Electrodynamics Exam 2 – Sample Exam Problems
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Problems

1. A sphere of radius $R$ carries a surface charge density of $k\cos \theta$. Find the potential and the field for this charge distribution a large distance $r$ from its center.

2. A sphere of radius $R$ carries a surface charge density of $k\cos^2 \theta$. Find the potential and the field for this charge distribution a large distance $r$ from its center.

3. A solid sphere of radius $R$ carries a volume charge density of $\rho = kr'\cos \theta'$. Find the potential for this charge distribution a large distance $r$ from its center.

4. A solid sphere of radius $R$ carries a volume charge density of $\rho = kr'\cos^2 \theta'$. Find the potential for this charge distribution a large distance $r$ from its center.

5. For $a >> d$, find the electric field for the charge configuration shown below for the following points: $(a,0)$, $(a,a)$, $(-a,-a)$, and $(0,-a)$.

6. Four charges are placed around the origin as shown below, forming a square having sides 2$d$. Calculate the force on a charge, $Q$, placed at a position $(a, a, 0)$. Assume $a >> d$.

7. A charge of $-q$ is placed at the origin. A second charge of $+q$ is placed a distance 2$d$ from the origin, its distance from the x-axis being $d$. Calculate the field at the point $x=0, y=a$ for $a >> d$.

8. A dielectric sphere has polarization $\vec{P} = k\vec{r}$. Find the electric field both inside and outside the sphere due to this polarization.
9. You have a polarized sphere of radius $R$, whose polarization is given by $\vec{P} = \alpha r^3 \hat{r}$. (a) Find $\sigma_B$ and $\rho_B$, and use Gauss’s Law for $E$ to determine the electric field everywhere. (b) Use Gauss’s Law for $D$ to determine the electric field everywhere.

10. A sphere of radius $a$ has a radial polarization given by $\vec{P} = \alpha r^n \hat{r}$ where $a$ and $n$ are constants and $n > 0$. Find the volume and surface densities of bound charge. Find $\vec{E}$ inside and outside the sphere.

11. A conducting sphere of radius $a$ and a conducting spherical shell of inner radius $b$ ($b > a$) are concentric with one another. A linear isotropic dielectric of electric susceptibility $\chi_e$ fills the space between the two conductors, from $a$ to halfway between $a$ and $b$. Find the capacitance of this system.

12. A linear ring of charge has a constant positive charge density halfway around (total charge, $Q$), then an equal constant negative charge density the other halfway around (total charge, $-Q$). Find its dipole moment.

13. A charge $q$ is located on the z-axis a distance $a$ away from the center of a grounded conducting sphere of radius $R$. Find the potential for all points on the z-axis. (Hint: use a negative image charge located inside the sphere; its magnitude is not $-q$.)

14. You have a sphere of radius $R$ with $V = V_0 \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right)$ on its surface. Find the potential as a function of $r$ and $\theta$, inside and outside the sphere.
Answers (not guaranteed, please let me know if you find any errors.)

1. \[ V = \frac{kR^3 \cos \theta}{3\varepsilon_0 r^2}, \quad \tilde{E} = \frac{kR^3}{3\varepsilon_0 r^3} \left(2\cos \theta \hat{r} + \sin \theta \hat{\theta}\right) \]

2. \[ V = \frac{kR^2}{3\varepsilon_0 r}, \quad \tilde{E} = \frac{kR^2}{3\varepsilon_0 r^2} \hat{r} \]

3. \[ V = \frac{kR^3}{15\varepsilon_0 \frac{\cos \theta}{r}} , \quad \tilde{E} = \frac{kR^3}{15\varepsilon_0 \frac{\cos \theta}{r^3}} \left(2\cos \theta \hat{r} + \sin \theta \hat{\theta}\right) \]

4. \[ V = \frac{kR^4}{12\varepsilon_0 r}, \quad \tilde{E} = \frac{kR^4}{12\varepsilon_0 r^2} \hat{r} \]

5. At \((a,0)\): \[ \tilde{E} = \frac{3}{8\pi \varepsilon_0} \frac{qd}{a^3} \left(2\hat{x} - \hat{y}\right) \] at \((a,a)\): \[ \tilde{E} = \frac{3}{8\sqrt{2}\pi \varepsilon_0} \frac{qd}{a^3} \left(\hat{x} + \hat{y}\right) \] at \((-a,-a)\):
\[ \tilde{E} = \frac{-3}{8\sqrt{2}\pi \varepsilon_0} \frac{qd}{a^3} \left(\hat{x} + \hat{y}\right) \] at \((0,-a)\):
\[ \tilde{E} = \frac{3}{8\pi \varepsilon_0} \frac{qd}{a^3} \left(\hat{x} - 2\hat{y}\right) \]

6. \[ \tilde{F} = \frac{qQ}{4\sqrt{2}\pi \varepsilon_0 a^4} \hat{r} \]

7. \[ \tilde{E} = \frac{1}{4\pi \varepsilon_0} \frac{qd}{a^3} \left(\hat{x} + 2\sqrt{3}\hat{y}\right) \]

8. inside: \[ \tilde{E} = -\frac{k}{\varepsilon_0} \hat{r} \] outside: \[ \tilde{E} = 0 \]

9. \[ \sigma_B = \alpha R^3, \quad \rho_B = -5\alpha \rho^2 \] \quad inside: \[ \tilde{E} = -\frac{\alpha \rho^3}{\varepsilon_0} \hat{r} \] \quad outside: \[ \tilde{E} = 0 \]

10. \[ \sigma_b = \alpha \rho^n, \quad \rho_b = -\alpha (n + 2) \rho^{n-1} \] \quad inside: \[ \tilde{E} = -\frac{\alpha \rho^n}{\varepsilon_0} \hat{r} \] \quad outside: \[ \tilde{E} = 0 \]

11. \[ C = 4\pi \varepsilon_0 \left(\frac{1}{a \varepsilon_r} + \frac{2}{a + b} \left(1 - \frac{1}{\varepsilon_r} \right) - \frac{1}{b}\right)^{-1} \]
12. \( \ddot{p} = \frac{4QR}{\pi} \dot{y} \)

13. \( V = \frac{1}{4\pi\varepsilon_0} \left( \frac{q}{|z-a|} - \frac{Rq}{|z-b|} \right) \)

14. inside: \( V = V_0 \left( \frac{r}{a} \right)^2 \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \); outside: \( V = V_0 \left( \frac{a}{r} \right)^3 \left( \frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) \)