\[ [110], \text{longitudinal} \quad \lambda \frac{\partial u}{\partial t} - \mu \nabla (\nabla \cdot u) \]

\[ U = U_0 e^{\frac{t}{\text{time}} \text{min}} \]

\[ V = V_0 e^{\frac{t}{\text{second}}} \]

\[ \rho \cdot \omega^2 u = \frac{1}{2} k^2 \left( C_{11} - C_{44} \right) u + \frac{1}{2} k^2 \left( C_{12} + C_{44} \right) \nabla \cdot u \]

\[ \rho \cdot \omega^2 V = \frac{1}{2} k^2 \left( C_{11} - C_{44} \right) V + \frac{1}{2} k^2 \left( C_{12} + C_{44} \right) \nabla \cdot V \]

Cauchy's:

\[
\begin{pmatrix}
\rho \omega^2 & 0 \\
0 & \rho \omega^2
\end{pmatrix}
\begin{pmatrix}
U \\
V
\end{pmatrix} = \frac{1}{2} k^2 \begin{pmatrix}
C_{11} & C_{12} \\
C_{12} & C_{44}
\end{pmatrix}
\begin{pmatrix}
U \\
V
\end{pmatrix}
\]

\[
\begin{pmatrix}
A - \rho \omega^2 & B \\
B & A - \rho \omega^2
\end{pmatrix}
\begin{pmatrix}
U \\
V
\end{pmatrix} = 0
\]

Only solution if matrix is invertible:

\[ \det \left( A - \rho \omega^2 \right) > 0 \]

\[ \left( A - \rho \omega^2 \right) \cdot B = 0 \]

\[ A - \rho \omega^2 = \pm B \]

\[ \frac{1}{2} k^2 \left( C_{11} \pm C_{44} \right) \rho \omega^2 = \frac{1}{2} \left( C_{12} + C_{44} \right) k^2 \]

\[ 2 \rho \omega^2 = k^2 \left( C_{11} \pm C_{44} \right) \Rightarrow \text{long} \]

\[ \frac{\rho \omega^2}{k^2} = \frac{3}{2} \rho \left( C_{11} \pm C_{44} + 2C_{44} \right) \rightarrow \text{transverse} \]

Why both long and transverse?

"Effective elastic constants"
Shortcut way (recommended for this problem)

Enforce $u = v$ from the start

\subsection{[110] Longitudinal}

Can be done with single eqn:

$$
\rho (-i \omega)^2 u = C_{11} \left( \frac{-k^2}{2} \right) u + C_{44} \left( \frac{-k^2}{2} + i \omega \right) u + (C_{12} + i \omega) \left( \frac{k^2}{2} \alpha + i \beta \right)
$$

\subsection{[110] Transverse}

Final term is $\frac{k^2}{2} u$

\subsection{Oscillatory [110] direction}

$$
\frac{\omega^2}{k^2} = \frac{1}{\rho} \left( \frac{C_{11} + C_{44}}{2} - \frac{C_{12}}{2} - \frac{C_{44}}{2} \right)
$$

$$
\sqrt{\frac{\omega^2}{k^2}} = \sqrt{\frac{1}{2\rho} (C_{11} - C_{12})}
$$
\[ n = n_0 e^{-\frac{x^2}{\alpha^2}} \]

\[ u, v = 0 \]

\[ -\beta \cos \omega t \]

\[ \frac{\omega}{k} = \sqrt{\frac{C_{44}}{\rho}} \]

III. For HW

Long: Coeff. = \( \frac{1}{3} (C_{11} + 2C_{12} + 4C_{44}) \)

Trans 1: Coeff. = \( \frac{1}{3} (C_{11} - C_{12} + 2C_{44}) \)

Trans 2: same as Trans 1.

Fig 20 pg 83
Dimension reduction

\[ \sqrt{\frac{v^2}{k}} \rightarrow \text{only true for long } \lambda \] (small k)

"acoustic waves"

\[ k \approx [100] \]

\[ k \approx [111] \]

Down \[ w \mid \text{ch } 3 \]

"Elastic Energy Density" \[ \rho \] 77-78

and "Bulk Modulus & Compressibility" pg 80
Chapter 4: Phonons

So far we've focused on very long wavelength oscillations (small k).

There was a straight line, and slope is speed of elastic waves (Problem 3.6)

Figure 11

Why is there a knee?

Now we'll answer some of these questions.

Basic answer: Leaving the continuum limit!

Consider $\lambda = 10a \Rightarrow k = \frac{3\pi}{5a}$

But what if $\lambda = a$? $k = 2\pi/a$

is there a wave?

$\lambda = a/2$ $k = 4\pi/a$

That's why there's a knee! For $\lambda < a$ these do not make sense.

Fig 11.93 solid curve gives no more info than dashed curve only $k \propto \beta^2$ is needed.