Reading quizzes: no talking, no looking in your books/notes

Q1. The smallest radius of a laser beam is known as the
   a. ankle
   b. knee
   c. neck
   d. pinky
   e. waist
   f. wrist

Q2. The distance over which the beam radius near a focus stays about constant is the ______ range
   a. Fresnel
   b. Gaussian
   c. Helmholtz
   d. Home_on_the
   e. Rayleigh

Q3. The ABCD law for Gaussian beams is used for:
   a. Finding the ABCD matrices for a Gaussian laser beam
   b. Finding the diffraction pattern in the near-field
   c. Finding the Gaussian beam parameters after an optical element
\[
\begin{align*}
t(x) &= \frac{1}{\sqrt{2\pi \sigma^2}} e^{-\frac{x^2}{2\sigma^2}} \\
E[x(I)] &= \frac{-\sigma^2}{4} e^{-\frac{x^2}{2\sigma^2}} \\
E[x^2(I)] &= \sigma^2 e^{-\frac{x^2}{2\sigma^2}} \\
E[x^4(I)] &= 3\sigma^4 e^{-\frac{x^2}{2\sigma^2}} \\
\end{align*}
\]
\[ I \propto |E|^2 \]

\[ I = Io \left( \frac{\omega_0}{\omega} \right)^2 e^{-2r^2/\omega_0^2} \]

\[ z_0 = \text{"Rayleigh range" aka "confocal parameter"} \]

\[ W = W_0 \sqrt{1 + \frac{z^2}{z_0^2}} \]

At \( z = z_0 \):
\[ I = \frac{I_0}{2} e^{-2r^2/\left(\frac{\omega_0}{\sqrt{2}}\right)^2} \]
\[ \text{width up by } \sqrt{2} \]
\[ \text{width down by } 1/2 \]

At \( z = 2z_0 \):
\[ I = \frac{I_0}{5} e^{-2r^2/\left(\frac{\omega_0}{\sqrt{5}}\right)^2} \]
\[ \text{width up by } \sqrt{5} \]
\[ \text{width down by } 1/5 \]

Large \( z \):
\[ W = W_0 \frac{z}{z_0} \]

Small \( W_0 \) (tight focus) → Small \( z \) (fast diverging)

Note: light backwards in time, get focusing (and then diverging beam after it focuses too)
phase factors of $E$

A) $e^{-ikz}$ obvious

B) $e^{itkz}$ correct wavefront

limits: 1) $z \to 0$

$R \to \infty$ this term goes away

2) $z \to \infty$ $R = z$

then $e^{ik(z + \frac{r^2}{2z})} = e^{ik\left(\frac{r^2}{2z} + \frac{r^2}{z}\right)}$

$= e^{i\frac{r^2}{2z + 2z}}$

$= e^{i\frac{r^2}{4z}}$

spherical wavefront!

C) $e^{it \sin \left(\frac{r}{zo}\right)}$

Jacobian phase shift between actual wave and plane wave

$= -\frac{i\pi}{2}$ when $z = -\infty$

$= \frac{i\pi}{2}$ when $z = +\infty$

also

D) when light goes through a focus,

it has an overall phase shift of $\pi$

"Gaussian shift" (look: see to the culprit, not just Gaussian)
Gaussian Wavefronts

Figure 3.1-7 Wavefronts of a Gaussian beam.

(Figures from Saleh and Teich, *Photonics*, 2nd edition, pg 82)
Higher-Order Modes

\[ m = 0, p = 0 \]

\[ E \]

\[ m = 1, p = 0 \]

\[ E^2 \]

\[ \text{TEM}_{0,0} \]

\[ \text{TEM}_{1,0} \]

\[ \text{TEM}_{2,0} \]

FIGURE 3.5. The field \( E \), intensity \( E^2 \), and "dot" pattern of various modes.


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Figure 3.3-2 Intensity distributions of several low-order Hermite-Gaussian beams in the transverse plane. The order \((l, m)\) is indicated in each case.
Gaussian beams + stable resonator

before

\[ \begin{align*}
&\text{infin} \text{ ity thin rays bounce back + forth} \\
&\text{if not, hide} \quad 0 < \left( 1 - \frac{L}{R_1} \right) \left( 1 - \frac{L}{R_2} \right) < 1 \quad \text{criterion (see p. 99.15)}
\end{align*} \]

Now, beam w/ width

\[ \begin{align*}
L & \quad \text{light mirror curvature to} \\
& \quad \text{match wave front curvature} \\
& \quad \text{at that point} \\
& \quad \text{a point } \quad \theta = \frac{2}{2}
\end{align*} \]

\[ \begin{align*}
1) \quad & \frac{2^2 + 2^2}{R_1} = R_1 \\
& \rightarrow \quad 2_1 = \frac{\sqrt{R_1^2 - 4L^2}}{2} \\
& \quad 2_1 = \frac{2}{2} \quad \text{match} \\
& \quad \frac{2^2 + 2^2 + L^2}{R_2} = R_2 \\
& \rightarrow \quad 2_2 = \frac{\sqrt{R_2^2 - 4L^2}}{2} \\
& \quad 2_2 = \frac{2}{2} \quad \text{match} \\
3) \quad & 2_1 + 2_2 = L
\end{align*} \]

\[ \begin{align*}
& \left( \frac{R_1}{2} + \frac{\sqrt{R_1^2 - 4L^2}}{2} \right) + \left( \frac{R_2}{2} + \frac{\sqrt{R_2^2 - 4L^2}}{2} \right) = L
\end{align*} \]

\[ \begin{align*}
2_0^2 &= -L \cdot \frac{(R_1 - L)(R_1 - R_2)(R_2 - L)}{(2L - R_1 - R_2)^2} \\
&= -L \cdot \frac{(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}{(R_1 - L + R_2 - L)^2}
\end{align*} \]

\[ \begin{align*}
\text{Num} &= X R_1 (1 - \frac{L}{R_1}) R_2 (1 - \frac{L}{R_2}) \left( \frac{R_2}{L} \left[ 1 - \left( 1 - \frac{R_2}{L} \right) \left( 1 - \frac{R_1}{L} \right) \right] \right)
\end{align*} \]

\[ \begin{align*}
&= R_1^2 R_2^2 (x)(1 - x) \\
&\quad \text{with } x = \left( 1 - \frac{L}{R_1} \right) \left( 1 - \frac{L}{R_2} \right)
\end{align*} \]

\[ \begin{align*}
\text{to } x \text{ must be positive} \\
\text{0 < } \left( 1 - \frac{L}{R_1} \right) \left( 1 - \frac{L}{R_2} \right) \text{ < 1}
\end{align*} \]

\[ \begin{align*}
\text{some condition as before!}
\end{align*} \]
Combine this info into one "Q parameter."

\[ q = z + i \frac{z}{\bar{z}} \]

How does \( q \) change as you pass through optical element? \( q_2 = \frac{A q_1 + B}{C q_1 + D} \)

When \( (A\ B) \begin{pmatrix} c \\ d \end{pmatrix} \) is normal matrix for the element.

"ABCD Law for Gaussian beams."

Lens system

New beam looks like it's coming from a new waist, with a new curvature.

Books eq 7, defines \( q = z + i \frac{z}{\bar{z}} \)

Note: eqn 11.13.1, (i) free space

1. Thin lens.
2. Thick window.
3. I won't grade.