Lecture 9: Mon, 28 Jan 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. Brewster’s angle occurs when $\theta_i$ and $\theta_t$
   a. Are equal
   b. Sum to $90^\circ$
   c. Sum to $45^\circ$
   d. Sum to $60^\circ$

Q2. During total internal reflection, the electric field in the air just outside the glass is called the
   a. reduction wave
   b. florescent wave
   c. evanescent wave
   d. vanishing wave
   e. nothing...there can be no E-field there.

Q3. Total internal reflection happens for:
   a. s-polarized light
   b. p-polarized light
   c. both polarizations
Also: \( \mathbf{k}_1 | \mathbf{y}_1 = \mathbf{k}_2 | \mathbf{y}_2 \)

Incident and reflected have same component!

1. Same magnitude, same y-component

\[
\mathbf{k}_1 \cdot \mathbf{y}_1 = \mathbf{k}_2 \cdot \mathbf{y}_2
\]

Optics only, not used for \( \omega \).

\[
\frac{\omega}{c} = \mathbf{k}_1 \cdot \mathbf{y}_1 = \mathbf{k}_2 \cdot \mathbf{y}_2
\]

Law of Reflection

\[
\theta_1 = \theta_2
\]

Law of Refraction; Snell's Law

\[
\frac{n_1}{n_2} = \frac{\sin \theta_1}{\sin \theta_2}
\]

3 Laws of Geometrical Optics

Gives deeper explanation: a beam of light bends because its fields propagate at a different speed (like density on shoulder).

Explains focusing, magnification, ray tracing, etc.

\(-\) only used most basic bending condition; not really any advanced.

Maxwell, Ray

So these are true for basically any waves?

\(-\) you're starting to realize why it was so hard to accept.

Einstein's photon idea!
Now let's assume a particular polarization: $p$-polarization

$E = \perp$ plane of incidence

$E_x = E_0 \cos \theta_1 + E_0' \cos \theta_2 = E_0 \cos \theta_2 \sin \theta_1$ (lost all exponential factors: because they're equal)

$E_y = E_0' \sin \theta_2 \cos \theta_1 - \frac{E_0}{\sin \theta_1}$

$E_z = E_0' \sin \theta_2 \sin \theta_1 - \frac{E_0}{\sin \theta_1}$

$sin \sin \theta_1 = \sin \alpha \cos \theta_2$

$E_0' = \frac{E_0}{\sin \theta_1}$

$E_0' = \alpha E_0' \cos \theta_2$

Solve system; easy with a little algebra

\[
\begin{align*}
E_{0x}' &= \frac{\alpha}{1 + \alpha} E_0 z \\
E_{0z}' &= \frac{\alpha}{\alpha + 1} E_0 z
\end{align*}
\]

Kernel Eqs. / 3.15-3.16

\[
[\text{some unspecified equation number}]
\]

Notice: $\alpha = 180^\circ$ beam shifts in reflected beam if $\beta = 0$

$\beta = 0$: special case, "Bremsstrahlung" $\to E_0 = 0$, no reflection

$E_{0x}' = \frac{\alpha}{1 + \alpha} E_0 z$

$\alpha = \frac{\sin \theta_1}{\cos \theta_2}$

$1 - \frac{\alpha^2}{\alpha + 1} \cos^2 \theta_2 = \frac{\alpha}{\alpha + 1} \sin^2 \theta_1$

$1 - \frac{\alpha^2}{\alpha + 1} \sin^2 \theta_2 = \frac{\alpha}{\alpha + 1} \cos^2 \theta_1$

$1 - \frac{\alpha^2}{\alpha + 1} \sin^2 \theta_1 = \frac{\alpha}{\alpha + 1} \cos^2 \theta_2$
\[ 1 - \frac{1}{\beta^2 \cos^2 \theta_1} = \beta^2 \cos^2 \theta_1 \quad (\times \pi) \]
\[ \beta^2 - \frac{1}{\beta^2 \cos^2 \theta_1} = \beta^2 \left(1 - \cos^2 \theta_1\right) \]
\[ \left\{ \begin{array}{l}
\theta_1^2 \cos^2 \theta_1 = \beta^2 - 1 \\
\cos^2 \theta_1 = \frac{\beta^2 - 1}{\beta^4 - 1}
\end{array} \right. \]
\[ \cos^2 \theta_1 = \frac{\beta^2 - 1}{\beta^4 - 1} \]
\[ \beta^4 - 1 = \frac{\beta^2}{\beta^4 - 1} \]
\[ \cos^2 \theta_1 = \frac{\beta^2 - 1}{\beta^4 - 1} \]
\[ \text{For } \theta_1 = 90^\circ \quad (\text{max}, \text{min}) \quad \theta = \frac{\pi}{2}, \text{(max)}, \text{(min)} \]
\[ \text{For } \theta_1 = 0^\circ \quad (\text{max}, \text{min}) \quad \theta = \frac{\pi}{2}, \text{(max)}, \text{(min)} \]

Reflection intensities: \[ I = \frac{1}{\cos^2 \theta_1} \]
\[ R - \frac{E_{01}}{E_{01}^2} = \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2 \geq 0, \quad \text{good} \]
\[ T = \frac{E_{01}^2}{E_{01}^2} \left(\frac{\alpha - \beta}{\alpha + \beta}\right)^2 \geq 0, \quad \text{good} \]
From *Griffiths*

For $n_2 = 1.5$

$n_1 = 1$

**Figure 9.16**

**Figure 9.17**
We do analysis for this!

We always have a phase shift in reflected if $n_2 > n_1$.

We get no Brewster's angle (always get some reflection) unless we put $\mu = 0$. 
Total Internal Reflection

When Bragg's Law predicts θ_2 > 90°
if it can't bend anymore
n, sθ_1 = n_c sθ_2

Critical sθ_1 = \frac{\pi}{m}

\[ \sin^{-1}(\frac{\lambda_1}{n}) \quad \text{TIR occurs} \]

Proof that it's total reflection

For p polar,
\[ r = \frac{E_{12}}{E_{22}} = \frac{a-\beta}{a+\beta} \]

\[ \alpha = \cos 2\theta, \quad \beta = \frac{22}{n} \]

a is now imaginary! But that's ok with equations

proof: \[ a = \sqrt{1 - \frac{s^2 \sin^2 \theta_1}{\cos \theta_1}} = \sqrt{1 - \left(\frac{n_1}{n_2}\right)^2 \sin^2 \theta_1} \]

= imaginary
= positive

What does that go to r?

\[ r = \frac{ix-\beta}{ix+\beta} \]

What is magnitude of r?

\[ |r| = \sqrt{r^*r} = \sqrt{\left(\frac{ix-\beta}{ix+\beta}\right)\left(\frac{-ix-\beta}{ix+\beta}\right)} \]

\[ = \sqrt{(-1)(-1)} = \sqrt{-1} \]

Ralph's comment: so we can be complex now!
- not only is a imaginary (complex?)
- but B is also!

All is reflected!
Lecture 10: Wed, 30 Jan 2008

Reading quizzes: no talking, no looking in your books/notes

Q1. In circularly polarized light, the two components of the \( \text{E-field} \) differ by a phase of
   a. 0
   b. \( \pi/3 \)
   c. \( \pi/2 \)
   d. \( 2\pi/3 \)
   e. \( \pi \)

Q2. The mathematical objects that describe polarization states are [ ]
   a. Fred
   b. Jones
   c. Newton
   d. Maxwell
   e. Smith

Q3. In the above vectors, the ones that use complex numbers represent
   a. linear polarized light
   b. elliptical light in general
   c. circular only
Polarization

We've been studying things like \( \mathbf{E} = E_0 e^{i(kx - wt)} \)

\( = (\text{say}) \quad E_0 \cos(kx - wt) \)

[Graph showing oscillation]

This is a "linearly polarized" wave, because the electric field goes back and forth in a line in the x-y plane.

What happens when we add two waves together?

Example: \( E_0 x \cos(kx - wt) + E_0 y \cos(kx - wt) \)

\( = E_0 \sqrt{2} \left( \frac{x+y}{2} \right) \cos(kx - wt) \)

Amplitude would be \( E_0 \sqrt{2} \)

[Graph showing superposition]

Still linear polarization!

Add a phase shift:

\( E_0 x \cos(kx - wt) + E_0 y \sin(kx - wt) \)

\( \phi: (kz - wt - 90^\circ) \)

Circular pol! See picture!

"Amplitude" = \( E_0 \), not \( E_0 \sqrt{2} \)

[Diagram showing circular polarization]

(right circular, to be precise)

Stop time, look at screw in PKS.
Same as regular screw + right-handed opposite \( \rightarrow \) left handed

also: right-handed \( \mathbf{E} \) rotates to right as seen from observer
$\hat{y} \sin(kx) + \hat{x} \cos(kx)$
writing circ. pol of complex number:

\[
E_0 = E_0^x \cos (kz - wt) + E_0^y \cos (kz - wt - 90°) \\
= E_0^x e^{i(kz - wt)} + E_0^y e^{-i(kz - wt - 90°)} \\
= \left[ E_0^x + E_0^y e^{i(90°)} \right] e^{i(kz - wt)}
\]

\[
E_0 \left( e^{i(90°)} \right) e^{i(kz - wt)} \\
= \begin{bmatrix} 0 & i(90°) \end{bmatrix} e^{i(kz - wt)} = -i
\]

the "Jones vector"

Convectin' vecor he magnitude, so add \( dE \)

What if the \( y \) component not as strong as \( x \) component?

\[
E_0 = \left( E_0^x \right. + \left. 0.2 E_0^y \right) e^{i(90°)} e^{i(kz - wt)}
\]

Jones = \[
\begin{bmatrix} 1 & i(90°) \\
1.0105 & 0.2 \end{bmatrix}
\]

\[
\text{Elliptically polarized} \quad \frac{E_{\text{max}}}{E_{\text{min}}} = \frac{E_{\text{max}}}{E_{\text{min}}} = \frac{1.0105}{0.2}
\]

Elliptically polarized (like the concept of elliptical, but not quite the same mathematically)

Summable Table 4.1

<table>
<thead>
<tr>
<th>( l )</th>
<th>( m )</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( \cos l ) ( \sin m ) ( \sin n )</td>
<td>( \cos k ) ( \cos \phi )</td>
<td></td>
</tr>
</tbody>
</table>

\[\frac{1}{\sqrt{\sin^2 \phi + \cos^2 \phi}} \] elliptical at some axis

\[\text{angle of gain}\]
Elliptically polarized light
E still rotates at a given point in space. But E is strongest when it points along a certain line, and weakest when it points 90 degrees from that line.
**Jones vectors summary (similar to P&W table 4.1)**

\[
\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{linearly polarized in x-direction}
\]

\[
\begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{linearly polarized in y-direction}
\]

\[
\begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix} \quad \text{linearly polarized at angle } \alpha \text{ from x-axis}
\]

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(-90^\circ)} \end{pmatrix} \quad \text{RCP}
\]

\[
\frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ e^{i(+90^\circ)} \end{pmatrix} \quad \text{LCP}
\]

\[
\begin{pmatrix} A \\ Be^{i(-90^\circ)} \end{pmatrix} \quad \text{RCP-like elliptical, with ellipse oriented along x- and y-axes } (A^2 + B^2 \text{ must } = 1)
\]

\[
\begin{pmatrix} A \\ Be^{i\delta} \end{pmatrix} \quad \text{elliptical oriented at angle } \alpha \text{ from } x- \text{ or } y-\text{axis (long equation for } \alpha \text{ as a function of } \delta \text{ given in textbook; } A^2 + B^2 \text{ must } = 1)
\]