}$ or $\boxed{y = 0.480\,\mathrm{m}}$

(b) Once again, we balance gravity with the buoyant force, but now we completely submerge the submarine (such that the volume of displaced water is just the volume of the sub) and solve for the volume of water that we have to add to our sub.

$$(m_{sub} + m_{water})g = V_{displaced} \cdot \rho_{water}g = V_{sub}\rho_{water}g$$

$$m_{water} = V_{sub}\rho_{water} - m_{sub}$$

$$V_{water}\rho_{water} = V_{sub}\rho_{water} - m_{sub}$$

$$V_{water} = L^3 - m_{sub}/\rho water = (5\,\mathrm{m})^3 - 12041\,\mathrm{kg}/(1000\,\mathrm{kg/\,m^3})$$

$\boxed{V_{water} = 113\,\mathrm{m^3}}$ (This works out to the same answer to three sig. figs. regardless of which mass you plug in).

(c) Ten atmospheres "gauge pressure" means eleven atmospheres absolute pressure. So we can cancel out the atmospheric pressure above the water.

$$P_{max} = 11\,\mathrm{atm}$$

$$P_{max} = P_0 + \rho_{water}gh$$

$$h = \frac{P_{max} - P_0}{\rho_{water}g} = \frac{11\,\mathrm{atm} - 1\,\mathrm{atm}}{\rho_{water}g} = \frac{10 \cdot 1.013 \times 10^5\,\mathrm{Pa}}{1000\,\mathrm{kg/\,m^3} \cdot 9.8\,\mathrm{m/\,s^2}}$$

$\boxed{h = 103\,\mathrm{m}}$

2. Water is flowing down a hill in a ditch with a rectangular cross section. Assume that the water is flowing like a non-viscous ideal fluid without turbulance, and that the ditch is open to the air. If the hill has a height y, and the depth and velocity of the flowing water in the ditch at the top of the hill is given by h_{top} and v_{top}, what is the depth and flow velocity at the botton of the hill? Give answers in terms of the given quantities and physical constants.

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We use two equations. First, Bernoulli's equation. The pressure at both the top and the bottom of the hill is atmospheric pressure.

$P_0 + \frac{1}{2}\rho v_t^2 + \rho g y = P_0 + \frac{1}{2}v_b^2 + \rho g(0)$

The P_0s cancel, and then we can cancel the ρs and simplify the equation to

$v_t^2 + 2gy = v_b^2$

Then we write down the equation of continuity and solve for v_b. Note that the width of the ditch, w, is the same at the top and the bottom.

$v_t A_t = v_b A_b \Rightarrow v_t w h_t = v_b w h_b$

$v_b = \frac{h_t}{h_b} v_t$

Now we plug this back into Bernoulli's equation and solve for h_b.

$v_t^2 + 2gy = \left(\frac{h_t}{h_b}\right)^2 v_t^2 \Rightarrow \left(\frac{h_t}{h_b}\right)^2 = 1 + \frac{2gy}{v_t^2}$

$$\boxed{h_b = \frac{h_t}{\sqrt{1 + \frac{2gy}{v_t^2}}}}$$

And from this we get...

$$\boxed{v_b = v_t\sqrt{1 + \frac{2gy}{v_t^2}}}$$

3. A sphere of aluminum, 1 cm in diameter, is painted with a thin coating of ultra-black paint. It is initially at a temperature of 23 °C.

(a) Assuming that deep space is devoid of radiation (It's not, but its blackbody temperature is really cold), if I place the sphere in deep space, how long will it take for the sphere's temperature to drop by 200 °C?

(b) Now instead of black paint, I polish the sphere so that it reflects 90 percent of the light that strikes it. How long does it take for it's temperature to drop by 200 °C now?

(c) The truth is, deep space is not devoid of radiation, but is bathed in radiation with a finite blackbody temperature (2.73 Kelvin to be precise). What is the net rate of heat flow (i.e. the power emitted minus the power absorbed) for our black sphere when it is at a temperature of 4 K, taking into account the finite temperature of deep space?

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(a) Using Stephan's radiation law we can find the power being emitted when the sphere is at a certain temperature. Unfortunately, as the sphere cools down its temperature drops, changing the radiated power. So, it looks like we need to integrate, adding together little bits of energy emitted in little slices of time...

$P = -\frac{dQ}{dt} = \sigma A e T^4$

Note the minus sign — in our formalism Q is the heat flowing *into* the system, but the radiation law tells us the power flowing *out* of the system. Our equation has three variables: T, Q, and t. We need to get rid of one of them so that we can integrate. We do this by relating dQ, a bit of heat leaving the system, to dT, the small change in temperature when the heat leaves:

$Q = C\Delta T \Rightarrow dQ = C dT$

Now I just plug this in and set up the integral.

$-\frac{C dT}{dt} = \sigma A e T^4 \Rightarrow \frac{C}{\sigma A e}\frac{dT}{T^4} = dt$

Now I integrate time from $t = 0$ to $t = t_f$ and temperature from $T = T_i$ to $T = T_f$ to find t_f.

$\int_0^{t_f} dt = \int_{T_i}^{T_f} \frac{C}{\sigma A e} T^{-4} dT \Rightarrow t_f = -\frac{C}{3\sigma A e}\left(T_f^{-3} - T_i^{-3}\right)$

We know that $T_i = 23\,°\mathrm{C} = 296.15\,\mathrm{K}$, so $T_f = T_i - 200\,\mathrm{K} = 96.15\,\mathrm{K}$. Since the sphere has a coating of ultra-black paint, the absorptivity is equal to 1. Since the emissivity is the same as the absorptivity, $e = 1$. Now we just need to know the surface area A and the heat capacity C.

$A = 4\pi r^2$

$r = d/2 = 1\,\mathrm{cm}/2 = 0.5\,\mathrm{cm} = 0.005\,\mathrm{m}$

$A = 4\pi(0.005\,\mathrm{m})^2 = 3.1416^{-4}\,\mathrm{m}^2$

$C = mc_{Al} = \rho_{Al} V c_{Al} = \rho_{Al}\frac{4}{3}\pi r^3 c_{Al} = 2700\,\mathrm{kg/\,m^3} \cdot \frac{4}{3}\pi(0.005\,\mathrm{m})^3 \cdot 900\frac{\mathrm{J}}{\mathrm{kg}\,°\mathrm{C}} = 1.2723\frac{J}{°\mathrm{C}}$

So, plugging this in...

$t_f = -\frac{1.2723\,\mathrm{J/\,°C}}{35.6696\times10^{-8}\,\mathrm{W/\,m^2\,K^4}\cdot 3.1416\times10^{-4}\,\mathrm{m^2}\cdot 1}\left((96.15)^{-3} - (296.15)^{-3}\right) \Rightarrow \boxed{t_f = 2.59\times10^4\,\mathrm{s}}$

Problem 3 continued...

(b) I just have to replace e in my equation above with the new e for this problem. Once again, the emissivity is equal to the absorptivity. The absorptivity is just one minus the reflectivity, so $e = 1 - 0.9 = 0.1$. So instead of dividing by one as I did in part (a), I will divide by 0.1. So the answer to part (b) is just 10 times the answer to part (a)!

$$\boxed{t_f = 2.59 \times 10^5\,\mathrm{s}}$$

(c) The rate at which the sphere emits blackbody radiation is simply given by Stephan's radiation law.

$$P_{out} = \sigma A e T^4$$

When the system is in thermal equilibrium, the rate at which the sphere absorbs energy from the blackbody background of deep space is equal to the rate at which the sphere emits energy back into it. But the rate that it absorbs power doesn't depend on the temperature of the sphere, just the temperature of space. So if the sphere is not in thermal equilibrium with the background temperature of space, we still know that it absorbs as much as if it were, which is equal to the rate at which it would be emitting if it were at the same temperature as deep space.

$$P_{in} = \sigma A e T_{deepspace}^4$$

The net rate of power flow out of the sphere is then...

$$P_{net} = P_{out} - P_{in} = \sigma A e \left(T^4 - T_{deepspace}^4\right) = (5.6696 \times 10^{-8}\,\mathrm{W/\,m^2\,K}) \cdot (3.1416 \times 10^{-4}\,\mathrm{m^2}) \cdot 1 \cdot \left((4\,\mathrm{K})^4 - (2.73\,\mathrm{K})^4\right)$$

$$\boxed{P_{net} = 3.57 \times 10^{-9}\,\mathrm{W}}$$

4. A wading pool contains a 200 kg sheet of ice floating on top of $0.250\,\mathrm{m}^3$ of water. The system is initially in equilibrium.

(a) What is the temperature of the system?

(b) if I add $0.200\,\mathrm{m}^3$ of water at $25\,°\mathrm{C}$, what will be the final temperature of the system and how much ice will be left?

Assume that no heat is exchanged with the environment.
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(a) If the ice and the water are in equilibrium, the temperature must be equal to $0\,°\mathrm{C} = 273.15\,\mathrm{K}$.

(b) The way that we do this depends on whether all of the ice melts or not. First we assume that it does or that it doesn't, and we proceed from there. If we assume that it does all melt but the final temperature of the water is below zero, or if we assume that only some of it melts but find that the quantity of ice that melts is greater than the amount we started with, then we know our assumption was wrong and we go back and do it the other way.

Lets first assume that all the ice melts. In real life, the actual path that heat flows can be complicated. But it doesn't really matter how it gets from one place to the other as we approach equilibrium, just so that energy is conserved along the way. Lets work the problem in two steps. First let's let the warm water interact with the ice and pass heat into it until it's all melted. We'll find what the temperature of the warm water is after this step, then we'll treat the water which once was ice and the cold water as one entity, and let it come into equilibrium with the warmer water.

First to melt the ice. To conserve energy, we simply state that the heat flowing out of the hot water is equal to the heat absorbed by the melting ice. We'll use the variables T_H to represent the hot water's initial temperature, and T_a to represent the temperature of the hot water after all the ice is melted.

$C_{hotwater} \cdot (T_H - T_a) = m_{ice} L_{ice} \Rightarrow T_H - T_a = \frac{m_{ice} L_{ice}}{C_{hotwater}} \Rightarrow T_a = T_H - \frac{m_{ice} L_{ice}}{C_{hotwater}}$

The heat capacity of the hot water is just $C_{hotwater} = m_{hotwater} c_{water} = V_{hotwater} \rho_{water} c_{water} = 0.2\,\mathrm{m}^3 \cdot 1000\,\mathrm{kg/\,m}^3 \cdot 4186\,\mathrm{J/\,kg\,°C} = 837,200\,\mathrm{J/\,°C}$, so...

$$T_a = 25\,°\mathrm{C} - \frac{200\,\mathrm{kg} \cdot 3.33 \times 10^5\,\mathrm{J/\,kg}}{837{,}200\,\mathrm{J/\,°C}} = -54.55\,°\mathrm{C}$$

This is less than the freezing point of water, so this can't be right. It must be that not all of the ice is melted. So lets make that assumption, and find out how much ice is melted as the hot water cools just to the freezing point of water. If this works, we know that the final temperature of the water is $0\,°\mathrm{C}$.

$C_{hotwater}(T_H - 0\,°\mathrm{C}) = m_{melt} L_{water} \Rightarrow m_{melt} = \frac{C_{hotwater}(T_H - 0\,°\mathrm{C})}{L_{water}} = \frac{837{,}200\,\mathrm{J/\,°C} \cdot 25\,°\mathrm{C}}{3.33 \times 10^5\,\mathrm{J/\,kg}} = 62.853\,\mathrm{kg}$

$m_{remaining} = 200\,\mathrm{kg} - 62.853\,\mathrm{kg}$

$$\boxed{m_{remaining} = 137\,\mathrm{kg},\ T_{final} = 0\,°\mathrm{C}}$$

5. A regulation #7 basketball has a diameter of about 24 cm, a mass (including the air inside) of about 0.6 kg, and is filled with approximately 1.5 atm of gas. Assume that the gas is an ideal gas with $\gamma = 1.4$, and that the ball is initially at a temperature of 23 °C. The ball is now dropped from a height of 2.5 m, and is compressed adiabatically when it strikes the floor. Assume that the gas (and not the wall of the ball) absorbs all of the ball's kinetic energy.

(a) What is the change in temperature (in °C) after the compression?

(b) What is the change in entropy?

(c) What is the ratio V_f/V_i where V_i is the volume before the adiabatic compression, and V_f is the volume after the compression.
....

(a) Since the compression is adiabatic, there is no heat flow. So all of the work done on the ball goes directly into internal energy. The work done is just enough to stop the ball, which is just equal to its kinetic energy when it hits the floor, which is equal to its potential energy before it is dropped... $W = mgh$. The change in temperature of an ideal gas is related to it's change in internal energy by $\Delta E_{internal} = nc_V \Delta T$. So...

$mgh = nc_V \Delta T \Rightarrow \Delta T = \frac{mgh}{nc_V}$

We can get the constant volume molar heat capacity from γ.

$\gamma = \frac{c_P}{c_V} = \frac{c_V + R}{c_V} \Rightarrow c_V \gamma = c_V + R \Rightarrow c_V(\gamma - 1) = R \Rightarrow c_V = R/(\gamma - 1) = R/0.4$

We find n using the ideal gas law (for the conditions before the ball is dropped), and then plug it into the equation above...

$PV = nRT \Rightarrow n = \frac{PV}{RT}$

$\Delta T = mgh \frac{RT}{PV} \frac{0.4}{R} = \frac{0.4mghT}{PV}$

The temperature is 23 °C = 296.15 K, and the pressure is $1.5 \cdot 1.013 \times 10^5$ Pa $= 1.5195 \times 10^5$ Pa. The volume of the ball is just $(4/3)\pi r^3$ (where $r = 0.24\,\text{m}/2 = 0.12\,\text{m}$), so $V = 7.2382 \times 10^{-3}\,\text{m}^3$. Plugging this in...

$\frac{0.4 \cdot 0.6\,\text{kg} \cdot 9.8\,\text{m/s}^2 \cdot 2.5\,\text{m} \cdot 296.15\,\text{K}}{1.5195 \times 10^5\,\text{Pa} \cdot 7.2382 \times 10^{-3}\,\text{m}^3}$

$\boxed{\Delta T = 1.58\,°\text{C}}$

(b) This is an adiabatic process, so $\boxed{\Delta s = 0}$.

(c) We use the fact that $TV^{\gamma-1}$ is constant to get...

$T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow \frac{V_f}{V_i} = \left(\frac{T_i}{T_f}\right)^{1/(\gamma-1)} = \left(\frac{296.15}{296.15+1.58328}\right)^{1/0.4}$

$$\boxed{\frac{V_f}{V_i} = 0.987 = \frac{1}{1 + 0.0134}}$$

6. Two moles of a monatomic gas of helium is contained in a cylinder with volume of 0.3 cubic meters at a temperature of 25 °C.

(a) What is the total heat capacity of the gas for constant volume processes?

(b) What is the *average* velocity of the atoms?

(c) What is the rate at which any given atom collides with other atoms?

Now I compress the gas at constant temperature to half its original volume.

(d) What is the average velocity and the collision rate of the atoms after the compression?

(e) How much did the internal energy and entropy change during the compression?

(f) How much work did I do on the gas during the compression?

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(a) $c_V = 3/2R,\ C = nc_V = 2 \cdot 3/2R$

$\boxed{C = 3R = 24.9\,\mathrm{J/\,kg}}$

(b) $\bar{v} = 1.60\sqrt{k_B T/m} = 1.60\sqrt{1.38065 \times 10^{-23}\,\mathrm{J/\,K}/cdot 298.15\,\mathrm{K}/6.6465 \times 10^{-27}\,\mathrm{kg}}$

$\boxed{\bar{v} = 1260\,\mathrm{m/\,s}}$

(c) We can pull this right from the book:

$f = \sqrt{2}\pi d^2 \bar{v} n_V$

or we can solve for the mean free path and use $\bar{v}$ to get the mean free time between collisions and invert it. If I use the equation above, I use $d = 2r$ and get the radius of a helium atom from the table, and I use Avogadro's number to find n_V

$n_V = N/V = nN_A/V = 2 \cdot 6.02214^{23}/0.3\,\mathrm{m}^3 = 4.01476 \times 10^{24}\,\mathrm{m}^{-3}$

$f = \sqrt{2}\pi(0.064 \times 10^{-9}\,\mathrm{m})^2 \cdot 1259\,\mathrm{m/\,s} \cdot 4.01475 \times 10^{24}\,\mathrm{m}^{-3}$

$\boxed{f = 92.0\,\mathrm{MHz}}$

(d) The velocity only depends on the temperature, so it doesn't change. All of the parameters upon which the collision rate depends didn't change except the volume. Cutting the volume in half means that the particles are twice as close together and the collision rate will go up by a factor of 2...

$\boxed{\hat{v} = 1260\,\mathrm{m/\,s},\ f = 184\,\mathrm{MHz}}$

(e) The internal energy only depends on the temperature, so $\boxed{\Delta E_{internal} = 0}$. To find the entropy change, we just use the formula for entropy change in reversible processes:

$\Delta s = nc_V \ln \frac{T_1}{T_2} + nR \ln \frac{V_f}{V_i} = 2c_V \ln(1) + 2R\ln(1/2) = 0 + 2R\ln(1/2)$

$\boxed{\Delta s = 2R\ln(1/2) = -2R\ln 2 = -11.5}$

(e) The work done comes right from another formula from the book (or if you didn't write it on your sheet, you would have to integrate PdV by using the ideal gas law to write P as a function of V and some constants). Since I asked for the work done by me, not the work done by the gas, I have to insert a minus sign.
$W = -nRT \ln \frac{V_f}{V_i} = -2R(298.15\,\mathrm{K})\ln(1/2) \Rightarrow \boxed{W = 3440\,\mathrm{J}}$