Physics 123 Class Schedule – Fall 2010

Note 1: In the reading assignments below, PpP refers to “Physics phor Phynatics”. All other reading assignments refer to Serway & Jewitt.

Note 2: Labs are set up and taken down on Saturday mornings. If a lab is due on a Saturday, you might not be able to do it that day.

<table>
<thead>
<tr>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
<th>Saturday</th>
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<tbody>
<tr>
<td>30</td>
<td>Lecture 1</td>
<td>Lecture 2</td>
<td>Lecture 3</td>
<td>Lecture 3</td>
<td>Begin Lab 1</td>
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<tr>
<td>31</td>
<td>Intro, Pressure</td>
<td>HW 1 Archimedes’ Principle</td>
<td>HW 2 Fluid motion</td>
<td>HW 4</td>
<td>(Pressure)</td>
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<tr>
<td>14.1-14.2</td>
<td>Reading: syllabus</td>
<td>Reading: 14.3-14.4</td>
<td>Reading: 14.5-14.7</td>
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<tr>
<td>6</td>
<td>Labor Day Holiday</td>
<td>Lab 1 ongoing</td>
<td>Lab 1 ongoing</td>
<td>Lab 1 ongoing</td>
<td>Lab 1 due</td>
</tr>
<tr>
<td>7</td>
<td>Lecture 4</td>
<td>HW 3 Thermal expansion, Ideal gas law</td>
<td>Lecture 5</td>
<td>HW 4</td>
<td>Begin Lab 2</td>
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<tr>
<td>8</td>
<td>Lab 1 ongoing</td>
<td>Reading: 19.1-19.5</td>
<td>HW 4 Kinetic Theory</td>
<td>Reading: 21.1, 21.5 (and 21.6 if your book has it)</td>
<td>(Specific Heat)</td>
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<td>20</td>
<td>Lecture 9</td>
<td>HW 8 Molar Specific Heats</td>
<td>Lecture 10</td>
<td>HW 9 Heat engines</td>
<td>HW 10 Refractors &amp; Carnot</td>
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<td>Reading: 21.2-21.4</td>
<td>Reading: 22.1, 22.5</td>
<td>Reading: 22.2-22.4</td>
<td>Reading: 22.2-22.4</td>
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<tr>
<td>27</td>
<td>Lecture 12</td>
<td>HW 11 Entropy</td>
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<td>HW 12 What is entropy?</td>
<td>Lecture 14</td>
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<td>28</td>
<td>Reading: 22.6-22.7</td>
<td>Reading: 22.8</td>
<td>Reading: 18</td>
<td>HW 13 Waves</td>
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<td>4</td>
<td>Lecture 15</td>
<td>HW 14 Waves on a string</td>
<td>Lecture 16</td>
<td>HW 15 Complex exponentials</td>
<td>18</td>
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<tr>
<td>5</td>
<td>Reading: 16.3-16.6</td>
<td>Reading: PpP 2.1-2.2</td>
<td>Reading: PpP 1.1-1.4</td>
<td>Reading: PpP 3.1-3.5, 5.1</td>
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<tr>
<td>11</td>
<td>Lecture 18</td>
<td>HW 17 Sound waves</td>
<td>Lecture 19</td>
<td>HW 18 Doppler, Superposition</td>
<td>HW 19 Standing waves</td>
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<td>12</td>
<td>Reading: 17.1-17.3</td>
<td>Reading: 17.4, 18.1</td>
<td>Reading: 18.2-18.3, 18.5</td>
<td>Reading: 18.2-18.3, 18.5</td>
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<td>13</td>
<td>Lecture 21</td>
<td>HW 20 Reson., Beats, Uncertain.</td>
<td>Lecture 22</td>
<td>HW 21 Fourier transforms</td>
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<td>14</td>
<td>Reading: 18.4, 18.6-18.7</td>
<td>Reading: PpP 4.1</td>
<td>Reading: PpP 6.1-6.5</td>
<td>Reading: PpP 6.6-6.7</td>
<td>HW 22</td>
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<tr>
<td>18</td>
<td>Lecture 24</td>
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<td>Lecture 25</td>
<td>HW 24 Reflection, Refraction</td>
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<td>19</td>
<td>Reading: PpP 7.1-7.3</td>
<td>Reading: PpP 35, 35-35</td>
<td>Reading: 35, 35-35</td>
<td>Reading: 35.6-35.8</td>
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<td>HW 26 Polarization, Brewster</td>
<td>Lecture 28</td>
<td>HW 27 Images from mirrors</td>
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<td>Reading: 38.6</td>
<td>Reading: 36.1-36.2</td>
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<td>Reading: 36.3-36.4</td>
<td>HW 28</td>
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<td>8</td>
<td>Lecture 30</td>
<td>HW 29 Aberrations, camera, eye</td>
<td>Lecture 31</td>
<td>HW 30 Magnifier, microscope, telescope</td>
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<td>Reading: 38.5-38.6</td>
<td>Reading: 38.6-38.10</td>
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<td>Lecture 33</td>
<td>HW 32 More interference</td>
<td>Lecture 34</td>
<td>HW 33 Diffraction from wide slits</td>
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<td>16</td>
<td>Reading: 37.4-37.6 (and 37.7 if your book has it)</td>
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<td>Reading: 38.3-38.5</td>
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<td>22</td>
<td>Lecture 36</td>
<td>HW 35 Waves in 3-dimensions</td>
<td>Lecture 37</td>
<td>HW 36 Intro to relativity</td>
<td>Lecture 38</td>
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<td>23</td>
<td>Reading: PpP 8</td>
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<td>30</td>
<td>Lecture 38</td>
<td>HW 37 Special relativity</td>
<td>Lecture 39</td>
<td>HW 38 Relativistic transform.</td>
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<td>Reading: 39.4</td>
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<td>Lecture 41</td>
<td>HW 40 E = mc²</td>
<td>Lecture 42</td>
<td>HW 41 Project Show &amp; Tell</td>
<td>Reading Day</td>
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<td>Reading: 39.8-39.9</td>
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<td>Project due in class</td>
<td>HW 41 Project Show &amp; Tell</td>
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<td>HW 4</td>
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<td>Reading Day</td>
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<td>Reading Day</td>
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<td>24</td>
<td>FRIDAY classes</td>
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<td>HW 41 Project Show &amp; Tell</td>
<td>Reading Day</td>
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<td>25</td>
<td>Thanksgiving Holiday</td>
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<td>HW 41 Project Show &amp; Tell</td>
<td>Reading Day</td>
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<td>26</td>
<td>Thanksgiving, cont.</td>
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<td>HW 41 Project Show &amp; Tell</td>
<td>Reading Day</td>
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<tr>
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<td>No classes</td>
<td>HW 41 Project Show &amp; Tell</td>
<td>Reading Day</td>
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<td>30</td>
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<td>Lecture 39</td>
<td>HW 38 Relativistic transform.</td>
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<td>Lecture 40</td>
<td>HW 39 Space-time diagrams</td>
<td>Reading: website handout</td>
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<td>HW 39</td>
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<td>Lecture 41</td>
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<td>Reading Day</td>
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<td>HW 41 Project Show &amp; Tell</td>
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<td>HW 41 Project Show &amp; Tell</td>
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<td>HW 41 Project Show &amp; Tell</td>
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<td>HW 41 Project Show &amp; Tell</td>
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<td>18</td>
<td>HW 41</td>
<td>HW 41</td>
<td>HW 41 Project Show &amp; Tell</td>
<td>Reading Day</td>
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Final Exam
7 AM – 10 AM
Physics 123 – Fall 2010 – Section 2 “Physics Majors and Minors”  
Dr. John S. Colton

Instructor: Dr. John S. Colton, john_colton@byu.edu  
Office: N335 ESC, Phone: 422-3669  
Instructor Office Hours: 2–3 pm MWF, in the Underground Lab common area, ESC. Individual (private) office hours are available +by appointment.  
Research Lab: U130 ESC, Phone: 422-5286  
Website: http://www.physics.byu.edu/faculty/colton/courses/phy123-Fall10/  
You can navigate there via www.physics.byu.edu → Courses → Class Web Pages → Physics 123 (Colton).

Prerequisites: Everyone should have had Physics 121 and some basic differential and integral calculus.

Textbooks: (both available in the bookstore)  
- Physics for Scientists and Engineers, by Serway & Jewitt (6th, 7th, or 8th editions). You will need a textbook, or combination of textbooks, that covers chapters 14, 16–22, and 35–39. Inexpensive used versions are perfectly acceptable.
- Physics phor Phynatics, by Dallin Durfee. This book contains supplementary material specific to this section of 123. It is a very inexpensive book, and Dr. Durfee does not receive any royalties.

Course Objectives: Students who successfully complete this course will:  
- Learn the basics about the physics of fluids, heat (thermodynamics), waves, sound (acoustics), light (optics), and special relativity.
- Learn and apply advanced mathematical methods, reasoning, and general problem solving skills.
- Recognize physics principles at work in the world around them.

I also hope that as you learn more about the physical laws governing the universe, your appreciation for the order, simplicity and complexity of God’s creations will increase. I sincerely believe that one can come to know the Creator better by studying His creations. I have been struck by these three quotes; hopefully they will be as meaningful to you as they are to me.

Brigham Young:  
Man is organized and brought forth as the king of the earth, to understand, to criticize, examine, improve, manufacture, arrange and organize the crude matter and honor and glorify the work of God’s hands. This is a wide field for the operation of man, that reaches into eternity; and it is good for mortals to search out the things of this earth.

Steve Turley (former BYU Physics Department chair):  
My faith and scholarship also find a unity when I look beneath the surface in my discipline to discover the Lord’s hand in all things (see D&C 59:21). It is His creations I study in physics. With thoughtful meditation, I have found striking parallels between His ways that I see in the scriptures and His ways that I see in the physical world. In the scriptures I see a God who delights in beauty and symmetry, who is a God of order, who develops things by gradual progression, and who establishes underlying principles that can be relied on to infer broad generalizations. I see His physical creations following the same pattern.

Dallin Durfee (former instructor of Physics 123):  
In addition to learning physics, I hope [Physics 123] will broaden your interest in and understanding of, well… life, the universe, and everything! My understanding of science and math has affected all aspects of my life, from the way I manage my finances to my understanding and appreciation of the gospel. It has sharpened my reasoning skills and awakened a fascination of the universe we live in.
Class Identification Number: Each of you will receive a personal identification number for this course, called a “Class ID” (CID). The purpose of this number is to protect your privacy. If you did not receive your CID by e-mail, you can obtain it from the link on the class website. Include this number—and not your name—on all work you turn in.

Where to turn things in:
Turn in assignments to the slot labeled “physics 123, section 2” in the boxes near room N375 ESC. Be sure to staple your assignments together with a real staple (not just a fold!) and write your CID number in at the top of each assignment. Assignments will be returned to the slots next to the box where homework is handed in, sorted by the first two digits of your class ID. Because these “turn back” slots are open, other students will be able to see your work—so to maintain confidentiality, please do not write your name on your assignments.

Student Email Addresses: I will periodically send class information via email to your email address that is listed under Route-Y. If that is not a current address for you, please update it.

Clickers: We will use “i-clickers” in class. On the reverse side of your clicker is an alphanumeric ID code for your transmitter. You must go to the course website as soon as possible and register your transmitter ID in order to get credit for your in-class quizzes. Please also write down your clicker number here:

(The numbers frequently wear off during the course of the semester, and if you want to sell back the clicker or use it in another class, you will need to know the identification number.)

Mathematica: Some of the homework problems will require numerical calculations and plots. Mathematica is a very useful program for this, and will be the major topic of Physics 230 if/when you take that. In the meantime, for a basic, concise introduction which contains everything you should need to know for this course, see my Basic Commands of Mathematica document on the course website.

If you are interested in more advanced material, you can view the Physics 230 text, Introduction to Mathematica, here: http://www.physics.byu.edu/Courses/Computational/phys230.aspx. Mathematica’s own help files are very detailed, as well.

Mathematica is found on all departmental computers; you can gain access to these computers by following the instructions given here: http://www.physics.byu.edu/ComputerSupport/ComputerAccounts.aspx

Grading: If you hit these grade boundaries, you are guaranteed to get the grade shown. I may make the grading scale easier than this in the end, if it seems appropriate, but I will not make it harder. Because the class is not graded on a curve, it is to your advantage to help each other!

<table>
<thead>
<tr>
<th>Grade</th>
<th>Percentage</th>
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<tbody>
<tr>
<td>A</td>
<td>93%</td>
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<tr>
<td>A-</td>
<td>89%</td>
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<tr>
<td>B+</td>
<td>84%</td>
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<tr>
<td>B</td>
<td>80%</td>
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<tr>
<td>C</td>
<td>73%</td>
</tr>
<tr>
<td>D+</td>
<td>60%</td>
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<tr>
<td>D</td>
<td>56%</td>
</tr>
<tr>
<td>D-</td>
<td>50%</td>
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Grades will be determined by the following weights:
- Clicker quizzes: 5%
- 3 Midterm Exams: 30%
- Final Exam: 19%
- Term Project: 8%
- Labs/In-class writing: 8%
- Homework: 30%
Your scores will be available online through the class web page. Please check regularly that your scores are recorded correctly.

**Clicker quizzes:** There will be two types of in-class clicker quiz questions: (1) graded questions which should not be very difficult if you have done the reading assignment listed on the schedule, and (2) ungraded “thought questions” which will be used to help me pinpoint misconceptions and encourage class discussion. For the graded questions, you will get 2 points each for a right answer and 1 point for a wrong answer (for participating). For the ungraded questions, you will get 1 point each as long as you attempt an answer.

All of the questions from a given class period constitute a single quiz which will be recorded in your grades. You will not be allowed to make up missed clicker quizzes **for any reason** (tardy, excused absence, unexcused absence, registered late, forgot/lost clicker, etc.). However, so that you are not penalized unduly for missing class when circumstances necessitate, you will get four free clicker quizzes: I will convert your four clicker quizzes with the most missed points into perfect scores. I will bend the “no make-up quizzes” rule only if circumstances out of your control have resulted in you missing more than four class periods.

**Midterm Exams:** Three midterm exams will be given in the Testing Center and will be available for the days indicated on the schedule. Exams will include worked problems similar to homework problems, as well as conceptual questions related to things we discussed in class such as thought questions, demonstrations, etc.

**Final Exam:** A comprehensive final exam will be given during the regularly scheduled time for our class, as indicated on the schedule.

**Term Project:** The term project is an opportunity for you to propose and conduct a simple experiment or to theoretically, mathematically, or computationally investigate an aspect of the course in more depth. Term project guidelines, as well as a list of possible projects and examples of projects done in prior semesters are available on the class web page. There are three parts to the term project: a proposal, a progress report, and a final report. Due-dates for each of the three parts are on the class schedule.

**Labs:** You will perform several short experiments. Most will be similar to the “walk-in labs” in Physics 121, and will be set up in room S415 ESC. Two of the labs will be computer simulations available through the class website. The availability and due-dates of the labs are listed on your schedule. Each lab has a worksheet with instructions and questions to be answered; the worksheets are located at the end of this syllabus packet. You are encouraged to work and discuss the labs in groups, but everyone must be present and participate, and all analysis must be your own work. Because you are given a week in which to do each lab, labs typically may not be made up.

**In-Class Writing:** At various times during class I will stop and give you a minute or two to write a short—but complete—paragraph summarizing a topic we have just discussed. I will then randomly pick a student to share his or her summary with the class. Writing about a topic helps you internalize ideas and find/fill in holes in your understanding. As a sign on one of my English teachers’ desks read: “How do I know what I think? I haven’t written it yet.” It may be surprising to you, but the ability to express ideas clearly in writing is a tremendously important skill for scientists and engineers. I spend hours every day writing: writing notes for classes, research papers/proposals, progress reports, homework problems, syllabi, emails to students, etc.

At the end of the semester you will receive a score based on my perception of your efforts. To simplify grade keeping, this score will be recorded as the final lab. You will receive full points if you have made an honest effort, even if your summaries contain incorrect information. If you don’t take the assignment seriously, fail to write, or refuse to present when called upon, you will lose points. If you have special circumstances or needs, I will of course do everything reasonably possible to accommodate you.
**Homework:** This will be a very homework-intensive class. The homework problems for this course are found later in this packet. Problems 1-1 through 1-6 belong to Homework 1, problems 2-1 through 2-7 belong to Homework 2, etc. Some problems require numeric answers which will be graded by the computer, others (labeled “Paper only”) do not require you to enter your answers into the computer. A few problems contain both computer-graded and paper-only questions in different parts of the problem.

Be they computer-graded or paper-only problems, you must turn in your work for all homework problems, and your work must be legible with all steps clearly presented. Practice good problem solving skills: draw pictures of the problems, write and solve equations with symbols as much as possible before plugging in numbers, write neatly, and use plenty of space. Substitute units with your numbers into your algebra, and check to see that the units work out right on your final answer. Think about whether your final answer makes physical sense before submitting it. If the grader has difficulty following your work you may lose points. Conversely, if your homework looks exceptionally neat and organized, or if you have otherwise gone the extra mile, you may be awarded bonus points.

You are strongly encouraged to work with other students to figure out the problems; however, don’t just copy others’ work or allow them to copy your work. Any assignment handed in must be your own work. If you do get help on a homework problem, be sure to learn the strategy, concepts and steps used to solve the problem, and think about how they would apply to related situations.

Assignments are due on the dates marked on the schedule. Your work on paper is due any time before the building closes; your computer-graded answers must be submitted via the website by 11:59 pm. To allow for emergencies or adding the class late, you will get four free late assignments; after that, late work only counts for half credit. I will bend this rule only if circumstances out of your control have prevented you from turning in more than four homework sets on time. No homework assignments will be dropped.

**Computer-graded homework details:** The computer-graded problems use a custom-designed system created by BYU Physics Department faculty members. This system offers several major advantages to students and professors:

- Students get instant feedback as to whether they did the problem correctly.
- Because the HW problems are not assigned directly from the textbook, students can purchase cheap older editions instead of all being forced to use the same, newest edition.
- Students get multiple tries to get the problems right. Specifically, I have arranged things so that you get two attempts at a problem for full credit; after that, you start losing points.
- Each student gets a slightly different—but closely related—problem to work; this makes copying off of other students nearly impossible. (Yes, sadly even at BYU this is sometimes a problem.)

**Data for the problems:** Each of you will do the problems using different numbers (“data”), resulting in different numerical answers. Blanks are left in the problem statements where you can write in your own data. Your data for the entire semester is available via the internet: once you have a CID, go to the class website, click on “Online Homework”, and then click on “Homework Data Sheet”. You can get your same personal data again anytime during the semester if you lose your original data sheet. Assume that the numbers given in the problem and in your data sheet are exact. If you are given 2.2 m/s, it means 2.2000000..., to as many digits as you wish to imagine.

**Answer ranges and precision:** At the end of the list of homework problems, there is information about the answers. You are given a range of possible values for each answer, along with the units in which you must submit your answer. For example, “400, 800 J” means that your answer will lie between 400 and 800 J, and that you must give your answer in Joules (not kJ, BTU, ergs, foot-pounds, or any other energy units). These numbers also indicate the accuracy to which you must calculate the answer. This is simply the number of
digits shown—for example, "400, 800 J" means that the answer must be given to the nearest 1 J. As another example: “15.0, 60.0 N” means that the answer must be given to the nearest 0.1 N. In some cases, the accuracy is indicated via a plus/minus sign. For example, “32000, 39000 ±100 km” means the answer must be given to the nearest 100 km. You can always submit a more precise answer with no penalty. Tip: When working a problem, do not round off any numbers until you get your final answer; rounding along the way can lead to compounded errors that cause the final answer to be outside the specified precision range. That is one reason I recommend you write and solve your equations with symbols before plugging in numbers.

How to submit answers: After working the problems, you must submit your answers over the internet. Go to the class website, click on “Online Homework”, and then click on the assignment number. Fill in the numerical answers as indicated. Do not put units on your answer, but make sure that the number you submit is given in the units specified by the answer range. If a very large or very small value needs to be written in scientific notation, as specified by the answer range, indicate the exponent of 10 with an “e”. For example, 2.998 × 10^8 would be written 2.998e8, and 1.6 × 10^{-19} would be written 1.6e-19. Do not put any spaces, commas, or “x”s in the number. Do put in negative signs where appropriate.

Grading and viewing correct answers: Your submission will be graded immediately: after submitting your answers, you should see a status window that shows you which problems you got right and which you got wrong. You can see your grade at any time thereafter by going to the class website, clicking on “Online Homework”, and selecting “Homework Status”. In addition to your score, the computer will show you the correct answers for the problems you missed; that should help you figure out where you went wrong.

Try again: You have 3 tries to get the problem right before the 11:59 pm deadline. After each try, a new set of data will appear at the bottom of the homework status page (because you will have been given the answers for the old set of data). Use this new data for the next try. You only need to resubmit the parts that you missed in the previous try. Retries will also be graded immediately.

Points per problem: You will receive 5 points for each part of each problem done correctly on the first or second tries, 3 points for the third try, and no points thereafter. Aside from judging legibility, neatness, and organization as mentioned above, the TA will not grade the problems—or portions of problems—that are in the “computer graded” category. (Problems, or parts of problems, that do not involve computer grading will be graded out of a maximum score to be set relative to the difficulty of the problem.)

Special case: Multiple choice questions: Some computer-graded problems are multiple choice. Each correct multiple choice answer is also worth 5 points. Multiple choice problems will have drop-down boxes for submitting your answers. There are no retries for multiple choice problems.

Late credit: Any points from computer-graded or paper-only problems received after the deadline will be marked late. You will receive full credit for late points on the three assignments with the most late points. That is, you get four free late assignments, chosen to maximize your points. You will receive half credit for all other late points. You will get no credit for any HW turned in after the deadline marked on the schedule (the second reading day).

Getting help: There are multiple ways for you to get help solving homework problems.

Dr. Colton’s Office Hours. You should take full advantage of my office hours, which are held directly after class in the Underground Lab. (The secret passageway to the Underground Lab is located on the ground floor of the ESC, on the north end of the building. There you’ll find a door without a lock which opens to a long, descending staircase going down to the Underground Lab.) I recommend that you get as far as you can on the homework before class, and then come down to the UGL study area directly after class. You will
find other students from the class to work with, and you will have ready access to me when you have questions that your classmates can’t answer.

**Other Students.** One of your first lines of defense should be the other students in the class. Introduce yourself to people you sit next to. Be proactive: call others to discuss the homework, form study groups to work on homework or review for exams, etc. It has been shown in several studies that personal contact with classmates (and with faculty members) is one of the most important factors in a student’s success in college. Students in this class in the past who have gotten to know their fellow students have formed friendships that have lasted well beyond Physics 123, and which have helped their studies in future courses as well.

**Tutorial Lab.** A physics tutorial lab is provided in N304 and N362 ESC (it changes each semester; check the signs on the doors). Teaching assistants will be available roughly from 9 am to 9 pm every weekday, and for several hours on Saturday. The exact schedule can be found via a link on our course website. The tutorial lab is a great place for study groups to meet. It is also a great place to work on your homework individually: if/when you get stuck, you can get help from the TAs and from classmates who may be working on their homework at the same time. One cautionary note, though: the TAs in the tutorial lab will likely focus on the 123 section 1 homework problems, so they may not always be able to help with the section 2 problems.

**Course TA.** The course TA will schedule regular office hours (times to be determined), where you can go to find out why you missed past homework problems and get help on upcoming homework problems. Unlike the Tutorial Lab TAs, the course TA will focus on the homework specific to our section.

**Extra Credit:** There will be several ways to earn extra-credit points during the semester.

**Extra mile points.** As mentioned above, the TA can assign extra-credit points for homework that is exceptionally neat and organized, or has otherwise gone the extra mile.

**Finding errors in the homework problems.** The first student find an error in any of the optional HW problems listed below—and to give me the correct solution/answer—will receive extra-credit points.

**Extra-credit papers.** Each of the three items below can be done twice for extra credit: once in the first half of the semester, and once in the second half of the semester. See the class schedule for deadlines.

1. **Physics of TV/movies paper.** This is a short paper, 1-2 pages maximum. In the spirit of the “Insultingly Stupid Movie Physics” website, http://www.intuitor.com/moviephysics, I’d like you to write a physics-based review of a movie or TV show. Click on the “Movie reviews” link on the left of that website to see what I mean. Your review does not need to be as extensive as their reviews. Do not review a show that is about physics, just review a regular (fictional) show. What did they get right? What did they get wrong, and what should the proper physics have been? Focus on physics learned in this class, but you can also mention other physics. The TA will grade your review out of 5 points based on the quality of the writing, the accuracy of the physics (yours, not the movie’s), and how interesting your paper was to read; the maximum score is the equivalent of +5 points on one of your midterms.

2. **Book review.** This is a book review of a physics-related book that you read during the semester, written in a style similar to book reviews that you find on amazon.com. A list of allowed books is included later in this syllabus packet; if you want to write a review of a book not on the official list, you must get my permission first. This report also has a 1-2 page maximum. At a minimum you must include this information in your review: (1) title and author of the book, (2) a rating out of five stars, (3) some description of what the book was about, and (4) your personal assessment of the quality of the book. The TA will grade your review out of 5 points based on the quality of the writing and helpfulness.
of the review, the maximum score being the equivalent of +5 points on one of your midterms. You can get an additional +1 point for actually submitting the review to amazon.com (provide proof in the form of a printed out page from their website).

3. **Physics-related lecture.** You may attend a physics-related lecture; to get extra credit you must turn in a brief report (1 page maximum) of what you learned. Include this information in your report: (1) name of speaker, (2) time/place of lecture, and (3) some info about what kind of physics was discussed, (4) at least one thing you learned that you (hopefully) found interesting. This could be one of the weekly Physics Department colloquia (warning: these often—but not always—get very technical), an honors lecture, a university forum, a planetarium show,* or any other physics-related science lecture that you can find. If you wonder if a certain lecture is appropriate, please ask me. The TA will grade your report out of 3 points, the maximum score being the equivalent of +3 points on one of your midterms.

**Final Thoughts from Dr. Colton:** In a recent BYU seminar for new faculty, experts on student learning taught that most student learning is done outside of the classroom. I expect this class to follow that same trend. For the most part, you learn physics by doing physics. As mentioned above, this will likely be a very homework-intensive class for you, with labs, extra credit assignments, and a term project in addition to the regular homework problems. The BYU Undergraduate Catalog states that “The expectation for undergraduate courses is three hours of work per week per credit hour for the average student who is appropriately prepared; much more time may be required to achieve excellence”. To me, for this particular three credit hour class, that means an average student should spend at least six hours per week on study and work outside of class, in order to get an average grade. Many of you will spend many more hours than that. However, I hope that will not be an undue burden: we have a lot of cool things going on in this class, in my opinion, and if you are in the right major/minor, you should find them cool, too!


**BYU Policies:**

* **Prevention of Sexual Harassment:** BYU’s policy against sexual harassment extends to students. If you encounter sexual harassment or gender-based discrimination, please talk to your instructor, or contact the Equal Opportunity Office at 801-422-5895, or contact the Honor Code Office at 801-422-2847.

* **Students with Disabilities:** BYU is committed to providing reasonable accommodation to qualified persons with disabilities. If you have any disability that may adversely affect your success in this course, please contact the University Accessibility Center at 801-422-2767, room 1520 WSC. Services deemed appropriate will be coordinated with the student and your instructor by that office.

* **Children in the Classroom:** The serious study of physics requires uninterrupted concentration and focus in the classroom. Having small children in class is often a distraction that degrades the educational experience for the entire class. Please make other arrangements for child care rather than bringing children to class with you. If there are extenuating circumstances, please talk with your instructor in advance.

* If you write “Physics-related lecture” reports twice in the semester, you may not do planetarium shows both times.
Book Review Extra Credit Book List

Important Note: if you want to get credit for reading a book not on this list, you must get prior approval from Dr. Colton first.

A Brief History of Time, by Stephen Hawking
A Briefer History of Time, by Stephen Hawking
A Short History of Nearly Everything, by Bill Bryson
Albert Einstein – A Biography, by Alice Calaprice and Trevor Lipscombe
Beyond Star Trek: Physics from Alien Invasions to the End of Time, by Lawrence Krauss
Einstein: His Life and Universe, by Walter Isaacson
From Clockwork to Crapshoot: A History of Physics, by Roger G. Newton
Front Page Physics, by Arthur Jack Meadows
Genius: The Life and Science of Richard Feynman, by James Gleick
In Search of Schrödinger’s Cat: Quantum Physics and Reality, by John Gribbin
Lise Meitner: A Life in Physics, by Ruth Lewin Sime
Measured Tones, by Ian Johnston
Miss Leavitt’s Stars: The Untold Story Of The Woman Who Discovered How To Measure The Universe, by George Johnson
Mr. Tompkins in Paperback/ Mr. Tompkins in Wonderland (essentially the same book), by George Gamow
Parallax: The Race to Measure the Cosmos, by Alan Hirshfeld
Physics for Future Presidents: The Science Behind the Headlines, by Richard Muller
Quantum: A Guide for the Perplexed, by Jim Al-Khalili
Six Easy Pieces, by Richard P. Feynman
Stephen Hawking: A Biography, by Kristine Larsen
Symmetry and the Beautiful Universe, by Leon M. Lederman and Christopher T. Hill
The Accelerating Universe: Infinite Expansion, the Cosmological Constant, and the Beauty of the Cosmos, by Mario Livio
The Elegant Universe: Superstrings, Hidden Dimensions, and the Quest for the Ultimate Theory, by Brian Greene
The Fabric of the Cosmos: Space, Time, and the Texture of Reality, by Brian Greene
The God Particle: If the Universe Is the Answer, What Is the Question? by Leon Lederman
The Making of the Atomic Bomb, by Richard Rhodes
The New Cosmic Onion: Quarks and the Nature of the Universe, by Frank Close
The Physics of Baseball, by Robert K. Adair
The Physics of Basketball, by John Joseph Fontanella
The Physics of NASCAR: How to Make Steel + Gas + Rubber = Speed, by Diandra Leslie-Pelecky
The Physics of Star Trek, by Lawrence Krauss
The Physics of Superheroes, by James Kakalios
The Quantum World: Quantum Physics for Everyone, by Kenneth Ford
The Universe and Dr. Einstein, by Lincoln Barnett
The Universe in a Nutshell, by Stephen Hawking
Thirty Years that Shook Physics: The Story of Quantum Theory, by George Gamow
Voodoo Science: The Road from Foolishness to Fraud, by Robert Park
How to Solve Physics Problems

Here's the “Colton method” for solving physics problems. It’s not just the way I do problems, though; if you look at the worked problems in the book, you’ll find they all follow this same sort of procedure.

**Picture** – Always draw a picture, often with one or more FBDs. Make sure you understand the situation described in the problem.

**Equations** – Work forward, not backward. That means look for equations that contain the *given* information, not equations that contain the *desired* information. What major concepts or “blueprint equations” will you use? Write down the general form of the equations that you plan to use. Only after you’ve written down the main equations should you start filling things in with the specific information given in the problem.

**Algebra** – Be careful to get the algebra right as you solve the equations for the relevant quantities. Use letters instead of numbers if at all possible. Even though you (often) won’t have any numbers at this stage, solving the algebra gives you what I really consider to be the answer to the problem. And write neatly!

**Numbers** – After you have the answer in symbolic form, plug in numbers to obtain numerical results. Use units with the numbers, and make sure the units cancel out properly. Be careful with your calculator—punch in all calculations twice to double-check yourself.

**Think** – Does your final answer make sense? Does it have the right units? Is it close to what you were expecting? In not, figure out if/where you went wrong.

**Example problem:** Using a rocket pack, a lunar astronaut accelerates upward from the Moon’s surface with a constant acceleration of 2.1 m/s². At a height of 65 m, a bolt comes loose. (The free-fall acceleration on the Moon’s surface is about 1.67 m/s².) (a) How fast is the astronaut moving at that time? (b) How long after the bolt comes loose will it hit the Moon’s surface? (c) How high will the astronaut be when the bolt hits?

**Colton solution:** (notice how I use the five steps given above)

When I first did part (c), I got 0 m. This didn’t seem right (using the final step, “Think”), so I had to figure out what went wrong. I had used the wrong acceleration.
**Some things to remember before you begin Homework #1:**

- Be sure to put your HW in the right box! If your HW is handed into the wrong box it will be counted late.
- Be sure to staple your assignments (with a REAL staple) or you will lose points.
- Work all numerical answers to the number of digits specified by the answer key (located at the end of the HW problems). Typically that is 3 significant figures, but sometimes it is more. For intermediate results, keep more sig figs than that so that you do not accumulate rounding errors.
- Use the system described on the previous page (you can call it the PEANuT system, if you like):
  - Picture
  - Equations
  - Algebra
  - Numbers
  - Think
- Don't be shy about asking for help from fellow classmates, the TA, or Dr. Colton.
- **DO ALL OF YOUR HOMEWORK.** This is how you will learn the material, and this is the BEST way to prepare for exams. You will learn far more by completing—and understanding—the homework problems than you will learn from (for example) listening to Dr. Colton in class.

OK, now you can go to the next page and start your homework.
1-1. A [01] ________-kg ballet dancer stands on her toes during a performance with 26.5 cm² in contact with the floor. What is the pressure exerted by the floor over the area of contact (a) if the dancer is stationary, and (b) if the dancer is jumping upwards with an acceleration of 4.41 m/s²?

1-2. What must be the contact area between a suction cup with [02] ________ atm inside and the ceiling in order to support a 127-lb student? Please note the handy conversion table inside the back cover of your textbook.

1-3. If a certain nuclear weapon explodes at ground level, the peak over-pressure (that is, the pressure increase above normal atmospheric pressure) is [03] ________ atm at a distance of 6.0 km. What force due to such an explosion will be exerted on the side of a house with dimensions 4.5 m × 22 m? Give the answer in tons (1 ton = 2000 lb).

1-4. Piston 1 in the figure has a diameter of [04] ________ in.; piston 2 has a diameter of 1.5 in. In the absence of friction, determine the force F necessary to support the 500-lb weight.

1-5. A U-tube of uniform cross-sectional area and open to the atmosphere is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in the figure, with \( h_2 = [05] ________ \) cm, determine the value of \( h_1 \).
1-6. The tank shown in the figure is filled with water to a depth of \( h = [06] \) m. At the bottom of one of the side walls is a rectangular hatch 1.00 m high and 2.00 m wide. The hatch is hinged at its top. Determine the net force exerted by the atmosphere and water on the hatch. Hint: Since the pressure is not constant, you will have to integrate in order to get the force. If you divide the dam into narrow horizontal stripes, \( P \times \text{width} \times dy \) will be the force on each stripe (since force = pressure \( \times \) area), where \( P \) is the pressure that is changing with depth.

**Extra problems I recommend you work (not to be turned in):**

- Visit the Cartesian diver exhibit on the north-west side of the lobby of the Eyring Science Center. Play with the diver, and read the explanation on the wall. Why is the diver inside the bottle affected when you squeeze the outside of the bottle?

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2-1. A rectangular air mattress is 2.1 m long, 0.48 m wide, and [01] m thick. If it has a mass of 2.3 kg, what additional mass can it support in water?

2-2. A raft is made of solid wood and is 2.31 m long and 1.59 m wide. The raft is floating in a lake. A woman who weighs [02] lb steps onto the raft. How much further into the water does the raft sink? You do not need the thickness of the raft or the density of the wood to solve this problem.
2-3. A light spring of constant $k = 163 \text{ N/m}$ rests vertically on the bottom of a large beaker of water. A 5.29-kg block of wood (density=$[03] \text{ kg/m}^3$) is connected to the spring and the mass-spring system is allowed to come to static equilibrium. (a) Draw a free-body diagram of the block. (b) What is the elongation $\Delta L$ of the spring?

![Free-body diagram of the block](image)

2-4. A 10.0-kg block of metal is suspended from a scale and immersed in water as in the figure. The dimensions of the block are $12.0 \text{ cm} \times 10.0 \text{ cm} \times [04] \text{ cm}$. The 12.0-cm dimension is vertical, and the top of the block is 5.00 cm below the surface of the water. What are the forces exerted by the water on (a) the top and (b) the bottom of the block? (Take atmospheric pressure to be $1.0130 \times 10^5 \text{ N/m}^2$.) (c) What is the buoyant force? Think about how your answers to (a) and (b) relate to your answer to (c). (d) What is the reading of the spring scale?

![Diagram of a block suspended from a scale and immersed in water](image)

2-5. A geological sample weighs 10.3 lb in air and [05] _______ lb under water. What is its density in g/cm$^3$?

2-6. An extremely precise scale is used to measure an iron weight. It is found that in a room with the air sucked out, the mass of the weight is precisely [06] _______ kg. If you add the air back into the room, by how many grams will the new measurement differ from the old? Use a positive answer to indicate the scale reading has increased, and a negative answer to indicate the scale reading has decreased. Use the densities of iron and air given in the book for $0^\circ \text{C}$ and 1 atm.
2-7. (Paper only.) A lead weight is placed on one end of a cylindrical wooden log having cross-sectional area $A$. The combined mass of the log and the weight is $m$. The log is then placed in a fluid with a density $\rho$. Because of the weight, the log floats vertically. (a) Show that if the log is pushed down from its equilibrium position, it will undergo harmonic motion. (b) What will the period of the motion be? Use the letter $g$ to represent the acceleration due to gravity. (Hint: You may need to review harmonic motion from Physics 121. To show that the log will undergo harmonic motion, you simply have to show that the force on the log as a function of the displacement from equilibrium has the same form as the force exerted by a spring when it is stretched or compressed from its equilibrium length.)

Extra problems I recommend you work (not to be turned in):

- A block of wood with density 0.615 g/cm$^3$ floats in water with only 20.5% of its volume above the surface because an aluminum mass is attached to its top side. Find the percentage of the wood submerged when the block turns over so that the aluminum is completely submerged. The density of aluminum is 2.70 g/cm$^3$. (Answer: 72.83%. Note: I did not double-check this answer, nor did I do very much double-checking of any of the answers given for the optional problems in this packet. There are almost certainly mistakes in the answers. This gives you added incentive to work the optional problems: the first student to tell me of an error and give me the correct solution/answer, will receive bonus points.)

3-1. A cowboy at a dude ranch fills a horse trough that is 1.53 m long, 61 cm wide, and 42 cm deep. He uses a 2.0-cm-diameter hose from which water emerges at \[01\] m/s. How long does it take him to fill the trough?

3-2. Suppose the wind speed in a hurricane is \[02\] mph (mi/h). (a) Find the difference in air pressure outside a home and inside a home (where the wind speed is zero). The density of air is 1.29 kg/m$^3$. (b) If a window is 61 cm wide and 108 cm high, find the net force on the window due to the pressure difference inside and outside the home.
3-3. What gauge pressure must a pump generate to get a jet of water to leave its nozzle with a speed of 5.2 m/s at a height of [03] ________ m above the pump? Assume that the area of the nozzle is very small compared to that of the pipe near the pump.

3-4. A U-tube open at both ends is partially filled with water, as in Figure (a). Oil \((\rho = 754 \text{ kg/m}^3)\) is then poured into the right arm and forms a column \(L = [04] \text{ ________ cm} \) high, as in Figure (b). (a) Determine the difference \(h\) in the heights of the two liquid surfaces. (b) The right arm is then shielded from any air motion while air is blown across the top of the left arm until the surfaces of the two liquids are at the same height, as in Figure (c). Determine the speed of the air being blown across the left arm. Assume that the density of air is 1.29 kg/m\(^3\).

3-5. (Paper only.) Imagine that you had a cylindrically-shaped paper cup filled to a height \(h\) with water sitting on a level table top. If you poked a small hole in the side of the cup, water would shoot out in an arc and hit the table. (a) If you want to maximize the distance that the water goes before hitting the table, how far from the bottom of the cup should you poke the hole? Hint: To maximize the distance, you will have to calculate a derivative and set it equal to zero. (b) If you place the hole at that location, how far will the water travel before hitting the table? Assume the hole is small enough that the height of the water in the cup doesn’t change significantly over the time that you make the measurements, and assume that you can neglect viscosity.
3-6. (Paper only.) Now let’s test it out: Find a paper or Styrofoam cup. A cylindrical one would be best, but those are hard to come by, so just get one as close to cylindrical as you can find. Punch a small hole at the correct height to maximize the distance that the water will go before hitting the table. The hole should be small so you can make your measurements before the height of the water in the cup changes appreciably, but not too small or viscosity will change your results. If you use a pencil to make your hole, you will probably do well, but you will have to watch what happens quickly before the water level in the cup drops. Now place your cup on the table and mark where you expect the water to hit the table. Put your finger over the hole, and fill the cup with water. Quickly remove your finger and note how close to your mark the water hits. (a) How close were you? (b) Now put tape over your hole and punch a new hole which is higher and try again. Did the water go farther or not as far? (c) Now do the same with a hole below the optimum height. Did the water go farther or not as far?

**Extra problems I recommend you work (not to be turned in):**

- I find that I can blow 1000 cm³ of air through a drinking straw in 2 s. The diameter of the straw is 5 mm. Find the velocity of the air through the straw. (Answer: 25.46 m/s.)

- A horizontal pipe 11.5 cm in diameter has a smooth reduction to a pipe 5.2 cm in diameter. If the pressure of the water in the larger pipe is 84.1 kPa and the pressure in the smaller pipe is 60.0 kPa, at what rate (kg/s) does water flow through the pipes? (Answer: 36.56 kg/s.)

4-1. Imagine that we want to invent a new temperature scale, called the BYU scale, where 0°C is the same as −40°C, and 100°C is the same as [01] ________°C. What would absolute zero be on the BYU scale?
4-2. The figure shows a circular steel casting with a gap. If the casting is heated, (a) does the width of the gap increase or decrease?  
(b) The gap width is 1.6000 cm when the temperature is 30°C. Determine the gap width when the temperature is [02] °C.

4-3. An underground gasoline tank at 54°F can hold [03] gallons of gasoline. If the driver of a tanker truck fills the underground tank on a day when the temperature is 90°F, how many gallons, according to his measure on the truck, can he pour in? Assume that the temperature of the gasoline cools to 54°F upon entering the tank. Use the coefficient of volume expansion for gasoline given in the textbook.

4-4. A grandfather clock is controlled by a swinging brass pendulum that is 1.3 m long at a temperature of 20°C. (a) By how much does the length of the pendulum rod change when the temperature drops to [04] °C? (b) If a pendulum’s period is given by \( T = 2\pi \sqrt{L/g} \), where \( L \) is its length, does the change in length of the rod cause the clock to run fast or slow? (c) Over the course of 24 hours, how many seconds does the clock gain or lose? Give your answer as a positive number.

4-5. Inside the house where the temperature is 20°C, we measure the length of an aluminum rod with a micrometer made of steel. (A micrometer is a device which measures distances very accurately.) We find the rod to be 10.0000 cm long. If we repeat this measurement outside where the temperature is [05] °C, what result would we obtain? Caution: the size of the micrometer is also affected by the temperature, so we no longer obtain the true length of the rod when we measure it with the micrometer. We want to find the length of the cold rod according to the cold micrometer.

4-6. A tank having a volume of 100 liters contains helium gas at 150 atm. How many balloons can the tank blow up if each filled balloon is a sphere [06] cm in diameter at an absolute pressure of 1.20 atm? Don't worry about the fact that when the pressure in the tank gets below 1.2 atm, the tank wouldn't be able to force the helium into any more balloons.
4-7. With specialized equipment, it is routine to achieve vacuums with pressures below $10^{-10}$ torr (1 torr = 1 mm of Hg = 133.3 Pa). However, special care must be taken in cleaning and baking the walls of the stainless steel chamber, or “outgassing” of contaminants will seriously increase the pressure (by orders of magnitude). If the pressure is $1.00 \times 10^{-10}$ torr and the temperature is [07] _________°C, calculate the number of molecules in a volume of 1.00 m$^3$.

4-8. If you push on an object from all sides, it will compress a bit. The amount it compresses is measured by the bulk modulus $B$. If a pressure increase of $\Delta P$ reduces the volume of the object from $V$ to $V + \Delta V$ (where $\Delta V$ is negative because the object is getting smaller), the bulk modulus is defined as:

$$B = -\frac{\Delta P}{\Delta V/V}.$$ 

Imagine that you make a copper sphere and embed it in a block of some super material which has an extremely high bulk modulus and a linear thermal expansion coefficient of [08] _________ °C$^{-1}$. Assume that the sphere is in contact with the block at all points on its surface. Assume that the sphere is a perfect fit for the cavity in the block—it’s a really snug fit, but the copper is not being compressed by the block. If you then heat the block and copper sphere by 20°C, with what pressure (in atm) will the copper push on the block? The bulk modulus and linear expansion coefficients of brass can be found in the textbook. Hint: Since $\alpha \Delta T \ll 1$, you can use the approximation that $\beta = 3\alpha$.

**Extra problems I recommend you work (not to be turned in):**

- The volume expansion coefficient for mercury is $1.82 \times 10^{-4}$/°C. So how can the mercury level in a mercury thermometer go from almost one edge of the tube to almost all the way to the other edge when the temperature changes by less than 100°C?

- An air bubble has a volume of 1.50 cm$^3$ when it is released by a submarine 100 m below the surface of a lake. What is the volume of the bubble when it reaches the surface where the atmospheric pressure is 1.00 atm? Assume that the temperature and the number of air molecules in the bubble remains constant during the ascent. (Answer: 16.01 cm$^3$.)
• A tire is filled to 35 psi (gauge pressure) on an unusually hot day in autumn (90°F). What will be the pressure on an unusually cold morning in December (−20°F)? Hint: Don’t forget to include the 14.7 psi of atmospheric pressure before computing the change. Then convert back to gauge pressure. Ignore any thermal contraction of the tire. (Answer: 25.05 psi gauge pressure.)

• The specifications on a particular scuba tank says that it should be filled to a pressure of 4350 psi (= 295.9 atm). It also claims that the volume of air that it holds is 90 cubic feet—but what they really mean is that the air that it holds at 4350 psi, if expanded at constant temperature until it was at atmospheric pressure, would fill that amount of volume. (a) What is the actual volume of the tank? (b) If the average mass of the molecules in the air is \(4.81 \times 10^{-26}\) kg, how much does the mass of the tank change when it is pressurized from 1 atm to 295.9 atm at 25°C? (Answers: 0.3042 cu ft, 3.006 kg.)

5-1. Suppose that Moses consumed on average \([01]\) _________ liters of water per day during his lifetime of 120 yrs. If this water is now thoroughly mixed with the Earth’s hydrosphere \((1.32 \times 10^{21}\) kg), how many of the same water molecules are found today in your 1-liter bottle of water?

5-2. In a 30.0-s interval, 492 hailstones strike a glass window with an area of 0.624 m\(^2\) at an angle of \([02]\) _________° to the window surface. Each hailstone has a mass of 5.00 g and a speed of 8.00 m/s. If the collisions are elastic, what are the average (a) force and (b) pressure on the window?

5-3. Twenty cars are moving in the same direction at different speeds on the highway. Their speeds are \([03]\) _________, 42, 44, 45, 49, 51, 52, 57, 59, 62, 66, 66, 67, 67, 71, 72, 77, 79, 81, and \([04]\) _________ mi/h. (a) What is their average (mean) speed? (b) What is their rms speed? Advice: use a computer program such as a spreadsheet or Mathematica; don’t do the calculations with a hand calculator.

5-4. (a) How many atoms are required to fill a spherical helium balloon to a diameter of 30.0 cm at a temperature of \([05]\) _________°C? Take the pressure to be 1.00 atm. (b) What is the average kinetic energy of individual helium atoms? (c) What is the root-mean-square speed of the atoms? (d) What is the average speed of the atoms?
5-5. The mean free path $l$ is the average distance a molecule travels between collisions. As discussed in the 6th edition of the textbook (but omitted in later editions), it is related to the number of molecules per volume $n$, and the average diameter of the molecules $d$, in this way:

$$l = \frac{1}{\sqrt{2\pi d^2 n}}$$

(That equation is derived in the 6th edition by visualizing the cylinder that is swept out by the motion of a molecule, and comparing it to the average spacing between molecules. And some hand-waving.)

The mean free path also relates to the average time between collisions $\tau$, through the average velocity $v_{\text{avg}}$:

$$v_{\text{avg}} = \frac{l}{\tau}$$

For an ultra high vacuum situation similar to that described in the previous homework assignment, suppose there are \[\text{[06]}\] molecules per cubic meter. The temperature is 300K. Determine (a) the mean free path and (b) the time between collisions for diatomic nitrogen molecules ($d \approx 10^{-10}$ m).

5-6. (Paper only.) Note: Use a program such as Mathematica for parts (b)–(e) of this problem.

(a) Show that the Maxwell-Boltzmann probability density function (which is the $N_v$ function given in the book, divided by $N$) for oxygen molecules at 500 K, can be written in this form:

$$f(v) = (0.0044 \text{ s/m}) \left(\frac{v}{510 \text{ m/s}}\right)^2 \exp \left[-\left(\frac{v}{510 \text{ m/s}}\right)^2\right]$$

(b) Make a plot of this function; go up to high enough velocities that you can see the full shape of the curve.

(c) Verify that this function is properly normalized: that the integral from 0 to infinity equals 1.

(d) Use these statistical definitions to calculate $v_{\text{mp}}$, $v_{\text{avg}}$, and $v_{\text{rms}}$ for this situation:

$$v_{\text{mp}} = \text{the velocity where } f(v) \text{ is a maximum (i.e., where the derivative } = 0)$$
\[ v_{\text{avg}} = \int_0^\infty v f(v) dv \]
\[ v_{\text{rms}} = \sqrt{\int_0^\infty v^2 f(v) dv} \]

Verify that the equations given for those quantities in the textbook produce the same numerical results.

(e) If there are \(10^{20}\) molecules in your distribution, how many will have speeds between 300 and 400 m/s? (This is the total number of molecules times how much area the probability density function has between 300 and 400 m/s.)

**Extra problems I recommend you work (not to be turned in):**

- If 2.4 mol of gas is confined to a 5.0 L vessel at a pressure of 8.0 atm, what is the average translational kinetic energy of a gas molecule? (Answer: \(4.206 \times 10^{-21}\) J.)

- The escape velocity for the Earth is 11.2 km/s. At what temperature will the most probable velocity in a gas of nitrogen molecules be greater than the Earth’s escape velocity? (Answer: \(2.113 \times 10^8\) K.)

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6-1. A 3000-lb car moving at \([01]\) mi/h quickly comes to rest without skidding the tires. The kinetic energy is converted into heat in each of the four 15-lb iron rotors. By how much will the temperature rise in the rotors?

6-2. Most electrical outputs in newer homes can deliver a maximum power of about 1800 W. Using this much power, how long would it take to heat up a bathtub containing \([02]\) m\(^3\) of water from 25\(^\circ\)C to 40\(^\circ\)C?

6-3. (Paper only.) Imagine an ideal aluminum calorimeter with a mass of 150 g (i.e., an aluminum cup that is thermally isolated from the rest of the world). The calorimeter contains 200 g of water in thermal equilibrium with the calorimeter at a temperature of 25.00\(^\circ\)C. You then heat an 80 g piece of an unknown metal to a temperature of 100\(^\circ\)C and put it into the water. The system comes to equilibrium some time later at a temperature of 27.32\(^\circ\)C. (a) What is the specific heat of the metal? (b) From the table in your book, determine what the metal is.
6-4. A [03] _______-g block of ice is cooled to $-78.3^\circ C$. It is added to 567 g of water in an 85-g copper calorimeter at a temperature of $25.3^\circ C$. Determine the final temperature. Remember that the ice must first warm to $0^\circ C$, melt, and then continue warming as water. The specific heat of ice is 2090 J/kg$^\circ C$.

6-5. What mass of steam that is initially at $121.6^\circ C$ is needed to warm [04] ________ g of water and its 286-g aluminum container from $22.5^\circ C$ to $48.5^\circ C$?

6-6. (Paper only.) The following may or may not work out well, depending on your microwave, etc. If it does work, you will get a wonderful feeling of awe for the power of physics. If it doesn’t, it’s still a good exercise and will help you understand the frustrations of an experimental physicist. As long as you do the calculations correctly, the grader will be sympathetic if your experimental results aren’t so good. You can work with others if you like, but your work must be your own.

Step 1: Get a microwave, a watch, a pencil, a microwave safe container, a measuring cup, water, and several pieces of ice.

Step 2: Put some ice into the microwave safe container and the measuring cup, then fill them with water. Stir the water with the pencil for several minutes until the water and ice are in equilibrium (if all the ice melts, add more). Since ice melts at $0^\circ C$, and water freezes at $0^\circ C$, we know that the water and ice equilibrium mixture will be at $0^\circ C$.

Step 3: Remove the ice from the measuring cup, and pour water from the microwave safe container into the measuring cup until you reach the desired volume of water (you can choose the volume, but I would suggest something near 1 cup). Pour out the rest of the water/ice from the microwave safe container, and refill it with the water from the measuring cup. You now have a known volume of water at a known temperature.

Step 4: Quickly put the microwave safe container (with water) into the microwave on high. With your watch measure the time it takes for the water to boil (when the water starts boiling it is at $100^\circ C$).

(a) Derive a symbolic expression for the heating power of the microwave $P$, in terms of the volume $V$ and density $\rho$ of water, the temperature change $\Delta T$, the time to make this temperature change $t$, and the specific heat of water $c$.

(b) Plug in the numbers to determine $P$ for your microwave (in Watts).

(c) Derive a symbolic expression for the time it will take to melt a piece of ice with volume $V_{\text{ice}}$, density $\rho_{\text{ice}}$, and latent heat $L$, in a microwave with heating power $P$. 
(d) Measure as best as you can the volume of an ice cube, then put it into the container. Use the measured volume of your ice cube, the measured value of $P$, and the equation that you derived for part (c), make a prediction about how long it will take the microwave to melt your ice cube.

(e) Now microwave it on high and measure how long it takes for the ice cube to melt. Potentially useful conversion factors: 1 liter = $10^{-3}$ m$^3$; 1 cup = 0.240 liter; 1 fluid oz = 0.0296 liters, 1 pint = 16 fluid oz, 1 quart = 32 fluid oz.

**Extra problems I recommend you work (not to be turned in):**

- Suppose your water heater is broken, so you plan to heat your bath water by converting potential energy to heat. You hoist buckets of water up really high, then tip them over so that the water falls down into the bathtub. If you want to increase the temperature of the water by 15°C, how high will you have to lift the buckets? (Answer: 6.407 km.)

- An aluminum rod is 20 cm long at 20°C and has a mass of 350 g. If 15.5 kJ of energy is added to the rod by heat, what is the change in length of the rod? (Answer: 0.2362 mm.)

- A 0.42-kg iron horseshoe that is initially at 652°C is dropped into a bucket containing 19 kg of water at 22°C. By how much does the temperature of the water rise? Neglect any energy transfer to or from the surroundings. (Answer: 1.487°C.)

- A 20 kg iron shell from a tank goes off course and lands in a frozen lake. If the shell is moving at 300 m/s and is at a temperature of 40°C when it hits the 0°C ice, how much ice will melt? (Answer: 3.779 kg.)

- A water heater is operated by solar power. If the solar collector has an area of 7.68 m$^2$ and the power delivered by sunlight is 550 W/m$^2$, how long does it take to increase the temperature of 1.00 m$^3$ of water from 20.0°C to 60.0°C? (Answer: 11.01 hours.)

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7-1. A Styrofoam box has a surface area of 0.832 m$^2$ and a wall thickness of 2.09 cm. The temperature of the inner surface is 4.8°C, and that outside is 25.5°C. If it takes [01] h for 5.54 kg of ice to melt in the container, determine the thermal conductivity of the Styrofoam.
7-2. Suppose you have two solid bars, both with square cross-sections of 1 cm$^2$. They are both [02] _________ cm long, but one is made of copper and one of iron. You place the two side by side and braze them together, making a composite bar with a cross-section of 2 cm$^2$. If one end of this rod is placed in boiling water and the other end in ice water, how much power will be conducted through the rod when it reaches steady state?

7-3. A sheet of copper and a sheet of aluminum with equal thickness are placed together so that their flat surfaces are in contact. The copper is in thermal contact with a reservoir at [03] _________°C, and the aluminum is in contact with a reservoir at 0°C. What is the temperature at the interface between the metals?

7-4. (Paper only.) The light from the sun reaches the Earth’s orbit with an intensity of 1340 W/m$^2$. Assuming that the emissivity of the Earth is the same for all wavelengths of light, calculate the temperature of the Earth in steady state. You should get something much colder than the actual average surface temperature, thought to currently be about 15°C. The primary reason why the Earth is not this cold is due to the fact that the emissivity of the Earth depends strongly on wavelength via the so-called “greenhouse effect”. Because of the atmosphere, the Earth absorbs and emits visible radiation better than infrared radiation. Since the sun is very hot, it emits a lot of visible light which is absorbed by the Earth. The colder Earth, however, emits mainly infrared light. The clouds are very reflective in the infrared, so the emissivity is small right where the Earth would be radiating most of its blackbody radiation otherwise. On the moon, however…

7-5. (Paper only.) A cylindrical insulating bucket is filled with water at 0°C. The air above the water has a temperature of 12°C. If the air remains at this temperature, how long will it take for a 1 cm layer of ice to form on the surface of the water? Hint: How much heat gets transferred through when the ice thickness is “$x$”. You will have to figure out how to set things up so that you can integrate from $x = 0$ to $x = 0.01$. I got 694 seconds as my answer.
7-6. (Paper only.) In your job as an intergalactic pizza deliverer, you accidentally deliver a pizza to the wrong location—so far off, in fact, that there aren’t even any stars nearby. The pizza, initially at 340 K, cools through emitting blackbody radiation.

(a) How warm is the pizza after 1 sec? 1 min? 1 hr? 1 day? 1 month (30 days)? Specify any assumptions you make to solve the problem. Hint: Combine the radiation equation (left hand side is \( dQ/dt \)) with the differential of the specific heat equation (left hand side will be \( dQ \)). Then move all of the temperature quantities to the left hand side, all of the time quantities to the right hand side, and integrate both sides with definite integrals.

(b) Use a program such as Mathematica to plot the temperature as a function of time for the first month. Force the vertical scale to go from 0 to 340 K.

**Extra problems I recommend you work (not to be turned in):**

- A typical 100 W incandescent light bulb has a filament which is at a temperature of 3000 K. Typically, of the 100 W that goes into the bulb, 97.4 W is conducted or convected away as heat, and only 2.6 W is radiated as light (and most of that is invisible infrared light—now you see why incandescent lights are so inefficient). (a) If you assume the emissivity of a tungsten filament to be about 0.4, what is the filament’s surface area? (b) If the temperature were raised, one would expect that the losses due to conduction and convection would go up by about the same factor as the temperature increase, but that the radiation power would scale as \( T^4 \). Given those scaling factors, if you could increase the temperature of the filament by 50% to 4500 K, how much light power would now be radiated? (Assume the same 100 W total power.) Unfortunately, if the filament gets too hot, it will melt or vaporize. This is why almost all incandescent bulbs run at about the same temperature—as hot as possible without quickly destroying the tungsten filament. This is also the secret to halogen bulbs: the halogen gas in the bulb reduces the rate at which the tungsten evaporates from the filament, allowing it to operate at higher temperatures for more brightness and efficiency. (Answer: 9.009 W.)

- Water is being boiled in an open kettle that has a 0.52-cm-thick circular aluminum bottom with a radius of 12.0 cm. If the water boils away at a rate of 0.355 kg/min, what is the temperature of the lower surface of the bottom of the kettle? Assume that the top surface of the bottom of the kettle is at 100.0°C. (Answer: 100.95°C.)
Hot water from a water heater passes through a section of insulated pipe. The pipe is made of iron with an inner diameter of 1.0 cm and an outer diameter of 1.3 cm. The pipe is surrounded by a thin layer of rubber out to a diameter of 1.4 cm. Assuming that the water is flowing quickly enough that the temperature of the inside of the pipe is always 40°C, and that the outer surface of the rubber is always at 25°C, (a) what is the temperature of the iron/rubber interface, and (b) how much power flows out of a section of pipe which is 2 m long? Hint: Because the area through which the heat is flowing is changing, you will have to integrate. Rewrite the thermal conductivity equation as 

\[ \frac{dT}{dx} = \frac{P}{kA} \]

then integrate both sides. Do this for the iron, then for the rubber. You will be left with two equations and two unknowns. (Answers: 39.87°C, 504.2 J/s.)

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8-1. We have some gas in a cylinder like that in the figure. The diameter of the cylinder is 8.1 cm. The mass of the piston is \( [01] \) _________ kg. The atmospheric pressure is \( 9.4 \times 10^4 \) Pa. (a) Find the pressure of the gas. (Both the weight of the piston and the pressure of the atmosphere on top of the piston contribute to the pressure of the gas inside the cylinder.) (b) If we heat up the gas so that the piston rises from a height of 12.3 cm to 15.6 cm (measured from the bottom of the cylinder), find the work done on the gas. Note that the pressure of the gas remains constant as it is heated up.

8-2. A gas expands from I to F along the three paths indicated in the figure. Calculate the work done on the gas along paths (a) IAF, (b) IF, and (c) IBF. \( P_i = [02] \) _________ atm and \( P_f = [03] \) _________ atm.
8-3. A monatomic ideal gas undergoes the thermodynamic process shown in the $PV$ diagram in the figure. Determine whether each of the values (a) $\Delta U$, (b) $Q$, (c) $W$ for the gas is positive, negative, or zero. (Note that $W$ is the work done on the gas.)

8-4. We have some gas in a container at high pressure. The volume of the container is [04] ________ cm$^3$. The pressure of the gas is $2.52 \times 10^5$ Pa. We allow the gas to expand at constant temperature until its pressure is equal to the atmospheric pressure, which at the time is $0.857 \times 10^5$ Pa. (a) Find the work done on the gas. (b) Find the change of internal energy of the gas. (c) Find the amount of heat we added to the gas to keep it at constant temperature.

8-5. (Paper only.) An ideal gas is initially at 1 atm with a volume of 0.3 m$^3$.
(a) The gas is then heated at constant volume until the pressure doubles. During this process 1200 J of heat flow into the gas. How much work does the gas do?
(b) What is the change in the internal energy of the gas as it is heated?
(c) Now the pressure of the gas is kept at 2 atm and the gas is heated while its volume increases to twice its initial volume. In the process, the internal energy of the gas increases by 1000 J. How much work does the gas do?
(d) How much heat flows into the gas during the expansion?
(e) Draw a P-V diagram of this sequence of processes. Label the initial state of the gas A, the state after the constant volume process B, and the state after the constant pressure process C.

**Extra problems I recommend you work (not to be turned in):**
- One mole of an ideal monatomic gas is at an initial temperature of 305 K. The gas undergoes an isovolumetric process, acquiring 728 J of energy by heat. It then undergoes an isobaric process, losing this same amount of energy by heat. What is the final temperature of the gas? (Answer: 328.3 K.)
• A sample of gas is taken through a single cycle as shown in the figure, where $P = 4.55 \text{ atm}$. (a) How much work must be done on the gas during the cycle? (b) How much heat is transferred out of the gas during the cycle? Hint: The ratio of the area of an ellipse to the area of the rectangle containing it is $\pi/4$. (Answers: 1130 J, 1130 J.)

• An ideal gas is contained inside a cylinder with a moving piston on the top. The piston has a mass $m$ which keeps the gas at a pressure $P_0$. The initial volume of the gas is $V_0$. For this whole problem give your answers in terms of $P_0$ and $V_0$. (a) The gas is heated until the volume has expanded to twice its initial volume. How much work is done by the gas during this process? (b) By what factor does the temperature increase during this expansion? (c) The piston is then locked in place and the gas is cooled back to its original temperature. What is the pressure of the gas after it is cooled? (d) How much work is done on the gas as it is cooled? (e) The cylinder is then placed in a bucket of water which keeps the temperature constant (at the original temperature), and the piston is released and allowed to slowly drop until the gas returns to its initial pressure $P_0$. How much work is done on the gas during this process? (f) Draw a P-V diagram of this sequence of processes. Label the initial state of the gas $A$, the state after expanding $B$, and the state after it is cooled $C$. (Answers to parts (a)–(e): $P_0V_0$, $\times 2$, $\frac{1}{2} P_0$, 0, $P_0V_0\ln 2$.)

9-1. [01] _________ moles of a monatomic ideal gas have a volume of 1.00 m$^3$, and are initially at 354 K. (a) Heat is carefully removed from the gas as it is compressed to 0.50 m$^3$, causing the temperature to remain constant. How much work was done on the gas in the process? (b) Now the gas is expanded again to its original volume, but so quickly that no heat has time to enter the gas. This cools the gas to 223 K. How much work was done by the gas in this process?
9-2. A diatomic ideal gas ($\gamma = 1.40$, $V = 4$ L) confined to a cylinder is subjected to a closed cycle. Initially, the gas is at 1.00 atm and at 300 K. First, its pressure is increased by a factor of ________ under constant volume. Then, it expands adiabatically to its original pressure. Finally, the gas is compressed isobarically to its original volume. (a) Draw a P-V diagram of this cycle. (b) Determine the volume of the gas at the end of the adiabatic expansion. (c) Find the temperature of the gas at the start of the adiabatic expansion. (d) What was the net work done by the gas in this cycle?

9-3. We have a container of a hot ideal monatomic gas. The volume of the container is 25 liters. The temperature of the gas is ________°C, and its pressure is $0.858 \times 10^5$ Pa. We allow the gas to cool down to room temperature, which at the time is 21°C. We do not allow the volume of the gas to change. (a) Find the final pressure of the gas. (b) Find the amount of heat that passed from the gas to its surroundings as it cooled (a positive number), by finding the change in internal energy and the work done on the gas, and using the First Law of Thermodynamics. (c) Find the amount of heat that passed from the gas to its surroundings as it cooled, by using $C_V$, the molar heat capacity for constant volume changes.

9-4. We have some air in a cylinder like that in the figure. Assume that air is an ideal diatomic gas, with $\gamma = 7/5$. The diameter of the cylinder is 5.3 cm. The mass of the piston is negligible so that the pressure inside the cylinder is maintained at atmospheric pressure which is 1.00 atm. The height of the piston is 9.7 cm, measured from the bottom of the cylinder. The temperature of the air is ________°C. (a) We heat the gas so that the piston rises to a height of ________ cm. The pressure of the air remains constant. (a) Find the final temperature of the air. (b) Find the amount of heat that was put into the air, by finding the change in internal energy and the work done on the gas, and using the First Law of Thermodynamics. (c) Find the amount of heat that was put into the air, by using $C_P$, the molar heat capacity for constant pressure changes.
9-5. One mole of a monatomic ideal gas is compressed adiabatically from an initial pressure and volume of 2.00 atm and 10.0 L to a final volume of [06] _________ L.

(a) Using \( W = - \int_{V_1}^{V_2} P \, dV \), find the work done on the gas. Be sure to include the sign if negative.

(b) Find the final pressure.

(c) Find the final temperature.

(d) Use the first law together with the knowledge of the initial and final temperatures to find the work done on the gas. HINT: Your answer should agree with part (a).

9-6. What are the number of degrees of freedom for

(a) helium at room temperature?

(b) oxygen at room temperature?

(c) water vapor at 200°C?

(d) hydrogen at a few thousand Kelvin?

9-7. (Paper only.) We’ve talked about degrees of freedom for molecules in gases, but how about for atoms in a solid? One view is that each atom in a solid should have 6 degrees of freedom: three translational and three vibrational. In other words, the total energy of an atom in a solid is \( \frac{1}{2}mv_x^2 + \frac{1}{2}mv_y^2 + \frac{1}{2}mv_z^2 + \frac{1}{2}kx^2 + \frac{1}{2}ky^2 + \frac{1}{2}kz^2 \), where \( k \) represents the “spring constant” of the restoring force holding each atom in place. If this view is correct, the molar heat capacity of all solids should be equal to \( C = 6R/2 = 3R \). That is called the Dulong-Petit law.

(a) Let’s test it out with real data. Make a list of the specific heats (units J/kg·°C) for the elements given in the table in your book. (The elements are on the left hand side of the specific heat table.) For each element, convert its specific heat \( c \) into its molar heat capacity \( C \) (J/mol·°C) by multiplying each specific heat by the appropriate molar mass (kg/mol). For each element, calculate the percent difference between the real value you obtained for \( C \), and the value predicted by the Dulong-Petit law. Feel free to use a spreadsheet program to do all these calculations. You should find very good agreement for all but two of the elements. Wikipedia has this to say about the Dulong-Petit law: “Despite its simplicity, the Dulong-Petit law offers fairly good prediction for the specific heat capacity of solids with relatively simple crystal structure at high temperatures. It fails, however, at room temperatures for light atoms bonded strongly to each other [because there is not enough thermal energy to excite the higher frequency vibrational
modes of the light elements].” Does that match what you found? Think about the
atomic weights of the two elements that did not fit the law well.
(b) Explain why I just used the symbol \( C \) to represent the molar heat capacity in the
problem above instead of \( C_V \) or \( C_P \).

**Extra problems I recommend you work (not to be turned in):**

- Consider a gas composed of 3.5 moles of nitrogen molecules (\( \text{N}_2 \)) at a temperature low
  enough that the vibration modes of the molecule are “frozen out”. In other words, the
  molecules have 5 degrees of freedom: 3 translational and 2 rotational. (a) What is the
  molar specific heat at constant volume? (b) What is the molar specific heat at constant
  pressure? (c) If the gas is in a rigid container, how much will the temperature of the gas
  change if 75 J of heat are added to the gas? (d) If the gas is in a container kept at a
  constant pressure, how much will the temperature of the gas change if that same amount
  of heat is added to the gas? (e) In which case will the gas do more work as it is heated?
  (Answers to parts (a)–(d): \( \frac{5}{2} R \), \( \frac{7}{2} R \), 1.031° C, 0.736° C.)

- (a) Explain in your own words why the molar specific heat at constant pressure should
  always be higher than the molar specific heat at constant volume. (b) Explain why the
  change in internal energy (\( \Delta E_{\text{int}} \)) for a gas always equals \( n C_V \Delta T \), even when it
  undergoes a process in which the volume changes.

- During the compression stroke of a certain gasoline engine, the pressure increases from
  1.00 atm to 18.4 atm. Assuming that the process is adiabatic and that the gas is ideal,
  with \( \gamma = 1.40 \), (a) by what factor does the volume change and (b) by what factor does the
  absolute temperature change? If the compression starts with 0.0160 mol of gas at 27° C,
  find the values of (c) \( Q \), (d) \( W \), and (e) \( \Delta E_{\text{int}} \) that characterize the process. (Answers:
  decreases by a factor of 8.006, increases by a factor of 2.298, 0, –129.6 J, 129.6 J.)

10-1. A heat engine performs \([01]\) J of work in each cycle and has an efficiency of
32.9%. For each cycle of operation, (a) how much energy is absorbed by heat and
(b) how much energy is expelled by heat?
10-2. A nuclear power plant has an electrical power output of 1000 MW and operates with an efficiency of 33%. If the excess energy is carried away from the plant by a river with a flow rate of [02] _________ kg/s, what is the rise in temperature of the flowing water?

10-3. One mole of an ideal monatomic gas is taken through the cycle shown in the figure, where $P_1 = [03] \text{ atm}$ and $P_2 = P_1/5$. The process $A \rightarrow B$ is a reversible isothermal expansion. Calculate (a) the net work done by the gas, (b) the energy added by heat to the gas, (c) the energy expelled by heat from the gas, and (d) the efficiency of the cycle.

10-4. Suppose your gasoline car has a compression ratio of [04] _________ to 1. The specs for the car indicate that the engine produces 105 hp when being operated at 6000 rpm. (a) Assuming that the air (or more properly, air-fuel mixture) is composed entirely of diatomic molecules with 5 degrees of freedom at these temperatures, and assuming that the actual cycle can be perfectly approximated as the ideal Otto cycle, find how much $Q_{in}$ per second is required to run the engine at that rpm. (b) If you can travel at 100 mph at that rpm (watch out for cops!), how many miles per gallon will your car get? Gasoline produces about 47000 kJ for each kg burned, and the density of gasoline is 0.75 g/cm$^3$.

10-5. (Paper only.) Show that the efficiency for an engine working in the Diesel cycle represented ideally below is

$$e = 1 - \frac{1}{\gamma} \left( \frac{T_D - T_A}{T_C - T_B} \right).$$

Diesel cycle: Adiabatic compression AB heats the gas until ignition at B when fuel is introduced (no spark plug needed). A constant pressure expansion BC takes place as combustion adds heat. Adiabatic expansion CD accomplishes additional work before the exhaust is exchanged for new air during what can be thought of as a constant volume cooling DA.
10-6. (Paper only.) (a) In the Otto cycle, the ratio of maximum volume to minimum volume is called the “compression ratio” $r$. Use a program such as Mathematica to make a plot of the Otto cycle’s efficiency vs. the compression ratio.

(b) In the Diesel cycle, the ratio of maximum volume to minimum volume is called the compression ratio $r$, and the ratio of the intermediate volume to the minimum volume is called the “cut-off ratio” $r_c$. The equation you derived for efficiency of the Diesel cycle can be written as:

$$e = 1 - \frac{1}{r^{\gamma-1}} \left( \frac{r_c^\gamma - 1}{\gamma (r_c - 1)} \right)$$

Use a program such as Mathematica to make plots of the Diesel cycle’s efficiency vs. the compression ratio, for cut-off ratios of 1, 2, 3, and 4. Plot all four on the same graph.

10-7. (Paper only.) Many people believe that a higher octane fuel means “more power”. That’s not quite correct; what higher octane means, is that the fuel does not self-ignite as easily as the fuel heats up during compression. Higher power engines often use higher compression ratios, the reason hopefully being clear from the results of the previous problem, so high power gas engines often require higher octane fuel to prevent the fuel from igniting before the spark plugs fire—hence the confusion. However, if the normal compression ratio is low enough that low octane fuel will not self-ignite, a higher octane fuel will provide absolutely no benefit. Some websites say that with 91 octane fuel, compression ratios up to about 11.5 can safely be used. Use this information to estimate the temperature at which an air-fuel mixture using 91 octane gasoline will spontaneously ignite. Assume an ambient air temperature of 25°C and a specific heat ratio $\gamma$ of $7/5$.

Extra problems I recommend you work (not to be turned in):

- Prove that the two Diesel cycle efficiency equations given above are equivalent.

- An engine absorbs 1678 J from a hot reservoir and expels 958 J to a cold reservoir in each cycle. (a) What is the engine’s efficiency? (b) How much work is done in each cycle? (c) What is the power output of the engine if each cycle lasts for 0.326 s? (Answers: 42.91%, 720 J, 2209 W.)
11-1. A refrigerator has a coefficient of performance equal to 5.21. Assuming that the refrigerator absorbs $[01]$ _________ J of energy from a cold reservoir in each cycle, find (a) the work required in each cycle and (b) the energy expelled to the hot reservoir.

11-2. A refrigerator keeps its freezer compartment at $-10^\circ$C. It is located in a room where the temperature is $20^\circ$C. The coefficient of performance (heat pump in cooling mode) is $[02]$ _________. How much work is required to freeze one 26-g ice cube? Assume that we put 26 g of water into the freezer. The initial temperature of the water is $20^\circ$C. The final temperature of the ice cube is $-10^\circ$C. The refrigerator removes the heat from the freezer compartment, maintaining its temperature at about $-10^\circ$C.

11-3. Consider a heat pump which is used to cool down a home during the summer. Its coefficient of performance in cooling mode is $[03]$ _________. On a particular hot day, the temperature outside the home is $90^\circ$F, and the temperature inside the home is maintained at $70^\circ$F. If the heat pump consumes 500 W of electrical power, at what rate does it remove heat from the home?

11-4. Suppose you want to keep the inside of your freezer at a temperature of $-5^\circ$C when your house is at $[04]$ ________°C. (a) What is the maximum possible coefficient of performance for a refrigerator operating between those two temperatures? (b) If 350 J of heat leak from the environment into your freezer each second, what is the minimum theoretical power that your freezer will consume to keep the temperature inside the freezer at $-5^\circ$C. (c) How much per year (365 days) would it cost you to operate such a freezer if you never open it up? Use 8 cents/(kilowatt-hour) as the price for electricity.
11-5. (Paper only.) A sample of a monatomic gas is taken through the Carnot cycle ABCDA. For your convenience, the cycle is drawn with the mathematical relationships of each part shown. Complete the table for the cycle.

<table>
<thead>
<tr>
<th></th>
<th>( P )</th>
<th>( V )</th>
<th>( T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1400 kPa</td>
<td>10.0 L</td>
<td>720 K</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>24.0 L</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>15.0 L</td>
<td></td>
</tr>
</tbody>
</table>

Avoid rounding intermediate steps so that errors do not accumulate. You may find it beneficial to solve for the unknowns in the order requested below.

First determine the number of moles from the data in row A.

(a) Find \( P_D \).
(b) Find the value of \( T_D \) and \( T_C \), which are equal.
(c) Find \( P_C \).
(d) Find \( T_B \).
(e) Find \( V_B \).

You should then be able to find that \( P_B = 875 \text{ kPa} \) (provided here as a check).

11-6. (Paper only.) For the parameters in previous problem, complete the table below.

<table>
<thead>
<tr>
<th></th>
<th>( Q )</th>
<th>( W )</th>
<th>( \Delta E_{\text{int}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BC</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CD</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DA</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Hint: Curves AB and CD are constant temperature, meaning that the internal energy is constant on the curves. Curves BC and DA are adiabatic, meaning that no heat flows into or out of the gas.
11-7. (Paper only.) (a) From the temperatures found in the problem before last, compute the theoretical maximum efficiency for this cycle. (b) From the heats and work found in the last problem, calculate the actual efficiency of the cycle using the definition of efficiency. It should match your answer to part (a).

**Extra problems I recommend you work (not to be turned in):**

- A reversible engine draws heat from a reservoir at 399°C and exhausts heat to a reservoir at 19°C. (a) Find the efficiency of the engine. (b) Find the heat required to do 100 J of work with this engine. (Answers: 56.53%, 176.9 J.)

- Would you save money if you were to somehow pipe the heat from your refrigerator’s heat-exchanging coils (in the back of the refrigerator) to outside the house?

- Let’s derive the efficiency for a general Carnot cycle. Take a look at the P-V diagram of the Carnot cycle as given in the figure. Efficiency is defined to be
  \[
e = \frac{W}{Q_h} = \frac{(Q_h - Q_c)}{Q_h}.
\]
  Unless otherwise noted, give all answers in terms of \(n, T_h, T_c, V_A, V_B, V_C, V_D, \gamma,\) and fundamental constants.
  (a) Find the heat that enters the gas during the adiabatic processes from B-C and from D-A. (In other words, what are \(Q_{BC}\) and \(Q_{DA}\)?)
  (b) Find the change in the internal energy of the gas during the isothermal processes. (In other words what are \(\Delta E_{AB}\) and \(\Delta E_{CD}\)?)
  (c) How much work is done on the gas during each isothermal process? (In other words, what are \(W_{AB}\) and \(W_{CD}\)?)
  (d) Use your results above to find \(Q_h\) and \(Q_c\).
  (e) Use the adiabatic transitions to find a relationship between \((V_B/V_A)\) and \((V_C/V_D)\).
  (f) Use what you found above to write the Carnot efficiency in terms of just \(T_h\) and \(T_c\).

12-1. We drop a [01] ________-g ice cube (0°C) into 1000 g of water (20°C). Find the total change of entropy of the ice and water when a common temperature has been reached. Caution: calculate the common temperature to the nearest 0.01°C.
12-2. We have 2.451 moles of air in some container at 25.2°C. Assume that air is an ideal
diatomic gas. We put \(02\) \(\text{J}\) of heat into the air. (a) Find the change of
entropy of the air if we hold the volume constant. (b) Find the change of entropy of the
air if we hold the pressure constant.

12-3. (a) A container holds 1 mol of an ideal \textit{monatomic} gas. A piston allows the gas to
expand gradually at constant temperature until the volume is \(03\) times
larger. What is the change in entropy for the gas?
(b) What is the change in entropy for the gas if the same increase in volume is
accomplished by a reversible adiabatic expansion followed by heating to the original
temperature?
(c) What is the change in entropy for the gas if the same increase in volume is
accomplished by suddenly removing a partition, which allows the gas to expand freely
into vacuum?

12-4. (Paper only.) Prove that \(\Delta S\) of the universe will always increase for calorimetry-type
situations if the two objects start off at different temperatures. Hint: Add together the
change in entropy for each object. Also, you may find what Wikipedia calls the “First
mean value theorem for integration” to be helpful.

12-5. (Paper only.) The goal of this problem is to figure out an equation for the change in
entropy of an ideal gas for an arbitrary state change from state A to state C. Since
entropy is a state variable, the entropy change of an \textit{arbitrary} process from A to C will be
the same as an entropy change of a \textit{specific} process going from A to C. So, let’s consider
a specific process made up of two sections: a constant volume change from A to B (B
having the same volume as A, and the same pressure as C) followed by a constant
pressure change from B to C. The gas has \(n\) moles of molecules and a molar heat
capacity at constant volume of \(C_V\).
(a) Draw a P-V diagram of the situation just described: pick two arbitrary points A and
C on the diagram, locate the appropriate point B, and draw arrows indicating the two
parts of the overall state change.
(b) How much will the entropy change if the gas undergoes a constant volume change
during which the temperature changes from \(T_A\) to \(T_B\)?
(c) How much will the entropy of the gas change if it undergoes a constant pressure
change during which the temperature changes from \(T_B\) to \(T_C\)?
(d) Use the ideal gas law to find a relation between the ratio of the temperatures before and after the isobaric process \((T_B/T_C)\) and the ratio of the volumes before and after the process \((V_B/V_C)\).

(e) Use what you have found in parts (b) through (d) to derive the general formula for the entropy change for any process (even irreversible ones) in an ideal gas:

\[
\Delta S = nC_v \ln \frac{T_f}{T_i} + nR \ln \frac{V_f}{V_i}
\]

**Extra problems I recommend you work (not to be turned in):**

- One mole of a diatomic ideal gas (5 degrees of freedom), initially having pressure \(P\) and volume \(V\), expands so that the pressure increases by a factor of 1.8 and the volume increases by a factor of 2.2. Determine the entropy change of the gas in the process. (Answer: 35.16 J/K.)

13-1. Assume that our classroom has a volume of [01] ________ m\(^3\) which is filled with air at 1.00 atm and 25°C.

(a) Calculate the probability that all of the air molecules will be found in the forward half of the room. Represent this remote possibility as 1 part in \(10^x\), where \(x\) is some large number. Give the value of \(x\). NOTE: \(2^N = 10^{N \log 2}\).

(b) How much *more* entropy is present when the air is distributed throughout the room rather than confined to the front half only?
13-2. (Paper only.) This problem involves flipping a fair coin and counting how many times you get heads, H, and how many times you get tails, T. You may want to refer to the similar example problem in the textbook where they describe choosing red and green marbles from a bag. The “microstates” are the specific ordered lists of heads and tails that you get (“HHTTHTTHH” would be one possible microstate for a collection of 8 flips); the “macrostates” are the overall number of heads (or tails) that you get. (The above microstate would belong to the “5 heads”, or “5H” macrostate.) Hopefully it’s obvious that each macrostate will likely be associated with many different microstates. The probability of a given macrostate occurring is proportional to how many microstates are associated with it. Specifically, the probability of a particular macrostate is the number of microstates associated with it, divided by the total number of microstates. That may sounds complicated, but should make much more intuitive sense as you start doing the problem below.

(a) Suppose you flip the coin once. List the 2 possible microstates. For each of the 2 possible macrostates (0H and 1H), list how many microstates are associated with it. (Don’t worry, this is not supposed to be complicated yet.)

(b) Suppose you flip the coin twice. List the 4 possible microstates. For each of the 3 possible macrostates (0H, 1H, and 2H), list how many microstates are associated with it.

(c) Repeat for three flips. There are 8 possible microstates and 4 possible macrostates (0H, 1H, 2H, and 3H).

(d) Repeat for four flips.

(e) Repeat for five flips. OK, that should be enough. Think about this question: what’s the probability of getting exactly 2 heads if you flip a coin five times? The answer is 10/32. Hopefully you can see why, from your list.

(f) Fill in a chart like this. Leave the table entries blank if not applicable.

<table>
<thead>
<tr>
<th></th>
<th>0H</th>
<th>1H</th>
<th>2H</th>
<th>3H</th>
<th>4H</th>
<th>5H</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 flip</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 flips</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 flips</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 flips</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 flips</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Do you see the pattern? Hopefully you recognize Pascal’s triangle. Each entry in the next row can be obtained by adding together two entries from the previous row. If you
don’t recall learning about Pascal’s triangle, Google it. Among other things, it gives you the coefficients to the expansion of \((x + y)^n\). Who would have thought that FOIL was related to flipping coins?

(g) Two important facts about Pascal’s triangle that you might not have run across before are: (1) the numbers in the \(n\)th row add up to \(2^n\). (For our situation, that’s the total number of microstates. Hopefully it’s clear to you why they must add up to \(2^n\).) (2) The \(k\)th number in the \(n\)th row is given by the “choose” formula, the left hand side of this equation being read as “\(n\) choose \(k\)”: 

\[
\binom{n}{k} = \frac{n!}{k!(n-k)!}
\]

\((k\) is the column label, which starts at 0 and goes to \(n\).) This is essentially what mathematicians call the “Binomial Theorem”. If you haven’t seen that before, you should verify the formula for a few entries in your table before proceeding.

Use those facts to easily answer this question: If you toss the coin 100 times, what is the probability you will get \textit{exactly} 50 heads and 50 tails? Give your answer as an exact expression as well as a numerical percentage.

13-3. (Paper only.) If you toss a fair coin, you should expect to get heads half the time, right? Well, hopefully the previous problem has convinced you that actually getting heads \textit{exactly} half the time is a pretty rare event. But you should expect to get heads \textit{close} to half the time. How close is close? Understanding that is the point of this problem. You are welcome to work in groups for this, just make sure you are a full participant and that you understand everything that’s going on.

(a) Toss a coin 100 times. After each toss write down how many total heads you have gotten, along with the fraction of total tosses which have resulted in heads. Here’s some sample data I made up, just to show you what I mean:

<table>
<thead>
<tr>
<th>Number of toss, (N)</th>
<th>Cumulative number of heads</th>
<th>Fraction of heads</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>0.667</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.75</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>0.6</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td>0.667</td>
</tr>
</tbody>
</table>
Plot the fraction of heads as a function of \( N \), the number of tosses. I’d use a spreadsheet program for all of this.

(b) Now calculate the difference between the fractions you calculated in part (a), and the expected value of 0.5. Plot this on another graph as a function of \( N \). On the same graph plot these two functions \( f_1(N) = 1/\sqrt{N} \) and \( f_2(N) = -1/\sqrt{N} \). Using statistical techniques similar to the previous problem, it can be shown that most of the time the absolute difference from the expected value will be less than \( 1/\sqrt{N} \); hopefully that’s what you found on your graph, that when \( N \) got large enough to avoid the initial large fluctuations, your measured difference was almost always between the \( f_1 \) and \( f_2 \) curves on your graph.

This type of thing becomes important time and time again in experimental physics. One situation that immediately springs to mind is in detecting light. In my lab we have detectors which can measure batches of individual photons. However, there are always statistical fluctuations present in the numbers of photons we detect, that are just like the fluctuations we saw above. Therefore, if we expect to see 1,000,000 photons each second, what we will actually see are photon numbers ranging from 1,000,000 + 1,000 down to 1,000,000 – 1,000 (because one thousand is the square root of one million). For very low light levels, this so-called “shot noise” becomes the dominant source of noise in most optical experiments.

13-4. (Paper only.) Imagine that you are doing exit polls to determine the winner of an election between two candidates. It is a very close election, with each candidate receiving very close to half the votes. You are very careful to poll a balanced cross-section of voters. You may assume that the fluctuations in the poll results will be approximately \( 1/\sqrt{N} \). (a) If you poll 100 people, how close can the election be (in percentage points) if you want to be reasonably certain that you predict the right winner based on your poll? (b) What if you poll 10,000 people?
13-5. (Paper only.) Imagine that I have a spring pressure gauge with a moving plate area of 1 cm$^2$. If it is placed in a container filled with air at room temperature and atmospheric pressure, on average about $10^{24}$ molecules will collide with this surface every second (by the way, you should know how to calculate that from the last chapter). Like any real gauge, our gauge won’t respond to pressure changes instantaneously. Using the expected $1/\sqrt{N}$ fluctuation discussed above, calculate how the readout of our gauge would fluctuate (as a fraction of the average pressure), if the gauge response time is one second, one $\mu$s, or one fs? (A fs or femtosecond is $10^{-15}$ seconds, about the time it takes light to travel 0.3 $\mu$m.)

14-1. (a) An AM radio station transmits at [01] ________ kHz. What is the wavelength of these radio waves? Radio waves travel at the speed of light, $3.00 \times 10^8$ m/s. (b) Repeat for an FM radio station which transmits at [02] ________ MHz.

14-2. At position $x = 0$, a water wave varies in time as shown in the figure. (The curve is at the 10-cm mark at both edges of the figure.) If the wave moves in the positive $x$ direction with a speed of [03] ________ cm/s, write the equation for the wave in the form, 

$$y(x, t) = A \sin(kx - \omega t - \phi).$$

Give the values of (a) $A$, (b) $\omega$, (c) $k$, and (d) $\phi$. (Give the value of $\phi$ between 0 and $2\pi$ rad.) HINT: When taking an inverse sine to find $\phi$, you must be careful to use the right quadrant. Your calculator by default will use the 1st and 4th quadrants. Check your final answer to make sure that it actually fits the curve everywhere. To change quadrants, use $\sin^{-1} x \rightarrow \pi - \sin^{-1} x$. 

![Diagram of water wave](attachment://water_wave.png)
14-3. Suppose you are watching sinusoidal waves travel across a swimming pool. When you look at the water right in front of you, you see it go up and down ten times in [04] ________ s. At the peaks of the wave, the water is [05] ________ cm below the edge of the pool. At the lowest points of the wave the water is 6.0 cm below the edge of the pool. At one particular moment in time you notice that although the water right in front of you is at its maximum height, at a distance [06] ________ m away the water is at its minimum height. (This is the closest minimum to you.) (a) What is the frequency $f$ for this wave? (b) What is $\omega$ for this wave (rad/s)? (c) What is $\lambda$ for this wave? (d) What is $k$ for this wave (rad/m)? (e) What is the amplitude $A$ of this wave? (f) What is the speed of water waves in this pool?

14-4. (Paper only.) A particular transverse traveling wave has the form,

$$y(x,t) = A \sin(kx - \omega t - \phi),$$

where $A = 1$ cm, $k = 0.15$ cm$^{-1}$, $\omega = 7$ s$^{-1}$, and $\phi = 1$ rad.

(a) What is the amplitude of the wave?
(b) What is the wavelength?
(c) What is the period?
(d) What is the direction of the velocity?
(e) What is the magnitude of the velocity?
(f) Use a computer program such as Mathematica to plot the shape of the wave, i.e., $y(x)$, at time $t = 0$, and also at a time one fifth of a period later, on the same graph. Label the two plots. The wave at $t = 0.2$ period should be offset in the direction corresponding to your answer in (d).
(g) Verify that the peaks of the wave at $t = 0.2$ period have shifted by the amount predicted by your answer to (e). (One method would be to combine Mathematica’s FindRoot command with its derivative command, in order to find out where a specific peak is.)
14-5. (Paper only.) Consider a transverse traveling wave of the form:

\[ y(x, t) = \frac{1}{(x - 10t)^4 + 1} \]

(a) Is the wave moving in the \(+x\) or \(-x\) direction?
(b) Write an equation for a wave which is identical to this wave, but which is moving in the opposite direction.
(c) What is the wave’s velocity?
(d) What is the transverse velocity of a section of the medium located at \(x = 0\), at \(t = 0.05\) s?

Extra problems I recommend you work (not to be turned in):

- As we will study in a future unit, light is a wave. Lasers can generate waves which are almost perfectly sinusoidal. The wavelength of light from a certain laser pointer is 620 nm. The speed of light is \(2.9979 \times 10^8\) m/s. Find the (a) wavenumber, (b) frequency, (c) period, and (d) angular frequency of the light from this laser. (Answers: \(1.013 \times 10^7\) rad/m, \(4.835 \times 10^{14}\) oscillations/sec, \(2.068 \times 10^{-15}\) s, \(3.038 \times 10^{15}\) rad/sec.)

15-1. A phone cord is 4.89 m long. The cord has a mass of 0.212 kg. A transverse wave pulse is produced by plucking one end of the taut cord. That pulse makes four round trips (down and back) along the cord in \([01]\) s. What is the tension in the cord?

15-2. Imagine a clothesline stretched across your yard. It has a mass of 0.113 kg and a length of 6 m. When you flick the line, the pulse you generate travels down the line at a speed of \([02]\) m/s. When the pulse gets to the end, it is completely absorbed without reflection by the flexible pole it is tied to. If you stand near the other end of the line and wiggle it sinusoidally for one minute with an amplitude of 10 cm at a frequency of 3 Hz, how much energy will the flexible pole absorb?
15-3. (Paper only.) Two triangular shaped pulses are traveling down a string, as shown in the figure. The figure represents the state of the string at time \( t = 0 \). The pulse on the left is traveling to the right, and the pulse on the right is traveling to the left, as indicated by the arrows. The speed of waves on the string is 1 m/s. Draw the shape of the string at the following times: \( t = 2 \) s, \( t = 2.5 \) s, \( t = 3.5 \) s, and \( t = 5 \) s.

15-4. (Paper only.) Imagine your slinky stretched to a length \( L \) and fixed at both ends.

(a) Write the slinky’s tension \( T \) and linear mass density \( \mu \) in terms of the mass \( m \), spring constant \( k \), and length \( L \). Assume that the stretched length of the slinky is long enough compared to the length when it is not stretched that the unstretched length is negligible. What is the wave speed for transverse waves on a slinky in terms of \( m \), \( k \), and \( L \)?

(b) Have someone hold one end of your slinky (or attach it to something like a doorknob). Take the other end and stretch the slinky until it is about five feet long. Now strike one end of the slinky to make a transverse pulse and watch as the pulse travels to the other end and then reflects back. Time how long it takes for the pulse to go out and back 10 times, and use this to calculate the wave speed for transverse waves on the slinky.

(c) Now predict what the wave speed would be if the slinky were stretched to about 10 feet.

(d) Stretch the slinky until it is about 10 feet long and measure the wave speed the same way you did before. Compare your answer to your prediction in (c).
15-5. (Paper only.) (a) If a transverse pulse travels down your slinky and reflects off of the end which is being held fixed by a friend, will the reflected pulse look the same as the incoming pulse, or will it be inverted?
(b) Test our your prediction by having someone hold one end of your slinky (or attach it to something like a door knob) while you take the other end and pull it back until the slinky is stretched about 10 feet (don’t stretch it too far or it won’t slink back together again and the slinky will be ruined). Quickly strike the top of the slinky with your hand to make a transverse pulse. Watch carefully as the pulse reflects off of the fixed end. Did it match your prediction?
(c) Now hold one end of your slinky up high and let the other end dangle downward (don’t let it touch the floor). If you whack the end of the slinky to make a transverse pulse, what do you think will happen to the pulse when it reaches the bottom? Will it reflect? Will the reflection be inverted? I want an honest educated guess; you won’t lose points if your prediction is incorrect.
(d) Try it and see what happens. Did the dangling end of the slinky act as a free end, fixed end, or something else?

15-6. (Paper only.) Let’s find the equation that describes the longitudinal vibrations in a rod of solid material. If the waves passing through the rod have a small amplitude, we can imagine that the rod is made up of tiny slices of mass each connected to the pieces on either side of it by tiny massless springs, as shown in the figure below. $s(x,t)$ is the function that tells you how much each slice of mass is displaced from equilibrium.

![Diagram](image)

The equilibrium length of each of these springs is $dx$, and each of the thin masses will have a mass of $m = \rho V = \rho A \, dx$ where $A$ is the cross sectional area of our rod. We’ll label each slice of the rod with the location $x$ of its center when it is in equilibrium (i.e. when no waves are passing through and all of the springs are relaxed, neither stretched nor compressed).
(a) Find the spring constant of our little springs, \( k \), in terms of \( dx \) and the Young’s modulus of the material. Young’s modulus \( Y \) is defined by the following equation:

\[
Y = \frac{\text{stress}}{\text{strain}},
\]

where the stress is the force per cross sectional area that’s being applied, and the strain is the fractional change in length \( \Delta L/L \), that occurs in response. Hint: You can find the spring constant of the entire rod by re-writing the definition of Young’s modulus as

\[
F = (\text{spring constant}) \Delta L.
\]

The little springs are all acting in line with each other to give you that overall spring constant; the number of springs in the whole rod is \( L/dx \). Do you remember how the overall spring constant changes if you attach springs end-to-end?

(b) If we look at a piece at position \( x \) while a wave passes through, the spring to its left will be compressed by an amount \( s(x - dx, t) - s(x, t) \), resulting in a force of

\[
F_{\text{left}} = k (s(x - dx, t) - s(x, t)).
\]

What is the force on that piece exerted by the spring to its right? (Note that if the spring on the right is compressed it will result in a force in the negative direction—be sure to get your signs right!)

(c) Remember the definition of a derivative, as \( dx \) goes to zero:

\[
\frac{\partial f}{\partial x} \bigg|_{x=x_0} = \frac{f(x_0 + dx) - f(x_0)}{dx}
\]

where the vertical bar and the subscript \( x = x_0 \) mean that we are evaluating the derivative at a particular position, \( x_0 \). If you plug in the equation for \( k \) from part (a) and apply the definition above, you can write \( F_{\text{left}} \) and \( F_{\text{right}} \) in terms of

\[
\frac{\partial s}{\partial x} \bigg|_{x=x} \quad \text{and} \quad \frac{\partial s}{\partial x} \bigg|_{x=x-dx}
\]

So, do it now! Find \( F_{\text{left}} \) and \( F_{\text{right}} \) in terms of these derivatives.

(d) Now add these together to find the net force on the thin mass, and set it equal to \( ma \) in accordance with Newton’s Second Law. \( (m \) is the amount of mass in our “thin mass”\).

Show that this simply becomes the one-dimensional linear wave equation and determine what \( v \), the speed of longitudinal waves in our rod, is equal to. Hint: don’t forget that
$A\,dx$ is the volume occupied by each mass, so that $m/(A\,dx)$ is the density of the material.

**Extra problems I recommend you work (not to be turned in):**

- (a) If you hold one end of your slinky up high and let the other end dangle downward without touching the floor, how will the wave speed change as a function of the distance from the bottom of the slinky? Hint: Pick a point on the slinky a distance $x$ up from the bottom of the slinky, and draw a free-body diagram for that point. There’s some weight (but not all the weight) pulling down and some tension pulling up. That should give you tension as a function of distance. You already know how the wave speed depends on tension. (b) Use your answer to predict the time it would take for a transverse pulse to travel from the bottom of the slinky to the top. Hint: Doing this requires some calculus. You should have found the speed $dx/dt$ as a function of $x$. The best way to solve this equation is to bring all of the $x$ quantities to the left hand side, all of the $t$ quantities to the right hand side, and integrate both sides of the equation. (c) Now test your prediction by measuring how long it takes for a transverse pulse to travel up and back down the slinky 5 times.

- You are abducted by aliens and placed in a holding cell on an unknown planet. Due to your diligent study of the Starfleet Planetary Guide, you know that if you could determine $g$, the gravitational acceleration on the planet, you would be able to figure out where you are. So you pull a thread from your uniform which is 1.55 m long and which weighs 0.500 grams. You tie the end to your shoe, which weighs 0.21 kg. You then hold the top of the string with the shoe hanging at the bottom, and you pluck the string near the top. The pulse takes 0.112 seconds to travel down to the shoe. (a) What is the value of $g$ predicted by the wave speed? (b) To double-check your results, you now start the shoe oscillating back and forth. You time 5 periods in 68 s. What is the value of $g$ predicted by the motion of the pendulum? Ignore the length of the shoe. (Answers: $0.29 \, m/s^2$, $0.33 \, m/s^2$.)
A light string of mass 15.2 g and length $L = 3.23$ m has its ends tied to two walls that are separated by the distance $D = 2.41$ m. Two objects, each of mass $M = 2.03$ kg, are suspended from the string as in the figure. If a wave pulse is sent from point A, how long does it take to travel to point B? (Answer: 33 ms.)

(a) Consider the function $y = Ae^{(x-vt)^2/a^2}$ (where $A$, and $a$ are constants, and $v$ is the speed of waves on the string). Plug this into the linear wave equation and show that it is a solution. (b) Show that $y = A\sin(bxt)$ is not a solution to the wave equation (where $A$ and $b$ are constants). (c) By plugging things into the wave equation, show that if $y_A(x,t)$ and $y_B(x,t)$ are solutions to the wave equation, $y_A + 2.13y_B$ is also a solution.

16-1. (Paper only.) Note: many students have calculators that can do the following types of complex number problems automatically. However, I don’t want you to use your calculator’s complex number functions for these problems—instead, do them by hand (addition and subtraction can be done in rectangular form; multiplication and division should be done by converting to polar form).

(a) If $z_1 = 2 + 3i$ and $z_2 = 3 - 5i$, what is $z_1 + z_2$ (in both rectangular and polar form)? What is $z_1 \times z_2$ (in both rectangular and polar form)?

(b) If $z_1 = 1 - i$ and $z_2 = 3 + 4i$, what is $z_1 - z_2$ (in both rectangular and polar form)? What is $z_1 \div z_2$ (in both rectangular and polar form)?
16-2. (Paper only.) (a) Use Euler’s formula to create a table of real and imaginary parts of the
given complex numbers, such as the one below. (b) Plot each of these points in the
complex plane. Use graph paper or a computer.

<table>
<thead>
<tr>
<th>$\tilde{z}$</th>
<th>Re($\tilde{z}$)</th>
<th>Im($\tilde{z}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5e^{i0}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5e^{i\pi/8}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5e^{i\pi/4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5e^{i3\pi/8}$</td>
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<td>$5e^{i\pi/2}$</td>
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<td>$5e^{i3\pi/4}$</td>
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<tr>
<td>$5e^{i\pi}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5e^{i5\pi/4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$5e^{i3\pi/2}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16-3. (Paper only.) Pick two random cosine functions of the form $A \cos(\omega t + \phi)$. They should
have different amplitudes and different phases, but the same frequency. (a) Use a
computer program such as Mathematica to plot the two functions, and also their sum.
You should find that their sum is a cosine function with the same frequency, but with a
still-different amplitude and phase. (b) Add the functions together using the complex
exponential technique discussed in class. (c) Plot the function you obtain through that
technique, and show that it really is the same as the combined function you plotted in
part (a).

16-4. (Paper only.) Use Euler’s formula to prove that $\cos(a + b) = \cos(a) \cos(b) - \sin(a) \sin(b)$
and that $\sin(a + b) = \sin(a) \cos(b) + \cos(a) \sin(b)$. Hint: First note that $e^{i(a+b)} = e^{ia} \cdot e^{ib}$.
Then apply Euler’s formula to each of the exponentials. Finally, note that the real part
of the stuff on the left side of the equation must be equal to the real stuff on the right
side, and the imaginary stuff on the left must equal the imaginary stuff on the right. This
lets you separate your equation into two equations which will lead to the two equations
you are trying to prove.
16-5. (Paper only.) (a) The equation of motion for a simple harmonic oscillator is:

\[
\frac{d^2x}{dt^2} = -\frac{k}{m}x.
\]

That simply comes from Newton’s 2nd Law, \(\Sigma F = ma\), where I’ve plugged in the spring force, reversed the left and right hand sides, and divided by \(m\). Guess a solution of the form \(x(t) = A\cos(\omega t)\). Take two derivatives, plug the appropriate things into the equation of motion, and show that you can easily solve for what \(\omega\) is in terms of \(k\) and \(m\). You should get a very familiar result. (Note: this isn’t a complete solution, since, for example, there could be a phase shift in the cosine function.)

(b) As given in Physics phor Phynatics, the equation of motion for a damped harmonic oscillator is:

\[
\frac{d^2x}{dt^2} = -\frac{\gamma}{m}\frac{dx}{dt} - \frac{k}{m}x.
\]

The difference is the damping term, a non-conservative force that is proportional to the velocity and measured by the “damping constant” \(\gamma\). Guess a solution of the form \(x(t) = Ae^{-t/\tau}\cos(\omega t)\). That is a decaying cosine function; \(\tau\) is the characteristic time it takes for the decay to occur. Take derivatives, plug things into the equation, and show that although it’s a pain, you can solve for what \(\tau\) and \(\omega\) are in terms of \(k\), \(m\), and \(\gamma\). Hint: You will get an equation with various sine and cosine terms in it. The sine terms on the left side of the equation must be equal to the sine terms on the right side; same for the cosine terms. This lets you separate your equation into two equations which will let you solve for \(\tau\) and \(\omega\). Another hint: Your answer for \(\omega\) should be your answer to part (a), times a factor of the form \(\sqrt{1 - \text{stuff}}\).

(c) Now guess a solution of the form \(x(t) = Ae^{-t/\tau}e^{i\omega t}\), realizing that the real solution will only be the real part of that. Take derivatives, plug things into the equation, and show that you it is much easier to solve for \(\tau\) and \(\omega\) than in part (b). Hint: If you write \(x(t)\) as \(Ae^{t(-1/\tau + i\omega)}\), the time derivatives are almost trivial.

Extra problems I recommend you work (not to be turned in):

- Get someone to hold the other end of your slinky or attach it to something. Then whack the slinky to send a pulse down it. Right as that pulse is reflecting off of the far end, whack it again to make a second pulse. Watch as the two pulses collide. Is the approximation that your slinky is a linear medium a good one?
• Use the techniques/principles of complex numbers to write the following as simple phase-shifted cosine waves (i.e. find the amplitude and phase of the resultant cosine wave). (a) \(5 \cos(4t) + 5 \sin(4t)\). (b) \(3 \cos(5t) + 10 \sin(5t + 0.4)\). (Answers: 7.07, \(-45^\circ\); 6.96, \(-7.61^\circ\).)

17-1. A beam of light crossing a boundary between two media at normal incidence (i.e. perpendicular to the boundary) shares many features with waves on strings reflecting and transmitting from boundaries. Among those features is the dependence of reflection and transmission coefficients on wave speed. Suppose you have a light ray going from air into glass. Light travels at \(2.9979 \times 10^8\) m/s in the air and at \([01]\) ________ m/s in the glass. What percent of the incident light power will reflect off of the surface of the glass?

17-2. Imagine that you have a copper wire with a round cross section, 0.411 mm in diameter. You splice the end of that wire to another wire with the same cross section, but which is made of an unknown metal with density of \([02]\) ________ kg/m\(^3\). (Copper has a density of 8920 kg/m\(^3\).) You then pull on the joined wires until they are under a tension of \(T = [03]\) ________ N. (a) What is the ratio of the wave velocity on the copper wire to that on the unknown wire (i.e., what is \(v_{\text{copper}}/v_{\text{unknown}}\))? (b) What is the ratio of the wave numbers for the two wires (\(k_{\text{copper}}/k_{\text{unknown}}\)) for a sine wave with an angular frequency 500 rad/s? (c) If I send a sine wave down the copper wire, what fraction of the power in the incident sine wave is transmitted to the unknown wire?

17-3. If you splice a copper wire with a round cross section, \([04]\) ________ mm in diameter, to an iron wire with a different diameter, what should the diameter of the iron wire be if you don’t want waves to reflect at the junction when the wire is pulled tight? Copper has a density of 8920 kg/m\(^3\), and iron has a density of 7860 kg/m\(^3\).

17-4. (Paper only.) Imagine that I have a string which I can use to transmit waves. I make various measurements of the speed of sine waves and determine that they travel at a velocity given by \(v_{\text{sine}} = (0.637 \text{ m} \cdot \text{s})\omega^2\). (a) Is this velocity the group velocity, the phase velocity, or neither? (b) Find the dispersion relation \(\omega(k)\) for the string. If I have a pulse with an average \(k\) of 1.42 radians per meter, what will be (c) the pulse’s phase velocity and (d) its group velocity? (e) At what speed would I expect the center of that pulse to travel down the string?
17-5. (Paper only.) Consider the following two functions, which are just sums of sinusoidal waves having different amplitudes.

\[
y_1(x, t) = \sum_{n=1}^{100} e^{-n^2/1000} \cos (2\pi n(x - t))
\]

\[
y_2(x, t) = \sum_{n=1}^{100} e^{-n^2/1000} \cos (2\pi n(x - n^{0.25}t))
\]

Notice that the components making up the first function all have the same velocity \(v = 1\), whereas the components making up the second function have velocities which depend on their frequencies.

(a) Estimate the phase velocity and group velocity of function \(y_1\). Using a computer program such as Mathematica, plot the function from \(x = -0.1\) to \(x = 0.1\) at times \(t = 0, 0.01,\) and \(0.02\), on the same graph. (Forcing a vertical range of something like \(-20\) to \(30\) might be a good thing to do. In Mathematica that’s done via the PlotRange option for the Plot command.)

(b) Repeat, for function \(y_2\). (What does it mean to estimate the phase velocity for this function?) Do you see the wave disperse?

Extra problems I recommend you work (not to be turned in):

- Show that in the limit as \(\mu_2 \to 0\) or \(\mu_2 \to \infty\), our equations for the transmitted and reflected amplitudes and powers are consistent with what we deduced earlier for a string with a fixed or a free end.

18-1. The intensity level of an orchestra is \([01]\) _________ dB. A single violin achieves a level of 68.2 dB. How does the intensity of the sound of the full orchestra compare with that of the violin’s sound? (Find the ratio of the intensities.)

18-2. A family ice show is held at an enclosed arena. The skaters perform to music with level 81.7 dB. This is too loud for your baby who yells at \([02]\) _________ dB. (a) What total sound intensity engulfs you? (b) What is the combined sound level?
18-3. A stereo speaker (considered a small source) emits sound waves with a power output of $03$ W. (a) Find the intensity $10.5$ m from the source. (Assume that the sound is emitted uniformly in all directions from the speaker.) (b) Find the intensity level, in decibels, at this distance. (c) At what distance would you experience the sound at the threshold of pain, $120$ dB?

18-4. A firework explodes $04$ m directly above you. You record the explosion with a microphone and you find that the average intensity of the sound was $05$ dB and that the sound lasted for $2.32$ ms. How much was the total sound energy released by the explosion (in Joules)? Assume that the sound waves were spherical.

18-5. You notice a supersonic jet flying horizontally overhead before the sonic boom arrives. As the jet recedes from view, you judge that its position makes a $06$-degree angle with the horizon when you finally hear the sonic boom. What is the Mach number (i.e., the speed of the plane divided by the speed of sound)?

18-6. A jet airplane flies with a speed of $1120$ mph (mi/h) at an altitude of $07$ ft. It passes directly over my head. How soon after it passes directly above me will I hear the sonic boom? The speed of sound is $343$ m/s. (Caution: I am not asking how much time it took for the sound to travel from the airplane when it was directly overhead. The sonic boom originates from the airplane sometime before it reached the point directly overhead.)

18-7. (Paper only.) Imagine a slinky which has a total mass $m$, cross sectional area $A$, and a spring constant $k$. In a previous problem we modeled waves in a solid rod as lots of tiny masses connected by springs. That model works pretty well for slinkies, also. You should have found that the wave speed of longitudinal waves in a rod is:

$$v = \sqrt{\frac{Y}{\rho}},$$

where $\rho$ is the density and $Y$ is the “Young’s modulus”, defined by

$$Y = \frac{F_{\text{applied}}/A_{\text{cross-section}}}{\Delta L/L}.$$
If we write $F_{\text{applied}}$ as $\Delta F$ then we obtain:

$$Y = \frac{\Delta F}{A \Delta L/L} = \frac{L \Delta F}{A \Delta L} = \frac{L dF}{A dL} \text{ (for small } \Delta L)$$

Also, from Hooke’s Law, the force on a slinky is $F = k(L - L_0)$, where $L$ is the total length and $L_0$ is the unstretched length.

(a) Use all of that information to derive an expression for the longitudinal wave speed on a slinky in terms of $m$, $A$, $k$, and $L$.

(b) Specifically, how does the speed of compression waves change with $L$?

(c) Stretch your slinky until it is 5 feet long (hook it to something or have a friend hold the other end), and then whack the end to make a compression wave. Measure the time that it takes for the pulse to travel back and forth 10 times and calculate the speed of compression waves on your slinky.

(d) Now do the same thing with the slinky stretched to 10 feet. Does the speed of compression waves vary with length as you predicted?

(e) Compare the speed of longitudinal waves to the speed of transverse waves that you measured and predicted in an earlier assignment.

Extra problems I recommend you work (not to be turned in):

- Remember that the bulk modulus tells us how the volume of a piece of some material changes when the pressure surrounding it changes by an amount $\Delta P$:

$$B = -\frac{\Delta P}{\Delta V/V}$$

(a) The speed of sound in water is 1493 m/s. The density of water is 1000 kg/m$^3$. Based on that data, what is the bulk modulus for water? (b) The bulk modulus for a gas depends on how the change in pressure occurs—i.e. whether the gas is compressed adiabatically, isothermally, etc. Imagine that I compress some air in a piston to measure the bulk modulus of air. Which process will result in the largest measured bulk modulus, an adiabatic compression or an isothermal one? Hint: Qualitatively, how does the pressure change in an adiabatic process vs. an isothermal one, for a given change in volume? I recommend you draw a P-V diagram. (c) Which bulk modulus should I use for sound waves: adiabatic, isothermal, or neither? (Answer to part (a): $2.229 \times 10^9$ Pa.)
Two small speakers emit spherical sound waves of different frequencies. Speaker A has an output of 1.51 mW, and speaker B has an output of 2.09 mW. Determine the sound level (in decibels) at point C (see figure) if (a) only speaker A emits sound, (b) only speaker B emits sound, (c) both speakers emit sound. Assume that the two waves are incoherent so that intensities add. (Answers: 66.82 dB, 69.20 dB, 71.18 dB.)

I am sitting 2.37 m from a speaker listening to some music. How close to the speaker should I sit if I want the music to be 14.3 dB louder? (Answer: 0.46 m.)

19-1. A bat flying at \(01\) \(\text{m/s}\) emits a chirp at 40.95 kHz. If this sound pulse is reflected by a wall, what is the frequency of the echo received by the bat? (\text{Hint: This is exactly the same as the situation where the source and observer are both moving towards each other.})\) Use 345 m/s for the speed of sound.

19-2. A train is moving close to and parallel to a highway with a constant speed of 22 m/s. A car is traveling in the same direction as the train with a speed of \(02\) \(\text{m/s}\). The car horn sounds at a frequency of 519 Hz, and the train whistle sounds at a frequency of 327 Hz. (a) When the car is well behind the train, what frequency does an occupant of the car observe for the train whistle? (b) When the car is well in front of the train, what frequency does a train passenger observe for the car horn after the car passes? Assume that the speed of sound is 343 m/s. There is no wind.

19-3. One day as I stepped out onto the street, I was very nearly hit by a car. Fortunately, the car honked its horn and I jumped out of the way. But I noticed that the frequency of the horn as the car approached me was a factor \(03\) \(\text{higher}\) than its frequency after the car passed me and was moving away from me. Calculate the velocity of the car. The speed of sound is 343 m/s.
19-4. When we analyze the light coming from a distant galaxy, we find a particular absorption line with a wavelength of \[04\] nm. This same absorption line in light from the sun has a wavelength of 625 nm. (a) Is the galaxy moving towards us or away from us? (b) Calculate the magnitude of the velocity of the galaxy relative to us. Note that for light waves, the Doppler shift is given by

\[ f' = f \sqrt{\frac{c \pm v}{c \mp v}}, \]

where \( c \) is the speed of light and \( v \) is the relative velocity of the source and observer. Use the upper signs when the source and observer are moving towards each other and the lower sign when the source and observer are moving away from each other.

19-5. A pair of speakers separated by \[05\] m are driven by the same oscillator at a frequency of 690 Hz. An observer, originally positioned at one of the speakers, begins to walk along a line perpendicular to the line joining the two speakers. (a) How far must the observer walk before reaching a relative maximum in intensity? (b) How far will the observer be from the speaker when the first relative minimum is detected in the intensity? (Take the speed of sound to be 345 m/s.)

19-6. Three senile old men are sitting on a park bench. One of them whistles for his dog (which has been dead for years) at a frequency of 1330 Hz. A long way from the bench, the sound waves from his whistling can be described by the equation

\[ \Delta P_1 = A_1 \cos(kx - \omega t + \phi_1), \]

where \( A_1 = [06] \) Pa and \( \phi_1 = 130^\circ \). (a) What is \( \omega \)? (b) Assuming that the speed of sound is 343 m/s, what is the wave vector \( k \) of the sound wave? (c) The second and third old men join in, whistling at the same frequency but with amplitudes and phases of \( A_2 = [07] \) Pa, \( \phi_2 = 225^\circ \), \( A_3 = [08] \) Pa, and \( \phi_3 = 40^\circ \). What is the amplitude of the resultant wave when all three are whistling? (d) What is the phase of the resultant wave? (e) What is the frequency of the resultant wave (in Hz)? Hint: Use complex number techniques. Be careful about the quadrant if you use an inverse tangent function on your calculator. For example, if you want to find the phase angle of \(-2 + 3i\), you can’t just take \(\tan^{-1}(-3/2)\), because your calculator will tell you the answer is \(-56.31^\circ\) when it should be \(123.69^\circ\).
Extra problems I recommend you work (not to be turned in):

- Imagine a lake on which water waves travel at a velocity \( v \). You and a friend are using magic shoes to hover motionless over the lake. Your friend, some distance away from you, starts dropping pennies into the lake at a frequency \( f \). Each penny makes a ripple which travels toward you, passing you at a frequency \( f \). (a) In terms of \( v \) and \( f \), what is the distance \( L \) between the ripples? (b) If your friend starts moving toward you with a velocity \( v_s \), how far apart will the ripples be? (c) Now, with your friend still moving across the lake toward you, you start to move toward him at a speed of \( v_o \) (relative to the lake). At what frequency will the waves pass you? You’ve now derived the equation for Doppler shifts.

- On a very quiet morning, you drop a tuning fork vibrating at 512 Hz from a tall bridge. How long until you will hear a frequency of 475 Hz? Take the speed of sound in air to be 343 m/s and the acceleration of gravity to be 9.80 m/s\(^2\). Hint: Don’t forget to include the time it takes for the sound to return to the point of release. (Answer: 2.832 s.)

- Two speakers emitting 651 Hz are separated by a distance of 2.0 m. A student is positioned directly in front of the first speaker and along a path 90° from the line that joins the two speakers. As she walks directly toward the first speaker she notices minima in the sound level. (a) How many minima does she experience if she begins from far away? (b) How far is she from the speaker for the first minimum she encounters? Use 343 m/s for the speed of sound. (Answers: 3, 3.5325 m.)
20-1. In the arrangement shown in the figure, an object of mass \( m = [01] \) ________ kg hangs from a cord around a light pulley. The length of the cord between point P and the pulley is \( L = 2.0 \) m. When the vibrator is set to a frequency of 150 Hz, a standing wave with six loops is formed. What must be the linear mass density of the cord?

![Diagram of vibrator and pulley system with length L and mass m]

20-2. A steel piano string is [02] ________ inches long. The diameter of the string is 0.0421 inches. When struck, this string produces the musical note B which has the pitch of 123 Hz. (a) Find the tension of this string. Give the answer in pounds. The density of steel is 7.86 g/cm\(^3\). (b) There are 228 strings in a piano. (Some notes use more than one string.) If we assume that the tension in every string is the same, find the total force (in tons, 1 ton = 2000 lb) exerted on the frame of the piano. (Add together the tensions of all of the strings.) Piano frames are made of steel so that they can withstand this kind of force.

20-3. A pipe open at both ends has a fundamental frequency of [03] ________ Hz when the temperature is 0\(^\circ\)C. (a) What is the length of the pipe? (b) What is the fundamental frequency at a temperature of 30\(^\circ\)C? Assume that the displacement antinodes occur exactly at the ends of the pipe. Neglect thermal expansion of the pipe.

20-4. A pipe is open at both ends. Its length is 22.81 cm. If I excite standing waves in the pipe by blowing air over one of its ends, I hear a pitch of [04] ________ Hz. (a) Find the distance between the antinodes at the two ends of the pipe. This is not the same as the actual length of the pipe, since the antinodes are not exactly at the ends of the pipe. The speed of sound is 343 m/s. (b) How far past the physical ends of the pipe do the antinodes extend? Assume that it is the same amount for each antinode. (c) If I close one end of the pipe, what pitch will I hear? The node at the closed end is exactly at the position of the closed end. The antinode at the open end extends past the physical end of the pipe by the same amount found in part (b).
20-5. (Paper only.) The high E string on Dr. Durfee’s guitar has a linear mass density of $3.93 \times 10^{-4}$ kg/m (he calculated it from information found at the string manufacturer’s web site) and a length of 25.5 inches (64.77 cm). The frequency of the fundamental mode of this string is 330 Hz. (This mode is also known as the “first harmonic”.) (a) What is the tension in the string? (b) When the string is plucked, a whole bunch of modes are excited. If the string is then touched right in the middle, all of the modes will be damped out except the ones that have a node at that point. After touching the string in the middle, what is the frequency of the lowest frequency mode which is still ringing? (c) What if the string is touched at a point $2/3$ of the way from one end to the other?

20-6. (Paper only.) Transverse standing waves on a slinky:

The picture represents a transverse wave on a slinky oscillating in the fundamental mode. The distance along the slinky is in the $x$-direction; because the wave is transverse, the displacement from equilibrium is in the $y$-direction.

(a) Make a sketch of the displacement vectors of this mode, at the time where the displacement is maximum, for seven evenly-spaced positions along the slinky. Make a similar sketch for the second, third, and fourth harmonics: the displacement as a function of position, and displacement vectors at seven evenly-spaced positions.

(b) Stretch your slinky to a fixed length feet of $5 - 10$ feet. Either measure the wave velocity for this length, or else go back to your notes from the problem in HW 15 where you measured the velocity and predicted the dependence as a function of length.

(c) Use the measured velocity along with the known wavelengths of the first four harmonics to predict the frequency of oscillation of the first two harmonics.

(d) Test it out: verify your sketches in part (a), and measure the frequencies of the first four harmonics. Do this by measuring about 10 oscillations and dividing the time for the ten oscillations by 10 to get the period for a single oscillation with greater accuracy. Then use the period to find the frequency. Compare with your predicted values from part (c).
20-7. (Paper only.) Longitudinal standing waves on a slinky: The following picture represents a longitudinal wave on a slinky oscillating in the fundamental mode. The distance along the slinky is in the \(x\)-direction; because the wave is longitudinal, the displacement from equilibrium is also in the \(x\)-direction.

(a) Make a sketch of the displacement vectors of this mode, at the time where the displacement is maximum, for seven evenly-spaced positions along the slinky. Make a similar sketch for the second, third, and fourth harmonics: the displacement as a function of position, and displacement vectors at seven evenly-spaced positions. Be careful with directions.

(b) Stretch your slinky to the same length as the last problem. Either measure the wave speed of longitudinal waves, or else refer to your notes from HW 18 as to how the longitudinal wave speed of a slinky compares to the transverse wave speed.

(c) Use the measured velocity along with the known wavelengths of the first four harmonics to predict the frequency of oscillation of the first two harmonics.

(d) Test it out: verify your sketches in part (a), and measure the frequencies of the first four harmonics. (The higher ones may be tricky; if too difficult, just do the first two harmonics.) Compare with your predicted values from part (c).

20-8. (Paper only.) Dr. Durfee recounts the following: One evening I took a tall glass from my cupboard and measured the frequency of the fundamental mode of the air in the glass. I did this by whacking the bottom of the glass while my wife held a microphone above the glass. The microphone was connected to my computer, and I measured the frequency using “Spectrum Lab”, a free program which you can download from the class web page. The inside of the glass is a cylinder which is 7 cm in diameter and 14.3 cm tall. The lowest frequency I measured when I whacked the glass was 570.8 Hz.

(a) If the glass were very narrow (i.e. if the diameter were much smaller than the height) then the waves would propagate almost as if they were one-dimensional. Otherwise you need to use a three-dimensional wave theory to get precise results. Assuming that the waves in the glass are one dimensional, calculate the velocity of sound using the height of the glass and the frequency of the fundamental mode. (Note: Although this assumption
is not really very good in this case, you should still get an answer which is within 5% of the “expected” answer of 343 m/s. This is due to the fact that the wideness of the glass introduces two errors which partially cancel each other. First, the fact that the waves propagate in the glass three-dimensionally means that the wavenumber $k$ is really the sum of three components $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$, resulting in a wavelength for the fundamental mode which is shorter than it would be if the diameter of the glass were smaller. Second, the wide mouth of the glass introduces an “edge effect”. Because the mouth is so wide, the oscillating wave pokes out of the cup and the pressure just outside the cup is not fixed at atmospheric pressure. This effectively increases the length of the cup, thereby increasing the wavelength of the fundamental mode.)

(b) I then filled the glass with water and found the frequency of the first harmonic to be 3168 Hz. Use this frequency to calculate the speed of sound in water. (Note: This answer won’t turn out as nicely. You still have the shorter wavelength due to the three-dimensional nature of the oscillation. But the “edge effect” is gone because there is no water outside of the glass: because the air above the glass has such a different wave speed than the water in the glass, the oscillation doesn’t penetrate out of the glass very far. Still, your answer should be within 25% of the expected value of 1480 m/s.)

(c) Then I put milk into the glass and measured a frequency of 3100 Hz. What is the speed of sound in milk?

(d) I then shook up the milk and found that the frequency of the first harmonic was cut in half. However, over the course of a few seconds the frequency drifted back up. This is because microscopic air bubbles in the milk decrease the density of the milk (air is less dense than milk) and decrease the bulk modulus (air is more compressible than milk). Which changed by a bigger factor, the density or the bulk modulus?

(e) (Optional.) This is a fun thing that you ought to try. You can do it with water and hear the pitch go down after you shake it. But water releases its air bubbles rather fast. It is easier in milk because the fat in milk increases the viscosity and holds onto the bubbles longer. And it works a lot better if you add ice cream. Make yourself a milkshake in the blender, pour it into a rigid cup (one made of glass works best), and then hit the bottom of the glass. You should hear a very deep, low frequency “thunk”.

Extra problems I recommend you work (not to be turned in):

• Suppose we excite a two-loop standing wave in a rope using a tension of 1.5 N. What tension should we apply to the rope if we want to excite a one-loop standing wave with the same frequency? (Answer: 6 N.)

• Two sine waves traveling down a string interfere to create a standing wave. The displacement of the string is given by \( y(x,t) = (3.21 \text{ cm}) \sin(0.342 \text{ rad/m} \cdot x) \cos(32.2 \text{ rad/s} \cdot t). \)
(a) What is the amplitude of each of the two interfering waves? (b) What is the wavelength of each of the two interfering waves? (c) What is the speed at which the two interfering waves are traveling? (Answers: 1.605 cm, 18.37 m, 94.15 m/s.)

• (a) Find the speed of sound in helium gas at 38.5°C. (b) If an organ pipe produces a tone (pitch or fundamental frequency of the pipe) with frequency 484 Hz in air at room temperature, find the frequency of its tone in helium gas at 38.5°C. The speed of sound in air at room temperature is 343 m/s. (Answers: 1038 m/s, 1465 Hz.)

• Two speakers emit sound waves with frequency 584 Hz. They are driven by the same oscillator so that they are in phase with each other. We place the speakers so that they are a few meters apart and facing each other. Along the line joining the two speakers, the sound waves from the two speakers are traveling in opposite directions. This creates a standing wave between the two speakers. How far apart are the antinodes in that standing wave? Neglect the effect of reflection of waves from the speakers. The speed of sound is 343 m/s. (Answer: 29.4 cm.)

• Download and install Spectrum Lab on a computer with a microphone. Measure resonances in a glass, yourself. Compare your measured values to Dr. Durfee’s.

21-1. Two identical mandolin strings under 205.6 N of tension are sounding tones with fundamental frequencies of 523 Hz. The peg of one string slips slightly, and the tension in it drops to [01] ________ N. How many beats per second are heard?
21-2. While attempting to tune the note C at 523.0 Hz, a piano tuner hears 2 beats/s between a reference oscillator and the string. (a) When she tightens the string slightly, the beats frequency she hears rises smoothly to [02] _________ beats/s. What is the frequency of the string now? (b) By what percentage should the piano tuner now change the tension in the string to bring it into tune?

21-3. A speaker at the front of a room and an identical speaker at the rear of the room are being driven at 456 Hz by the same sound source. A student walks at a uniform rate of [03] _________ m/s away from one speaker and toward the other. How many beats does the student hear per second? (Take the speed of sound to be 345 m/s.)

21-4. We place a speaker near the top of a drinking glass. The speaker emits sound waves with a frequency of [04] _________ kHz. The glass is 14.1 cm deep. As I pour water into the glass, I find that at certain levels the sound is enhanced due to the excitation of standing sound waves in the air inside the glass. Find the minimum depth of water at which this occurs (distance from surface of water to bottom of glass). The standing sound wave has a node at the surface of the water and an antinode at the top of the glass. Assume that the antinode is exactly at the top of the glass. The speed of sound in air is 343 m/s.

21-5. Suppose that your shower stall is [05] _________ m tall. (a) As you sing in the shower, how many frequencies in the range 300–1500 Hz will resonate? Ignore side-to-side sound waves and take the shower stall to be closed at both ends. Use 343 m/s for the speed of sound. (b) What is the lowest resonant frequency in that range? (c) What is its harmonic number? (d) What is the highest resonant frequency in that range? (e) What is its harmonic number?

21-6. (Paper only.) Find a piano. A real one with strings, not an electronic keyboard.
(a) Gently push down middle C lightly enough that no sound is made. Keep holding it down. Now strike C an octave above. Let go of the higher note while still holding down middle C. Can you still hear the higher note? That’s because the second harmonic of the middle C strings has nearly the same resonance frequency as the fundamental of the C an octave higher. This means that the middle C strings can absorb energy at this frequency and begin to oscillate. (b) Now try holding down middle C and playing different notes to see which ones share harmonics with middle C. Report on your findings.
Extra problems I recommend you work (not to be turned in):

• Download the program “Spectrum Lab” from the class web page, if you haven’t already. Install it and run it. Click on “View/Windows” and then on “Spectrum Lab Components.” In the top left-hand corner of the window that opens is a box called “Signal Generator.” Make sure that the switch below it points to the right. Make sure the “Mono” box on the far right hand side is green and set for “DAC”. (Click it once if it’s set to “(off)”.) Now click on “View/Windows” and open the “Test Signal Generator”. You can use this new window to make different tones. (a) Turn it on and generate two sine waves (with no AM or FM) at 440 and 441 Hz. Listen to the beats. (b) Play around with the program on your own. Try combining many different frequencies. What happens when you combine frequencies that are closer? Farther away? That are multiples of each other? Can you simulate the beat effect by using a single frequency but with amplitude modulation on? Etc.

• The wavelength of one sound wave is 0.81 m. The wavelength of a second sound wave is a little bit longer. When the two sound waves are superimposed on each other, we hear a 2.3 Hz beat. Find the difference in their wavelengths. The speed of sound is 343 m/s. (Answer: 4.4 mm.)
22-1. (Paper only) At a particular time, a wave has the shape shown in the figure \((y = 0.5x\) from \(0 \leq x \leq 2\), repeated). You might get this type of wave, albeit not repeated, by (for example) plucking a guitar string very close to the right end.

(a) Calculate the Fourier coefficients, and prove that this wave can be represented by:

\[
y(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin(n\pi x)
\]

(b) Give a brief argument through symmetry as to why all of the \(b_n\) terms except the DC offset were zero. (If you want to save yourself a little bit of work, you can do this part before part (a).)

(c) Verify that that function reproduces the above graph, by using a program such as Mathematica to plot the function with the summation having 1, 10, and 100 terms. (The Sum command in Mathematica should make this easy.)

22-2. (Paper only.) We have an equation for the time-averaged power carried by a sine wave traveling down a string. For a more complicated wave, the power is just the sum of the powers carried by each of the sine waves that make it up. Show that the square wave whose Fourier coefficients were solved in the text (see PpP equation 6.18) carries infinite time-averaged power. (This is one reason that you can’t ever make a true square wave on a string, you can only make an approximate one.)
22-3. (Paper only.) The power that comes out of the electrical outlets in your home is AC or “alternating current” power, characterized by a voltage that oscillates sinusoidally. Conversely, most electronic devices require a constant voltage (known as DC or “direct current” power). One way to generate DC power from AC power is to use a device called a diode to “kill” the negative part of the sine wave, resulting in a wave like the one shown below. Then the wave is filtered to keep only the DC component (i.e. the $b_0$ term of your Fourier series). The wave shown below is known as a “half-wave rectified” wave (“rectified” because it is only positive, and “half” because only half of the wave is left). If I wanted to make a 5 V DC power supply this way, what should the amplitude of the oscillating sine wave be before it is rectified?

![Graph of half-wave rectified wave](image)

22-4. (Paper only.) Some guitar amplifiers produce a heavy distortion of the signal from the guitar by applying the same type of half-wave rectification discussed in the previous problem. This type of distortion is very popular in heavy metal music. For simplicity let’s imagine that a guitar is generating a sine wave of the form $V_{\text{guitar}} = V_0 \cos(\omega t)$. This signal is then rectified to make a wave like the figure in the previous problem. Give all of your answers in terms of $V_0$ and $\omega$.

(a) What is $b_0$ for the rectified signal?

(b) Give a brief argument through symmetry as to why all of the $a_n$ terms should be zero.

(c) Find $b_1$. Hint: Instead of integrating from 0 to $T$, integrate from $-T/2$ to $T/2$, then use the following integral:

$$\int_{-\pi/2}^{\pi/2} \cos^2(u) \, du = \frac{\pi}{2}$$

By the way, that integral follows easily from a fact that you should memorize if you haven’t yet: the average value of $\cos^2 x$ (and $\sin^2 x$, for that matter) is $\frac{1}{2}$. Then, the area over one period is just the average value, times the period. (Hopefully you know that the period of $\cos^2 x$ and $\sin^2 x$ is $\pi$, not $2\pi$.)
(d) Plug the wave into the appropriate Fourier integral and show that all of the other odd \( b_n \) terms are zero. Find a general expression for the even \( b_n \) terms. You may need the following integral:

\[
\int_{-a}^{a} \cos(px) \cos(qx) \, dx = \frac{\sin((p-q)a)}{p-q} + \frac{\sin((p+q)a)}{p+q}
\]

Also note that \( \sin(m\pi/2) \) is equal to zero if \( m \) is even, and is equal to \((-1)^{(m-1)/2}\) if \( m \) is odd. To make the grader’s life easier, please write your answer as something times \((-1)^{n/2}/(n^2 - 1)\).

**Extra problems I recommend you work (not to be turned in):**

- Imagine that we send the rectified signal from the last problem to a speaker. The power delivered to a speaker by a constant voltage \( V \) (the \( b_0 \) term) is \( P = V^2/R \), and the time-averaged power delivered by a sine wave is equal to \( P = V_0^2/2R \), where \( R \) is the impedance of the speaker (which has units of Ohms) and \( V_0 \) is the amplitude of the sine wave (which has units of Volts; one Volt squared per Ohm is a Watt). Make plots of the magnitude of the amplitude \( \sqrt{a_n^2 + b_n^2} \) and the power spectrum of the Fourier coefficients of the rectified wave. Use \( V_0 = 0.5 \) V, the frequency of the wave generated by the guitar = 1 kHz, and \( R = 8 \) Ohms (a typical value for a speaker).

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23-1. (Paper only.) A string of length \( L \) is fixed at both ends. At time \( t = 0 \) the string is stretched into the shape shown in the figure.

![String Shape](image)

Mathematically, the shape of the string is given by the equation:

\[
y(x) = \begin{cases} 
0, & \text{if } 0 < x < \frac{7}{16}L \\
h, & \text{if } \frac{7}{16}L < x < \frac{9}{16}L \\
0, & \text{if } x > \frac{9}{16}L
\end{cases}
\]
We want to use a Fourier transform to write $y(x)$ as a sum of the harmonic modes of the string. (These are sine functions only.)

(a) If we want to perform a Fourier transform to write $y(x)$ in terms of the harmonic modes of the string, what should be the size and shape of the basic repeating unit? (Hint: if you just repeat the given shape, the periodic function will be even and you will have cosine terms only.)

(b) Calculate $b_0$. If it equals zero due to a symmetry argument, state the argument.

(c) Calculate $a_n$. If it equals zero due to a symmetry argument, state the argument.

(d) Calculate $b_n$. If it equals zero due to a symmetry argument, state the argument.

(e) Write $y(x)$ as a sum of harmonic modes on the string (similar to PpP equation 6.23, but with the parameters you calculated above inserted for $A_n$).

(f) Verify that the function you obtained reproduces the above graph, by using a program such as Mathematica to plot the function with the summation having 1, 10, and 100 terms. Use $h = L = 1$, and force the y-axis to display from $−0.2$ to $1.2$.

23-2. (Paper only.) Several of the BYU physics department faculty members have Ti:sapphire lasers (pronounced “tye-saff”; short for “titanium-doped sapphire”, the active medium) which produce pulses of light as short as about 25 femtoseconds long ($1 \text{ fs} = 10^{-15}$ seconds). Use the uncertainty principle to estimate the minimum bandwidth that the laser gain medium needs to have to produce such pulses. The bandwidth is the maximum frequency of light that it will amplify minus the minimum frequency it will amplify, labeled $\Delta f$.

23-3. (Paper only.) Find a piano. A grand piano would be best. Push down the sustain pedal, then sing a note into the strings. What do you hear after you stop singing? The piano is doing a Fourier transform of your voice. Each string only resonates with certain frequencies. Those strings which have a harmonic at the same frequency as one of the frequencies present in the note you sang will absorb sound and begin to oscillate, largely reproducing the sound of your voice.

23-4. (Paper only.) Find a piano. A grand piano would be best. Push down the sustain pedal, then clap your hands near the strings. Listen carefully to see which is the highest note excited by your clap. It may take several tries before you are convinced you have identified the correct note. Determine the frequency of that note. Now use the uncertainty principle to estimate the duration of your clap.
24-1. (Paper only.) I found a few websites that say the lowest note a piccolo can play is d\textsuperscript{2}, the D that is an octave and a note above middle C. The highest note a piccolo can play is c\textsuperscript{5}, the note that is four octaves above middle C. (a) What is the frequency ratio of these two notes? (b) What is the wavelength of the d\textsuperscript{2} note? Assume the speed of sound is 343 m/s. (c) Does that wavelength make sense with this homework problem I found in Serway, that states: “The overall length of a piccolo is 32.0 cm. The resonating air column vibrates as in a pipe open at both ends.” Why/why not? (d) When the piccolo is playing its highest note, what is the distance between adjacent antinodes?

24-2. (Paper only.) While you are practicing the piano, a car races past outside. You notice that as the car approached, the engine made a noise which was exactly in tune with a middle A (440.0 Hz). After the car passed, the pitch dropped down two half steps to a G. How fast was the car going? Assume that the piano is tuned to an equal temperament scale and that the speed of sound is 343 m/s.

24-3. (Paper only.) Guitar players often tune their instrument using harmonics. Imagine that I use an electronic tuner to tune my low E string to 164.814 Hz, precisely the correct frequency for an equal temperament scale referenced to a frequency for middle A of 440.000 Hz. Now I tune my next string, the A string which is 5 half-steps higher in frequency, using harmonics. Since 5 half-steps is a musical fourth, which corresponds to a frequency ratio of 4/3, I can do this by lightly touching the E string 1/4 of the way from the end of the string and lightly touching the A string 1/3 of the way from the end of the string. (a) Why is that? (b) I then play both strings and adjust the A string to make the beat frequency as low as possible. If I tune the beats completely away, how far off (in Hz) will the frequency of the A string be from the ideal frequency for this string according to the equal temperament scale?

24-4. (Paper only.) Find a piano. Push lightly on one of the low notes such that it doesn’t make a sound. While holding down that key, loudly strike the note which is one octave higher. You should now hear the lower strings oscillating at the fundamental frequency of the higher strings. (You already observed that in a previous assignment.) Now lightly strike and hold the higher note again. Chances are if you listen carefully, you will be able to hear beats. (a) Why? (b) How many beats did you hear, and what specifically does that tell you? (If you don’t hear any beats, try a different set of notes.)
Extra problems I recommend you work (not to be turned in):

• The lowest string on a 6-string guitar is usually tuned to an E at a frequency of 82 Hz (not including the decimal places). (a) If this E is referenced to an equal temperament scale for which middle A is exactly 440 Hz, give the frequency of this E to four decimal places. (b) Sometimes guitar players will loosen the tension in this string to drop it down two half-steps to a D to hit lower notes in a particular song. This is known as “drop D tuning”. What is the frequency of the D just below the E you found above? (to four decimal places) (c) By how much do you need to reduce the tension in the string to go from the E down to the D? (Answers: 82.4069 Hz, 73.4162 Hz, reduce by 20.6%).

25-1. A ray of light passes through a pane of glass which is 1.0 cm thick. The index of refraction of the glass is 1.53. The angle between the normal to the surface of the pane and the ray in the air as it enters the pane is [01] ________°. (a) Find the angle between the normal to the surface of the pane and the ray inside the glass. (b) Find the angle between the normal to the surface of the pane and the ray in the air after it exits the pane.

25-2. Light passes through a glass prism, as shown in the figure.
The cross-section of the prism is an equilateral triangle. (a) Find the incident angle, if we want the light ray inside the prism to be parallel to the base of the prism. Use [02] ________ for the index of refraction of glass. Remember that the incident angle is measured with respect to a line normal to the surface of the prism. You may use $n = 1$ for the index of refraction of air. (b) The index of refraction of glass for blue light is 1.528. Using the incident angle from part (a), find the angle at which blue light exits the prism. This is not the angle of deviation $\delta$ shown in the figure. We want the angle between the light and a line normal to the surface from which the light exits. (c) Repeat part (b) for red light, for which the index of refraction is 1.511. Caution: In parts (b) and (c), the light ray inside the prism is no longer parallel to the base of the prism.
25-3. (Paper only.) An optics researcher sets up two mirrors as shown by the black lines in the figure below. She shines a laser towards the mirrors along the path marked by the gray arrows. Find the angle between the incoming and outgoing beams of light, $\phi$, in terms of the angle between the mirrors, $\theta$. Perhaps surprisingly, the angle $\phi$ does not depend on how the two mirrors are oriented relative to the incident light ray. Hint: There are likely many ways to do this problem. The way I chose, was to call the incident angle $x$ (relative to the surface, not relative to the normal). Then I worked out all of the other angles in the picture in terms of $x$. In the end, when I solved for $\phi$, all of the $x$’s canceled out.

25-4. (Paper only.) When the sun heats a hot desert, the air near the ground heats up and becomes less dense than the air above it, such that the density of the air increases with the distance from the ground. Explain why this creates a mirage.

Extra problems I recommend you work (not to be turned in):

- Some rooms (such as many sealing rooms in LDS temples) have two parallel mirrors facing each other, so that when people look in the mirrors they see images of themselves “forever”. Estimate sizes and positions of the mirrors in a typical configuration. Based on the angle that you need to use in order to look past the side of your head, about how many images, at most, will you actually be able to see?

- The speed of sound in air is 343 m/s. In water it is 1480 m/s. If a directed sound wave in air strikes the surface of a lake at an angle of 23° from the normal, at what angle from the normal will the transmitted sound wave travel in the water? (You would likely need an array of speakers, properly phased relative to each other, to produce such a sound wave. As we have discussed, sound waves typically travel outwards in all three dimensions rather than going in a specific direction like a laser beam.)
26-1. You are a fish in deep, dark water with index 1.33. As you look up from
[01] ________ m below the smooth surface, you see a bright circle through which light enters from the outside world.
(a) What is the radius of the circle?
(b) Will you be able to see a fisherman 2.00 m tall standing at the water’s edge 4.00 m away?

26-2. A rectangular block of clear plastic is sitting on the ground. A beam of light traveling horizontally enters one face of the ice at an angle \( \theta \) from the normal. The transmitted beam then strikes the front face of the block, as shown in the figure below. What is the maximum angle \( \theta \) which will result in total internal reflection off of the top surface? The index of refraction for the plastic is [02] ________.

26-3. (Paper only.) Fermat’s Principle of Least Time. Fermat realized that if you imagine all possible paths that light rays could take between the source and the destination, the actual path is the one that takes the least amount of time. It’s kind of like this situation: suppose you are a lifeguard at position A below and must rescue a drowning swimmer at position B? What path should you take to get there the quickest? Let’s prove that the fastest time is given by the path predicted by Snell’s Law. Consider a light ray traveling from point A to point B, from a lower index of refraction \( n_1 \) to a higher index of refraction \( n_2 \). Find the time it takes for a hypothetical ray to travel from A to B when it enters the \( n_2 \) medium an arbitrary distance \( x \) from the left, in terms of the \( n \)'s and \( c \). Then, to find the \( x \) that produces the minimum time, take the derivative and set it equal to zero. Show that this gives you Snell’s Law.
Huygen’s Principle. Huygen’s principle says that each point of an advancing wave front can be considered as a point source of new circular (spherical, really) waves. The new waves add up to propagate the wave front. It’s often used to describe/explain diffraction through a slits, but it can be also used to describe/explain refraction.

In the Wikipedia article on Huygen’s principle, this picture is provided in order to graphically illustrate refraction. Stare at the picture until you can visualize that the green lines tangent to the circles (the parallel lines at the bottom of the picture) connect “matching” wavefronts. That is, if you label the wavefronts 1, 2, 3, etc., from the top of the picture on down, the first green line is tangent to all of the waves originating from wavefront #11, the second green line would be tangent to the waves from wavefront #12 (if those were drawn in), and so forth.

I want you to produce the same sort of picture, being as precise as you can with rulers/compasses/etc, to show that the graphical prediction of refracted angle from the Huygens’ principle picture matches the numerical prediction from Snell’s Law for at least one incident angle (you pick the angle). Draw an interface between an \( n = 1 \) and an \( n = 2 \) material. Draw the wavefronts of a wave hitting the interface at an angle. Just like the Wikipedia picture, treat each point where the wavefronts strike the interface as the source of circular waves propagating into the \( n = 2 \) material. Key: the wavelength of the circular waves (distance between wavefronts) must be exactly half the wavelength of the incident light because \( \lambda \) is reduced in the material by a factor of \( n \). Draw many circular waves going into the \( n = 2 \) material from at least four point sources and connect the matching wavefronts by drawing tangent lines like the green lines in the Wikipedia picture. Then measure the incident angle and the refracted angle and (hopefully!) prove that the refracted angle constructed this way is just the same as what Snell’s law would predict.
26-5. (Paper only.) (a) A swimmer is a distance $h$ under smooth water, a distance $L$ from shore, and is named Jane. Is it possible for light from the swimmer to reach the eyes of her boyfriend on the shore (i.e., can he see her?), or will this be prevented by total internal reflection? Explain why or why not. (Yes, you have been given all of the information you need.) (b) In light of your answer to this problem, why is it often difficult to see someone who is swimming under water? (c) If the boyfriend cruelly shines a powerful laser at the water, will he always be able to hit her with the light (assuming he has excellent aim), or will this be prevented somehow? Explain why or why not.

**Extra problems I recommend you work (not to be turned in):**

- A particular optical fiber is made from a glass core which is 2 microns in diameter with an index of refraction of 1.7, covered by a thin “cladding” (an outer shell made out of a different material) with an index of refraction of 1.5. Calculate the radius of the smallest cylinder you could wrap the fiber around without destroying total internal reflection at the core/cladding interface and allowing light to leak out of the fiber core and into the cladding.

  Hint: If the fiber/cladding combo (gray-blue, diameter $d$) were wrapped around a white cylinder of radius $r$, it would look something like the figure shown. The blue reflections on the upper-left would have no problem doing TIR. The red line on the right, however, represents a worst-case scenario that might run into trouble because of its steeper angle. (Answer: 15.0 $\mu$m.)

27-1. You wish to attenuate a polarized laser beam by inserting two polarizers. The second polarizer is oriented to match the original polarization of the beam to ensure that the final polarization remains unchanged. The first polarizer is then rotated through various angles to control the intensity. Through what angle should the first polarizer be rotated to reduce the final intensity by a factor of [01] _______? Assume perfect polarizers with no losses.
27-2. Light from incandescent light bulbs is unpolarized. I shine light from a light bulb with an intensity of \(02\) W/m\(^2\) onto a perfect linear polarizer. (a) What is the intensity of the light which passes through the polarizer? (b) If a second polarizer is placed after the first one, with its transmission axis rotated 35° from the transmission axis of the first polarizer, what will be the intensity of the light after passing through the second polarizer? (c) If I add a third polarizer after the second one, with its transmission axis rotated 90° from the transmission axis of the first polarizer (i.e., an extra 55° from the second polarizer), what will be the intensity of the light after passing through the third polarizer? (d) If I now remove the second polarizer, what will be the intensity of the light exiting the third polarizer?

27-3. You are looking across the surface of a very large, very calm lake. You are wearing polarized sunglasses, and you notice that your sunglasses nearly completely cut out the glare from the sun reflecting off of the lake. (a) Do your polarizing sunglasses block vertically or horizontally polarized light? (b) How far above the horizon is the sun in the sky? The index of refraction for water in the lake is \(03\).

28-1. An object is placed \(01\) cm in front of a concave mirror with a focal length equal to 3.00 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 2.00 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

28-2. An object is placed \(02\) cm in front of a concave mirror with a focal length equal to 3.00 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 1.00 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

28-3. An object is placed \(03\) cm in front of a convex mirror with a focal length equal to \(-2.0\) cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 5.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.
28-4. A concave mirror has a focal length of \( f = 40.0 \text{ cm} \).
(a) Find the position of the object that gives an image that is upright and \([04] \square \text{ times larger.}\)
(b) Draw a ray diagram using different colors for the real and virtual rays.
(c) Repeat part (a) for a convex mirror with focal length \( f = -40.0 \text{ cm} \).
(d) Draw a ray diagram for part (c).

28-5. (Paper only.) At the south end of the foyer in the Eyring Science Center, there is a demonstration called “The Illusive Dollar”. A large concave mirror produces a image of a dollar bill. The bill is 1.73 m from the mirror. (a) Is the image real or virtual? (b) What is the magnification of the dollar bill? (c) What is the mirror’s focal length? (d) How far should you put a nickel from the mirror if you want to make a real image which is twice as big as the nickel, and will that image be upright or inverted? (d) Test out your predictions to part (c).

28-6. (Paper only) Let’s derive the focal length of a mirror. The curved line in the figure below is a spherical mirror. The dotted line runs from the center of curvature and has a length \( R \). The red horizontal line at the top represents a beam of light traveling parallel to the principle axis. It makes an angle \( \theta \) with respect to the normal of the mirror. (a) In terms of \( \theta \) and \( R \), what is \( \alpha \)? (b) What is \( \beta \)? (c) Find \( f \) in the limit that \( \theta \) is very small (i.e., such that the light represents a paraxial ray).

Extra problems I recommend you work (not to be turned in):
• (Paper only.) If I place an object in front of a concave mirror, under what conditions will a real image be created? Under what conditions will a virtual image be created? Under what conditions will the image be inverted? What about a convex mirror?
29-1. You are trapped in the wilderness and must spear fish in order to survive. While looking into the water from directly above, a fish appears to be \(01\) cm below the surface.
(a) How far below the surface is it in actuality?
(b) Draw a ray diagram, using different colors for the real and virtual rays.

29-2. A fortune teller gazes into her crystal ball and sees a scene of sorrow and tragedy. The scene appears to be inside the ball 4.31 cm from the front surface of the ball. But, of course she is really seeing the image of the scene of sorrow and tragedy. The actual scene of sorrow and tragedy is embedded in the ball at a different location. (a) How far from the front surface of the ball is the actual scene of sorrow and tragedy? The ball, \(02\) cm in diameter, is made of quartz with an index of refraction of 1.54. (b) Is the image of the scene real or virtual?

29-3. An object is placed \(03\) cm in front of a converging lens with a focal length equal to 2.00 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 1.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

29-4. An object is placed \(04\) cm in front of a converging lens with a focal length equal to 6.0 cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 1.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.

29-5. An object is placed \(05\) cm in front of a diverging lens with a focal length equal to \(-5.0\) cm. (a) Find the location of the image. (b) Find the magnification. (c) If the height of the object is 5.0 cm, find the height of the image. (d) Draw a ray diagram, using different colors for the real and virtual rays.
29-6. A lens of focal length \( f_1 = 10.0 \) cm is placed a distance \( x = 6 \) cm before a lens of focal length \( f_2 = -10.0 \) cm. An object is positioned 15.0 cm before the positive lens.

(a) At what position relative to the location of the negative lens does the final image occur? (Enter a negative number if on the left.)

(b) Calculate the overall magnification \( M_{\text{overall}} = \frac{h_{\text{final image}}}{h_{\text{object}}} \).

29-7. The lens and mirror in the figure have focal lengths of 7 cm and \(-50.0 \) cm, respectively. An object is placed 1.00 m to the left of the lens, as shown. (a) Find the distance between the lens and the final image which is formed by light that has gone through the lens twice. (b) Is the image upright or inverted? (c) Determine the magnitude of the overall magnification.

Extra problems I recommend you work (not to be turned in):

- (Paper only.) If an object is a distance \( p \) in front of a converging lens (\( p \) could be negative if the “object” is an image formed from another lens), under what conditions will a real image be created? Under what conditions will a virtual image be created? Under what conditions will the image be inverted? What about a diverging lens?
• A cube which is 1 cm in length is placed 15 cm from a lens with a focal length of 2 cm. Draw the 3D image of what the cube will look like. Hint: Figure out where the images of the front and back surfaces will form, then connect them.

30-1. A near-sighted woman cannot focus on any object farther away than a distance \( x = [01] \) ________ cm. Find the focal length of the lens which will correct her vision. (If an object is very far away, the lens should produce an image a distance \( x \) in front of her eyes so that she can focus on it.) Neglect the distance between the lens and the eye.

30-2. A farsighted man cannot focus on anything closer than \([02]\) ________ m away from him. If he wants to be able to hold his book at 25 cm, find the focal length of the lens which will correct his vision. Neglect the distance between the lens and the eye.

30-3. Consider a camera with film in it. The focal length of the lens is \([03]\) ________ mm.
(a) If we want to take a picture of some distant object, where should we put the film? (Consider the distance to the object to be infinite.) (b) If we next want to take a picture of an object a distance \( x = [04] \) ________ m away, by how much should we change the distance between the film and the lens? (c) Suppose we didn’t change the position of the film, but left it in the position for taking a picture of a distant object, as in part (a). A “point” of light a distance \( x \) away would not be “focused” properly on the film but instead would produce a “dot” on the film. Find the diameter of the dot. The diameter of the lens is 1.0 cm. (d) If we cover up part of the lens so that light can only enter through a hole 3 mm in diameter, find the diameter of the dot in part (c).
30-4. Let’s take a look at spherical aberration. Imagine a plano-convex lens (meaning that one side is flat and one side is convex) made of a glass with an index of refraction of \[ n \]. The magnitude of the radius of curvature of the curved side is 30 cm. Two rays of light strike the flat side of the lens, both traveling parallel to the principle axis, as shown in the figure below. Suppose one beam hits the lens a distance of 0.5 cm from the principle axis, and the other a distance of 10 cm from the principle axis. (Note: the lower ray on the figure is NOT drawn at the right height.) After being bent by the lens, the two rays both cross the principle axis. If the lens were free of aberrations, they would cross the principle axis at the same point. But, in fact, they don’t. What is the distance \( \Delta x \) between the points where the two rays cross the principle axis? All you need is Snell’s Law, and some geometry/trigonometry.

Hint: Here’s a sketch for one of the rays, with the lens’s radius of curvature expanded to a full circle, which should help you think about how to calculate the right distances.

Final note: If you were to reverse the lens, so that the rays strike the curved side first, you would find \( \Delta x \) to be smaller. That gives rise to the first half of this optics rule about positioning these common “plano-convex” lenses: “Parallel rays to curved, diverging rays to flat.”
30-5. You shine red light on a penny. You place a lens exactly 1 meter from the penny, and a red image of the penny forms at a distance of [6 cm] from the lens, on the opposite side of the lens. You then shine blue light on the penny. How far from the lens will the blue image form? The index of refraction for this particular glass is 1.500 for the red light and 1.530 for the blue light.

Extra problems I recommend you work (not to be turned in):

- You want to take a picture of an ant. You place your camera such that the film is 250 mm from the ant. The lens has a focal length of 50 mm. (a) Show that there are two possible positions for the lens which will produce a focused image of the ant on the film. Find \( p \) and \( q \) for both cases. (b) What is the magnification of the image for the case where \( p > q \)? (c) What is \( M \) for the case where \( q > p \)? (Answers: 69.1 mm, 180.9 mm; \(-0.38, -2.62\).)

31-1. (a) I hold a flat mirror [1 cm] in front of my face. There is a freckle on my face 1 mm in diameter. Find the angular size of the freckle on the image of my face as viewed by my eye. (b) Repeat for a concave mirror which has a focal length of 39 cm. (c) What is the angular magnification of the concave mirror, as compared to the flat mirror?

31-2. Imagine that you are using a lens with a focal length of [2 cm] as a magnifying glass to look at (not cook!) an ant which is sitting on your finger. You put your eye up to the lens and adjust the position of the ant until the image of the ant is 25 cm from you. (a) What is the lateral magnification \( M \) of the image? (b) What is the angular magnification \( m \)?

31-3. A hobby telescope has an objective lens with a focal length of [3 cm] and an eyepiece with a focal length of 8.2 mm. We view the planet Jupiter with this telescope. We do this at the time of year when we are closest to Jupiter. Data about the solar system can be found in the front inside cover of the textbook. (a) Find the diameter of the image of Jupiter produced by the objective lens. (b) Find the angular size of Jupiter as viewed through the eyepiece. (c) How far should I place a marble (1-cm diameter) from my eye to obtain the same angular size as in part (b)?
31-4. This is a common trick for expanding (or reducing) laser beams: two converging lenses are set up such that the first lens’s right-hand focus is at the same point as the second lens’s left-hand focus. (The beam is traveling left to right.) That forces the laser to emerge collimated from the second lens, but with a different beam diameter. (a) Draw a ray diagram for this situation, to show that if the laser beam is collimated going into the first lens, it really does emerge collimated from the second. (b) If the laser beam diameter before the first lens is \( f_1 = 50 \) mm, and \( f_2 = \) \text{mm}, what is the laser beam diameter after the laser emerges from the second lens?

31-5. (Paper only.) In a Galilean telescope, the eyepiece lens has a negative focal length. This has the advantage of producing a positive angular magnification (i.e. the image is not inverted). The layout of such a telescope is shown in the figure. The focal points of the objective are marked with + signs; the focal points of the eyepiece are marked with \( \times \) signs. As you can see, the lenses are spaced such that the right-hand focal point of the objective is on top of the right-hand focal point of the eyepiece.

Before you begin, consider what would happen if you stood in front of the telescope and looked at some object (Mars, for example) with your eyes at the location of the left-hand focal point of the objective. If the optical axis runs into the bottom of Mars, and the two solid red lines on the left come from the top of Mars, the angle marked \( \theta \) in the figure is the angular size of Mars as viewed with the naked eye.

(a) The upper solid red line, which runs parallel to the optical axis, is bent by the objective such that it is traveling toward the focal point a distance \( f_o \) away. Because of the way we’ve placed the lenses, this point is a distance \( -f_e \) to the right of the eyepiece (this is a positive number, because the eyepiece has a negative focal length). After passing through the second lens, what angle does this beam make with respect to the optical axis?

(b) The lower ray passes through the focal point on the left side of the objective. In the limit as \( \theta \) is really small, in terms of \( \theta, f_e, \) and \( f_o \), what angle does this ray make with
respect to the optical axis after passing through the eyepiece? (Note that for small $\theta$, $\sin \theta \approx \tan \theta \approx \theta$).

(c) The dotted lines illustrate that by tracing backwards, we can determine the position and size of the virtual image formed by the telescope. In the limit as the object (and therefore the virtual image) are extremely far away from you, what is the angular magnification of this telescope in terms of $\theta$, $f_c$, and $f_o$?

32-1. Two loud speakers are 2.63 m apart. I am standing ______ m from one of them and 3.58 m from the other. The two speakers are driven by a single oscillator. If the frequency of the oscillator is swept from 100 Hz to 1000 Hz, find the lowest frequency at which I will hear an enhancement of the sound intensity due to constructive interference of the waves from the two speakers. Use 343 m/s for the speed of sound. (DO NOT use the double-slit equation, $d \sin \theta = m \lambda$. This equation is only valid for observations far from the two slits, compared to the distance between the two slits.)

32-2. We pass a laser beam through a double slit. On a screen ______ m away, we observe a series of bright lines which are 3.4 mm apart. The wavelength of the laser light is 633 nm. What is the distance between the two slits?

32-3. The figure shows an interference pattern from two slits. If the slits are 0.17 mm apart and the observed picture is seen on a screen ______ m from the slits, find the wavelength of the light used. Assume that the photograph in the figure is life-size. To obtain an accurate value for the distance between fringes, measure the distance between the topmost and bottommost fringes and divide by 4. Note: Sometimes printers do not faithfully reproduce the actual size of a photograph. The box displayed below should be 5.0 cm wide. If not, then scale the size of the photograph accordingly. ______ cm
32-4. Two narrow slits separated by 0.85 mm are illuminated with [04] ________-nm light. The peak intensity on a screen 2.80 m away is 0.1 W/cm². What is the intensity at a distance 2.50 mm from the center of the central peak?

32-5. (Paper only.) (a) Coherent monochromatic light with a wavelength \( \lambda \) passes through three parallel slits spaced evenly from each other with a distance \( d \). Use phasor addition/complex numbers to show that the intensity in the interference pattern at an angle \( \theta \) is given by:

\[
I(\theta) = I_0 \left( 1 + 2 \cos \left( \frac{2\pi d \sin \theta}{\lambda} \right) \right)^2
\]

where \( I_0 \) is a constant. Hints: At a given point in the pattern on the screen, the amplitude of the oscillating electric field will be a sum of the electric fields from each slit. Those will only vary by their phase, which phase difference arises from a difference in path length. Using complex numbers, you can easily include phase shifts by terms such as \( E_0 e^{i\phi} \). Recall that \( \cos \phi = \frac{e^{i\phi} + e^{-i\phi}}{2} \). The overall intensity is proportional to the electric field, squared.

(b) Using a program such as Mathematica, plot the intensity pattern for angles between \(-30^\circ\) and \(30^\circ\), with \( I_0 = 1 \) and \( d/\lambda = 5 \). Tip: if you use Mathematica, you cannot use a capital I as the function name; that’s a reserved symbol for the imaginary constant.

(c) You should observe that this pattern has two different types of “bright” fringes. The higher intensity maxima are known as primary maxima, and the lower intensity ones are known as secondary maxima. For the general case, what is the ratio of the intensity in a primary maximum to the intensity in a secondary maximum? Assume that \( \lambda < d \). Hint: The answer is the same for the two types of maxima in the graph of \( f(x) = (1 + 2 \cos x)^2 \).

Extra problems I recommend you work (not to be turned in):

• Two coherent sources of light are both shining down onto a piece of paper. If I block source “A” the electric field at a given point on the paper is given by the equation \( E_B = 1.5E_0 \sin(\omega t + 64^\circ) \), where \( E_0 \) is a constant. If I block source “B”, the electric field at the same point on the paper is given by the equation \( E_A = 2.5E_0 \sin(\omega t - 14^\circ) \). When both sources are unblocked, the electric field is given by \( E_{both} = A \times E_0 \sin(\omega t + \phi) \). Find (a) \( A \) and (b) \( \phi \) (in degrees). (Answers: 3.17, 13.56°.)
33-1. A pool of water is covered with a film of oil which is \[01\] \text{nm} thick. For what wavelength of visible light (in air) will the reflected light constructively interfere? The index of refraction of the oil is 1.65. Visible light (in air) has wavelengths between 430 nm (blue) and 770 nm (red) in air. Assume that the incident light is normal to the surface of the oil.

33-2. Solar cells are often coated with a transparent thin film of silicon monoxide (SiO, \(n = 1.85\)) to minimize reflective losses from the surface. Suppose that a silicon solar cell \((n = 3.5)\) is coated with a thin film of SiO for this purpose. If the thickness of the coating is \[02\] \text{nm}, find the maximum wavelength of ultraviolet light (in air) for which the reflection will be maximized instead of minimized. (The wavelength of ultraviolet light is shorter than that of visible light.)

33-3. A wedged air space is created between two plates of glass, and a sample of hair is inserted at one edge. The plates are illuminated from above with \[03\] \text{nm} light and \text{reflections} from the air wedge are observed.

(a) Will the fringe near where the plates touch be bright or dark?
(b) If a total of 25 dark fringes are observed (including the one in part (a) if it is dark), what is the diameter of the hair?

33-4. One leg of a Michelson interferometer contains an evacuated cylinder of length \(L = 13.2\) cm having glass plates on each end. A gas is slowly leaked into the cylinder until a pressure of 1 atm is reached. If \(N = [04]\) \text{bright fringes} pass on the screen when light of wavelength \(\lambda = 632\) nm is used, what is the index of refraction of the gas?
Extra problems I recommend you work (not to be turned in):

- A Michelson interferometer is used to carefully measure the wavelength of a single spectral line from a source. A computer connected to a scanning motor and a detector records 3400 fringes as the motor causes the mirror to move by 1.000 mm. What is the wavelength? (Answer: 588.2 nm.)

34-1. The second-order bright fringe in a single-slit diffraction pattern is \[01\] \[\text{mm}\] from the center of the central maximum. The screen is 81 cm from a slit of width 0.78 mm. Assuming that the incident light is monochromatic, calculate the light’s wavelength.

34-2. Babinet’s principle says that the diffraction pattern from an opaque object blocking the light will produce the same diffraction pattern as a slit of the same size/shape allowing the light to pass. Only the overall intensity of the pattern will be different. This can be seen by noting that the resulting light field from an opaque object is just the field made by the laser beam by itself, minus the field that would be produced by a slit in the shape of the opaque object: \( E = E_{\text{laser beam}} - E_{\text{slit}} \). In places where the undisturbed beam would not have reached, \( E_{\text{laser beam}} = 0 \). Therefore \( E = -E_{\text{slit}} \) over most of the diffraction pattern. Of course your eye can’t see the sign of the electric field (you only see the intensity), so you see essentially the same pattern that would be present if you used a slit rather than an opaque object.

Suppose you are working in a forensics lab, and you have a human hair whose diameter you need to measure. You decide to use Babinet’s principle: you shine a HeNe laser (\( \lambda = 633 \text{ nm} \)) at the hair and see a single-slit diffraction pattern. You look at this pattern on a screen which is 1.6 m away from the hair. The width of the central peak on the screen turns out to be \[02\] \[\text{mm}\], measured from the dark spot just to the left of the peak to the dark spot just to the right of the peak. What is the diameter of the hair?
34-3. (Paper only.) In the three-slit problem from the last assignment, you found the total electric field in the diffraction pattern at the screen by adding up the phase shifts from three separate slits. This gave rise to the formula for the three slit pattern. The same technique can be used to find the formula from a single slit having finite width. According to Huygen’s principle, a single slit behaves like an infinite number of point sources, spaced infinitely closely together. Thus, instead of adding three separate phase factors, like you did with three infinitely narrow slits, to solve the single finite width slit problem you must add an infinite number of phase factors: you must integrate.

(a) In the figure below, determine the phase shift of the light coming from an arbitrary \( y \), relative to the light coming from \( y = 0 \) (i.e., the difference in phase of the two dashed lines). Show that this phase shift is equal to \( \frac{2\pi}{\lambda} y \sin \theta \). (\( \theta \) is the angle from the slit to the screen, which is essentially the same for the two dashed lines because the slit is actually much smaller than indicated in the figure.)

(b) Add up an infinite number of phase shifts (i.e., integrate), for points ranging from \( y = -a/2 \) to \( y = a/2 \).

(c) Show that your answer can be written as \( a \text{sinc}(x) \), where \( x = \frac{\pi a \sin \theta}{\lambda} \). The “sinc” function is probably not one you are familiar with yet. It is defined as: \( \text{sinc} \, x = \frac{\sin x}{x} \).

Hint: Recall that \( \sin \phi = \frac{e^{i\phi} - e^{-i\phi}}{2i} \).

(d) The intensity pattern, being the square of the electric field pattern, is thus

\[
I(\theta) = I_0 \left( \text{sinc} \left( \frac{\pi a \sin \theta}{\lambda} \right) \right)^2.
\]

Using a program such as Mathematica, plot the intensity pattern for angles between \(-30^\circ\) and \(30^\circ\), with \( I_0 = 1 \) and \( a/\lambda = 10 \). Set your vertical scale to be from 0 to 1. Note: \( \text{Sinc}[x] \) is a built-in function in Mathematica, just like \( \text{Sin}[x] \) or \( \text{Cos}[x] \).
(e) Use the intensity pattern formula you just derived to prove the formula given in the book for the angle of the \( m \)th minimum in a single slit pattern:

\[
\sin \theta_{\text{dark}} = \frac{m \lambda}{a}
\]

34-4. (Paper only.) Below is a photograph of the interference pattern (intensity) on a screen for a particular two-slit experiment, and a plot of what the detector measured as it scrolled across the pattern. The \( x \) axis is in centimeters, the \( y \) axis has arbitrary units. A 633 nm laser was used. The intensity pattern is simply the two-slit (infinitely narrow) pattern, times a single finite-width slit pattern.

If the screen/detector was positioned 0.75 m away from the slits, (a) what was the separation between slits, and (b) what was the width of each slit? Hint: You will have to read some quantities off of the graph. The grader will allow some margin of error when judging whether your answers are correct or not, but be as accurate as you can.

35-1. On the night of April 18, 1775, a signal was sent from the Old North Church steeple to Paul Revere who was 1.8 miles away: “One if by land, two if by sea.” If in the dark, Paul’s pupils had [01] \[_______\]-mm diameters, what is the minimum possible separation between the two lanterns that would allow him to correctly interpret the signal? Assume that the predominant wavelength of the lanterns was 580 nm. NOTE: Although the wavelength is modified by the refractive index within the eye, the angles between incident rays are also modified by a similar amount. The two effects cancel each other, so you need not worry about it.
35-2. (a) Two stars have an angular separation of \(0.2\) \(\mu\)rad as viewed from Earth. Assuming that most of the light from the star is yellow with a wavelength around 580 nm, how big must the diameter of a telescope be in order to see that there are two stars, not one? (b) If my eye has a pupil which is \(4\) mm in diameter, what must the angular magnification (magnitude) of the telescope be, for me to be able to see that there are two stars rather than one?

35-3. (a) Grote Reber, a pioneer in radio astronomy, constructed a radio telescope with a \(10\) m diameter receiving dish. What was the telescope’s angular resolution when observing \(3\) m radio waves? (b) The Hubble telescope has a primary objective with approximately \(1\) m diameter. What is the angular resolution when observing visible light with wavelength \(4\) nm?

35-4. In this problem, we will find the ultimate resolving power of a microscope. First of all, in order to obtain a large magnification, we want an objective lens with a very short focal length. Second, in order to obtain maximum resolution, we also want that lens to have as large a diameter as possible. These two requirements are conflicting, since a lens with a short focal length must have a small diameter. It is not practical for a lens to have a diameter much larger than the radius of curvature of its surfaces. Otherwise, the lens starts looking like a sphere. So, let us assume that the objective lens has a diameter \(D\) equal to the radius of curvature of the two surfaces, like the lens in the figure to the right. (a) If the lens is made of glass with index of refraction \(5\), find the focal length \(f\) in terms of the diameter \(D\) of the lens. (b) The distance between the sample to be observed and the objective lens is approximately equal to the focal length \(f\). Find the distance between two points on the sample which can be barely resolved by the lens. Use the result from part (a) to eliminate \(f\) from the expression. You should find that \(D\) is also eliminated from the expression and that the answer is given entirely in terms of the wavelength \(\lambda\) of the light. You may use the small angle approximation, \(\sin \theta \approx \tan \theta \approx \theta\).
35-5. A diffraction grating contains 15,000 lines/inch. We pass a laser beam through the grating. The wavelength of the laser is 633 nm. On a screen 0.6 m away, we observe spots of light. (a) How far from the central maximum \((m = 0)\) is the first-order maximum \((m = 1)\) observed? (b) How far from the central maximum \((m = 0)\) is the second-order maximum \((m = 2)\) observed? DO NOT use the “small-angle approximation,” 

\[
y_{\text{bright}} = \left(\frac{\lambda L}{d}\right)m.
\]

The angles are too large for \(\sin \theta \approx \tan \theta\) to be a very good approximation.

35-6. The diagram depicts a standard spectrometer setup. A lens (or concave mirror) following a slit creates collimated light that strikes the grating (usually a reflective grating rather than a transmission grating as shown). Suppose that you observe light from a sodium lamp which has two strong emission lines at \(\lambda_1 = 589.0\) nm and \(\lambda_2 = 589.6\) nm. Your grating has 7 lines/mm. 

(a) In the first diffraction order, what is the difference between the diffraction angles of the two wavelengths (ignore the second lens)? Don’t use the small angle approximation. 

(b) To spatially separate the two wavelengths, it is necessary to let the light travel to a faraway screen. Because this can require very large distances, it is convenient to image what would have appeared on the faraway screen to a closer screen using a lens (or concave mirror). As proved in one of the additional problems below, the image appears at the focus of the lens and the angular separation is preserved, referenced from the position of the lens. If the final lens has a 30 cm focal length, how far apart on the detector screen are the two sodium wavelengths?
As mentioned in the 6th edition of Serway and Jewett (but not in subsequent editions), gratings are characterized by their resolving power $R$, defined as their ability to distinguish between two nearly equal wavelengths: $R = \lambda_{\text{ave}} / \Delta \lambda$ (\(\Delta \lambda\) is how close the wavelengths can be to each other without “blurring” together). As more and more grating lines contribute towards the overall interference pattern, the diffraction spots sharpen and the resolving power increases. By calculating the interference pattern for a very large number of contributing slits, like we did in a previous homework problem for three slits, a simple relationship can be derived: $R = Nm$, where $N$ is the number of slits being illuminated, and $m$ is the order of the diffraction spot being used. In order for the two sodium peaks to be cleanly separated from each other in the first diffraction order, what is the minimum number of slits that must be illuminated?

Extra problems I recommend you work (not to be turned in):

- A hobby telescope uses a concave mirror with a diameter of 12.4 cm. Find the distance between two points on the moon that can be resolved by this telescope. Use 550 nm for the wavelength of visible light. (Answer: 2078 m.)

- A television screen generates images which are composed of little red, green, and blue dots. Far from the screen, the dots blend together. (a) If there are 28 dots per cm, how close would you have to put your eye to the screen in order to see the individual dots? (b) If you can’t focus on anything closer than 25 cm from your eye, how close must the dots be such that you cannot possibly distinguish them from each other? Use a wavelength of 520 nm, and assume that your pupil has a diameter of 4 mm. (Answers: 2.252 m; 39.65 \(\mu\)m, which is 252 dots/cm.)
Coherent light with wavelength 500 nm passes through a round hole and propagates to a faraway screen. (a) Draw a picture of the setup and resulting pattern. (b) If a converging lens is placed close to the hole (after the hole), where will the diffraction pattern appear? Hint: Use the image from the hole (at infinity, or close to it) as the object for the lens, then find the image of the lens. (c) Show that the new diffraction pattern is just the same as the old pattern, but with distances scaled down by $f/L$, where $L$ was the original distance to the screen where the pattern was viewed. Hint: find the magnification. (d) Show that the angular separation between features on the new diffraction pattern (angles measured relative to the center of the lens) is the same as the angular separation was on the previous diffraction pattern (angles measured relative to the center of the hole). Hint: think similar triangles.

A HeNe laser as a wavelength of 633 nm. If the collimated beam has a diameter of 0.50 cm, estimate the radius of the beam after it travels 10 km. (Answer: 1.545 m.)

In a sodium chloride crystal, NaCl (the kind of salt we put on food), the sodium ions form a cubic lattice. The lattice constant $a$ (the distance by which each sodium ion is separated from its nearest neighbors) is 0.565 nm at room temperature. You send a beam of x-rays at a crystal of NaCl and find that the $m = 1$ diffraction order off of the Bragg planes which are spaced by $a$ is traveling in a direction which is $20^\circ$ from the direction of the incident beam of x-rays. You then shoot the same beam of x-rays at a crystal of potassium chloride, KCl, which has a similar structure to NaCl. You find that this particular diffraction order now makes an angle of $17.9^\circ$ with respect to the incident beam (as shown below). (a) What is the wavelength of the x-rays? (b) What is the lattice constant for the KCl crystal? (Answers: 0.386 nm, 0.629 nm.)

36-1. (Paper only.) A drop of water falls into a perfectly calm pond generating ripples that travel out in circular rings. Prove that as each ring expands, the amplitude of the ring drops off as $1/\sqrt{r}$, where $r$ is the radius of the ring. Assume that no energy is lost, and that the waves propagate non-dispersively (i.e., the radial thickness of a ring doesn’t change as the ring expands).
36-2. (Paper only.) Write down the proper mathematical expression for a plane wave traveling in the \( \frac{8+5\hat{x}}{\sqrt{26}} \) direction, oscillating at a frequency of 10000 Hz, having a wavelength of 5 meters. It is a transverse wave, with its amplitude oscillating back and forth from the \( \frac{-5\hat{x}+\hat{y}}{\sqrt{26}} \) direction to the \( \frac{5\hat{x}-\hat{y}}{\sqrt{26}} \) direction. Don’t worry about phase shifts.

Happy Thanksgiving!

This was a short assignment so that you can get a head start on homework 37. I don’t want you to have to work on that assignment during the entire Thanksgiving break. (Just during most of it, ha ha.)

37-1. A ball is thrown at [01] _________ m/s inside a boxcar moving along the tracks at 45.8 m/s. What is the speed of the ball relative to the ground if the ball is thrown (a) forward? (b) backward? (c) out the side door?

37-2. (Paper only.) A car of mass \( m_1 = 1000 \) kg is sliding without friction on ice at a velocity \( u_1 = 20 \) m/s when it strikes another car of mass \( m_2 = 1200 \) kg which was standing still. The two cars lock together and slide together without friction after the collision. (a) Use the principle of momentum conservation to find the velocity of the two cars after the collision. (b) An observer riding on a bicycle in the same direction as the cars watches the collision while traveling at a velocity \( v = 10 \) m/s. Find the initial and final velocities of the two cars as measured in the reference frame of the bicyclist. (c) Show that the velocities that you found in part (b) satisfy momentum conservation in the bicycle rider’s reference frame.

37-3. (Paper only.) Now consider the same car crash as viewed by a bicycle rider who is moving at a speed \( v = 10 \) m/s at a direction perpendicular to the direction that the cars are moving. Let the bike rider be moving in the \( x \) direction and the cars moving in the \( y \) direction. (a) Use the Galilean transformations to find the \( x \)- and \( y \)-components of the cars’ velocities before and after the collision. (b) Show that momentum is still conserved in the reference frame of the bike rider.
37-4. (Paper only.) An observer at rest with respect to the Earth finds that objects falling under gravity accelerate at a constant rate of 9.8 m/s. According to Galilean relativity:

(a) Will an observer on a train moving at 10 m/s see that objects falling under gravity accelerate at a constant rate in his frame? If so, what is that rate? (b) Will an observer on a rocket moving vertically away from the surface of the earth at 10 m/s see that objects falling under gravity accelerate at a constant rate in his frame? If so, what is that rate? (c) Will an observer on a rocket accelerating vertically away from the surface of the earth at 10 m/s² see that objects falling under gravity accelerate at a constant rate in his frame? If so, what is that rate?

37-5. (Paper only.) (From Peatross and Ware, *Physics of Light and Optics.*) Ole Roemer made the first successful measurement of the speed of light in 1676 by observing the orbital period of Io, a moon of Jupiter with a period of 42.5 hours. When Earth is moving toward Jupiter, the period is measured to be shorter than 42.5 hours because light indicating the end of the moon’s orbit travels less distance than light indicating the beginning. When Earth is moving away from Jupiter, the situation is reversed, and the period is measured to be longer than 42.5 hours.

(a) If you were to measure the time for 40 observed orbits of Io when Earth is moving directly toward Jupiter and then several months later measure the time for 40 observed orbits when Earth is moving directly away from Jupiter, what would you expect the difference between these two measurements be? Take the Earth’s orbital radius to be $1.5 \times 10^{11}$ m. To simplify the geometry, just assume that the Earth moves directly toward or away from Jupiter over the entire 40 orbits. (See the figure.) Hint: Find the Earth’s orbital speed from its orbital radius (given) and period (you should know!). You will need to determine how far the Earth moves closer to Jupiter in the $40 \times 42.5$ hrs observation period when it’s moving straight towards Jupiter (imagine straight-line motion during this time).

(b) Roemer did the experiment described in part (a), and experimentally measured a 22 minute difference. What speed of light would one deduce from that value?
38-1. A jet plane is \([01]\) ________ m long. How much shorter is the plane when it travels at the speed of sound, 343 m/s? You will find the following approximation useful:

\[ \sqrt{1-x} \approx 1 - \frac{1}{2}x \text{ when } x \ll 1.\]

38-2. How fast must a meter stick move for it to appear to be only \([02]\) ________ cm long?

38-3. In a laboratory, a muon is found to have an expected lifetime of 2.2 \(\mu\)s before it decays. Suppose that a muon is created by a cosmic ray \([03]\) ________ km above the Earth. What fraction of the speed of light must the muon travel if it is to reach the ground in its expected lifetime? Express your answer as a number times \(c\). Give your answer to 6 significant figures.

38-4. Astronauts travel at 0.950\(c\) from Earth to a star which is \([04]\) ________ ly (light years) away.

(a) How long does it take them to reach the star as observed by people on Earth? (Earthlings know how much time it takes for a signal to reach them from the star, and they do not include it as part of the travel time for the astronauts.)

(b) How long does the trip take from the perspective of the astronauts?

(c) How far apart are Earth and the star from the perspective of the astronauts as they travel?

38-5. You run a red light. You are pulled over. You explain to the traffic officer that you didn’t know that the light was red—because you were moving, the red light \(([05]\) ________ nm) was Doppler-shifted to appear green \(([05]\) ________ nm). If you were telling the truth, how fast were you going?

38-6. Ever since the Big Bang, different parts of the universe have been flying away from each other. Astronomers can figure out how fast a star is moving away from us by looking at atomic emission lines and measuring how much the lines have been Doppler-shifted from the wavelength of the lines which we measure in experiments on Earth. They often characterize the amount of Doppler shift using a parameter \(z\), frequently just called the “redshift”. It is defined as

\[ z = \frac{\lambda_m - \lambda_0}{\lambda_0} \]

where \(\lambda_m\) is the wavelength they measure for the light coming from the star, and \(\lambda_0\) is the wavelength measured in an experiment in which the atoms emitting the light are at rest.
(a) Use the equation for the Doppler shift for light to show that the parameter $z$ and the speed of the astronomical object $v$ are related by:

$$z = \sqrt{\frac{1 + v/c}{1 - v/c} - 1}$$

Hint: recall that the astronomical object is moving away from the Earth.

(b) The largest red shifts measured are those of quasars. Quasars are the most luminous objects in the universe, so they are the farthest-away objects that can be seen. Hence they are the fastest-moving objects that can be seen. Imagine that you look at a particular quasar and measure its redshift to be [01] _________. How fast is that quasar moving relative to the Earth? (According to Wikipedia, the record for the largest redshift as of Dec 2007 was a quasar with $z = 6.43$. Its speed is 0.907$c$.)

(c) Edwin Hubble discovered that (as one would expect in an explosion), the velocity at which various galaxies are receding from the Earth—as measured by their redshift—is very consistently proportional to their distance from us. Specifically, Hubble’s Law says $v = H_0d$. $H_0$ is the “Hubble constant”, which according to the best current measurements is about 70.8 km/s per Mpc. (1 Mpc, a “megaparsec”, is $3.2616 \times 10^6$ light years.) According to Hubble’s Law, how far away from us is the quasar? (Side note: as I understand it, the distance $d$ in Hubble’s law corresponds to the distance the light actually traveled. That is larger than the distance the quasar was from us when the light was emitted, because the space between the Earth and the quasar has been expanding the whole time the light has been traveling. However, it is smaller than the distance the quasar is from us now, because—due to this same expansion—distances covered by the light ray at the start of its path are now larger than they were then.)

(d) How long did it take the light from your quasar to reach us? For the record-holding quasar mentioned above, the answer is about 13.3 billion years. That sets a lower limit on how long it has been since the Big Bang. (The best current estimates for the Big Bang are that it occurred about 13.7 billion years ago.)

38-7. (Paper only.) Just because I can’t travel faster than the speed of light, it doesn’t mean that I can’t travel any further than about 100 light years from Earth before I die (where 100 years is about the longest someone might live). Why not?
Extra problems I recommend you work (not to be turned in):

- A moving rod is observed to have a length of 2.00 m, and to be oriented at an angle of 30° with respect to the direction of motion (see figure). The rod has a speed of 0.995c. (a) What is the proper length of the rod? (b) What is the orientation angle in the proper frame? (Answers: 17.4 m, 3.3°.)

- Deriving the time-dilation equation: Imagine that your good friend Albert is flying past you in a spaceship at velocity $v$ (relative to you) while you are floating out in deep space. Albert has a mirror glued to the roof of the bridge of his spaceship, a distance $L$ above him. He pulls out a laser pistol and shoots a burst of light upward at the mirror, it reflects back, and burns a bald spot in his hair. (a) How long does it take the light to travel from the pistol to Albert’s hair? We’ll call that time $\Delta t_p$. (b) In your reference frame, the mirror is moving at a velocity $v$. If you observe that the light took a time $\Delta t$ to go up and back to Albert, how far did the mirror move in this time? (c) Your answer to part (b) implies that the light traveled a path which looks like two sides of a triangle. How long is that path (in terms of $\Delta t$, $L$, $v$, and $c$)? (d) Knowing that the speed of light is constant, find the time it takes to travel this path, $\Delta t$, in terms of $L$, $v$, and $c$. (e) How does $\Delta t_p$ compare to $\Delta t$? In other words, find the time dilation equation!
At Stanford Linear Accelerator, Dr. Peatross’s former Ph.D. advisor was involved in experiments where a high-intensity laser is aimed at electrons which approach almost straight-on with a velocity of \( v = (1 - 3 \times 10^{-10})c \). In the lab frame, the time interval necessary for an electron to cross sequential wavecrests of the light wave is

\[ T_{\text{lab}} = \frac{\lambda_{\text{lab}}}{c + v}, \]

as you would expect. (a) Show that in the rest frame of the electron, that time interval is

\[ T_e = \frac{\lambda_{\text{lab}}}{c} \sqrt{\frac{1 - v/c}{1 + v/c}}, \]

and hence the wavelength experienced by the electron is

\[ \lambda_e = \lambda_{\text{lab}} \sqrt{\frac{1 - v/c}{1 + v/c}}. \]

This formula describes the Doppler shift of light, where \( v \) is the relative motion between the source and observer. Hint: \( 1 - v^2/c^2 = (1 + v/c)(1 - v/c) \).

(b) If the laser’s wavelength in the lab frame is 800 nm, what is the wavelength in the rest frame of the electrons?

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39-1. A red light flashes at position \( x_R = 3.31 \text{ m} \) and time \( t_R = 0.01 \text{ s} \), and a blue light flashes at \( x_B = 5.15 \text{ m} \) and \( t_B = 9.03 \times 10^{-9} \text{ s} \) (all values are measured in the S reference frame). Reference frame \( S' \) has its origin at the same point at \( S \) at \( t = t' = 0 \); frame \( S' \) moves constantly to the right. Both flashes are observed to occur at the same place in \( S' \). (a) Find the relative velocity between \( S \) and \( S' \). (b) Find the location of the two flashes in frame \( S' \). (c) At what time does the red flash occur in the \( S' \) frame?
39-2. Suppose we are on our way to Proxima Centauri, which is 4.2 ly away (in the reference frame of the earth). We are in a space ship which is traveling with a speed of \[02\] \(c\). When we are half-way there, we send a signal to both the earth and Proxima Centauri. The signals travel at the speed of light \(c\). In the reference frame of the earth and Proxima Centauri, both signals travel 2.1 ly and thus arrive at their destinations at the same time. (a) In our reference frame, how long does it take for the signal to reach earth? Remember that in our reference frame, the distance between earth and Proxima Centauri is less than 4.2 ly because of length contraction. (b) In our reference frame, how long does it take for the signal to reach Proxima Centauri? (c) These two events (the signal reaching earth and the signal reaching Proxima Centauri) are simultaneous in the earth’s reference frame. How much time elapses between these two events in our reference frame?

39-3. If two objects are both traveling at \[03\] \(c\) but in opposite directions, find the speed of one object in the reference frame of the other object.

39-4. Suppose we are on a space ship generating a beam of electrons. In our reference frame, the electrons are traveling in the \(+x\) direction with a speed of 0.87\(c\). Our space ship passes by the earth. In the earth’s reference frame, the space ship is traveling in the \(+x\) direction with a speed of \[04\] \(c\). Find the speed of the electrons in the earth’s reference frame.
39-5. (Paper only.) (Modified from Griffiths, *Introduction to Electrodynamics.*) A policeman 
\((v = \frac{1}{2}c, \text{ relative to the ground})\) is chasing an outlaw \((v = \frac{3}{4}c, \text{ relative to the ground})\). The
policeman fires a bullet whose speed is \(\frac{1}{3}c\) (relative to the policeman). Find the speed of each object: ground, policeman, outlaw, and bullet, in each of the four reference frames. Be careful with positives and negatives. Show that in each frame of reference the bullet does not catch up to the outlaw because its speed is less than the outlaw’s. (That’s good! If there were a reference frame where the bullet hit and killed the outlaw, relativity would be much more confusing than is already the case.) Present your results in this sort of summary table:

<table>
<thead>
<tr>
<th>Speed of →</th>
<th>Ground</th>
<th>Policeman</th>
<th>Outlaw</th>
<th>Bullet</th>
<th>Escape?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative to</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ground</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Policeman</td>
<td></td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Outlaw</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Bullet</td>
<td></td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**Extra problems I recommend you work (not to be turned in):**

- Read the short book *Mr. Tompkins in Wonderland*, by George Gamow (also sold under the name *Mr. Tompkins in Paperback*). You will enjoy it, and you will better understand relativity (and quantum mechanics!). The HBLL library has several copies under the following call numbers: QC 71.S775 1999, QC 71.G25 1965, QC 173.5.G36x, and QC 6.G23 1940.
Jimmy Neutron is returning from a trip to the center of the galaxy, traveling at a speed of 0.871c relative to the Earth. Carl, one of Jimmy’s friends standing still on Earth, is monitoring Jimmy’s trip. Right as Jimmy flies past the Earth Carl verifies that his watch is synchronized with Jimmy’s. In both Carl’s and Jimmy’s frames the Earth is at a location $x = 0, y = 0,$ and $z = 0$ at time $t' = t = 0$. Carl sets up his coordinate system such that Jimmy is moving in the $+x$ direction, and Jimmy sets his up such that his $y$ and $z$ directions are the same as Carl’s and such that Carl is moving in the $-x$ direction. After Jimmy has passed Earth, Carl observes a supernova looking through a telescope. Taking into account the time that it took for light to reach him, he determines that the supernova occurred at a time $t = -1.45 \times 10^9$ seconds at a location of $x = 3.29 \times 10^{17}$ m, $y = 1.53 \times 10^{17}$ m, and $z = 1.69 \times 10^{17}$ m. (a) As measured in Carl’s reference frame, how far is Jimmy from the supernova when it occurs? (b) As measured in Jimmy’s reference frame, how far is Jimmy from the supernova when it occurs? (Answers: $7.434 \times 10^{17}$ m, $1.458 \times 10^{18}$ m.)

40-1. A physics professor on the Earth gives an exam to her students, who are on a rocket ship traveling at speed $[01] \text{c}$ relative to the Earth. The moment the ship passes the professor, she signals the start of the exam. She wishes her students to have 50.0 min (rocket time) to complete the exam. At the appropriate time, she sends a light signal telling the students to stop. (a) Draw a space-time diagram from the perspective of the Earth, containing the world-lines of the professor, the students, and the light signal. Mark the exam start and stop events, along with the “professor sends signal” event. (b) Draw a similar space-time diagram from the perspective of the rocket. (c) How long did the professor wait (Earth time) before sending the light signal?
40-2. (Paper only.) Barn paradox: Suppose that Lee is carrying a 20 m long ladder (rest length) and running extremely fast towards a 10 m long barn (rest length). Cathy is watching the process and is stationary with respect to the barn. Because of length contraction, Cathy sees the ladder fit completely inside the barn. She flips a switch and “instantaneously” closes the front and back barn doors. Is the ladder completely inside the barn now? If the answer is yes, it seems paradoxical, because from Lee’s point of view it is the barn that has shrunk because of length contraction. This is discussed in the textbook before the Lorentz transformation equations are given, where it is called the “pole in the barn paradox”. Like all good physics “paradoxes”, there is no paradox when you look at things the right way. In this case, the solution to the paradox is that from Lee’s point of view, the front and back doors of the barn do not close simultaneously and so the 20 m (to Lee) ladder is not trapped inside the really narrow (to Lee) barn. You will prove that in this problem.

(a) How fast must Lee be going so that his ladder is 10 m long to Cathy?

(b) Suppose Lee is going that speed. In Cathy’s frame of reference, Lee’s ladder can “fit” inside the barn at a particular instant in time, because both the ladder and the barn are 10 m long. Let’s call that instant “t=0”; let’s call the middle of the barn “x=0”. Thus we can talk about two events in Cathy’s frame of reference: event 1 = “front end of the ladder gets to the end of the barn,” which is at \((ct, x) = (0, 5)\); event 2 = “back end of the ladder gets to the start of the barn,” which is at \((0, -5)\). Determine the times and places these two events occur, in Lee’s frame of reference. You should find that event (2) occurs after event (1), so that while Cathy knows that Lee’s ladder is completely inside the barn at a particular moment, to Lee himself, the ladder is not! Thus the paradox is resolved. How much later does event 2 occur than event 1 to Lee?

(c) Sketch Cathy’s world line, Lee’s world line, and the two events on a space-time diagram for (1) Cathy’s frame of reference, and (2) for Lee’s frame of reference.
40-3. (Paper only.) John, Lee, and Josh are in a train which is speeding past Cathy at 0.4c. Lee and Josh are standing still, 10 m apart, but John is running forward in the train at 0.3c (relative to the train). Right as John passes Lee on the train, the two also pass Cathy outside the train. Call this instant \( x = 0 \) and \( t = 0 \). John passes Josh a short instant later. Make three accurate space-time diagrams: one from Cathy’s point of view, one from Lee’s, and one from John’s. On each diagram draw the world lines of John, Lee, Josh, and Cathy. On each diagram accurately label event 1 (John passes Lee) and event 2 (John passes Josh).

**Extra problems I recommend you work (not to be turned in):**

- Suppose our Sun is about to explode and we escape in a spaceship toward the star Tau Ceti, which is 12 light years away (not including Lorentz contraction). We travel at \( v = 0.82c \). When we reach the midpoint of our journey, we see our Sun explode and, unfortunately, we see Tau Ceti explode as well (we observe the light arriving from each explosion). (a) Draw two space-time diagrams: one from the Sun’s frame of reference, and one from our frame of reference. In each diagram, include world lines for the Sun, Tau Ceti, our spaceship, and the light rays arriving from each explosion. Mark these three events on each diagram: Sun explodes, Tau Ceti explodes, and the instant we observe both explosions. (b) In the rest frame of the Sun (and Tau Ceti), did the explosions occur simultaneously? (c) In the spaceship frame of reference, did the explosions occur simultaneously? (d) In the spaceship frame of reference, how long before we saw the Sun explode did it actually explode? (e) In the spaceship frame of reference, how long before we saw Tau Ceta explode did it actually explode? (Answers to parts (d) and (e): 1.89 years, 19.08 years.)
- A deep space probe is launched from the Earth and passes a deep space station on its way into the unknown. The probe travels at a constant velocity of 0.811c relative to the station. It has an on-board atomic clock connected to a computer which is programmed to send a microwave signal back to the station exactly one year later (as measured in the probe’s frame of reference). (a) Draw two space-time diagrams: one from the station’s frame of reference, and one from the probe’s frame of reference. In each diagram, include world lines for the station and the probe, and the microwave signal sent by the probe. Mark these two events on each diagram: probe sends signal, and station receives signal. (b) From the reference frame of someone on the space station, how much time elapses from the time the probe passes by, to when the microwave signal arrives back at the station? (Answer to part (b): 3.10 years.)

- The Lorentz transformation equations transform a line in Cathy’s space-time diagram to a line in Lee’s space-time diagram. Prove this: given a line in Cathy’s frame, 
  
  \[ ct = mx + b, \]

  determine its equation in Lee’s frame. Lee is moving to Cathy’s right, with speed \( v \). Specifically, in terms of \( \beta (= v/c) \), \( \gamma (= 1/\sqrt{1-\beta^2}) \), \( m \), and \( b \), prove that the slope and the y-intercept of the line in Lee’s frame of reference are:

  \[
  m_{\text{Lee}} = \frac{m - \beta}{1 - m\beta}
  \]

  \[
  b_{\text{Lee}} = \frac{b}{\gamma(1 - m\beta)}
  \]

41-1. A particle with a rest mass of \( m \), traveling at a speed of [01] \( \ldots \)c to the right has a collision with a particle of mass 3m which is initially at rest. The larger mass moves away after the collision with a velocity of 0.89c to the right. (a) Will the smaller mass be traveling to the left or to the right after the collision? (b) What is the final speed of the smaller mass (in terms of \( c \))? 

41-2. An electron has a kinetic energy [02] \( \ldots \) times greater than its rest energy. Find (a) its total energy and (b) its speed.
41-3. Find the work required to increase the velocity of an electron by $0.010c$ (a) if its initial velocity is $[03] \quad c$ and (b) if its initial velocity is $[04] \quad c$. Remember that the work done is equal to the change of kinetic energy.

41-4. Suppose that a particle accelerator accelerates two electrons in opposite directions, such that they each have a speed of $[05] \quad c$ (in the laboratory frame of reference). (a) In the laboratory frame, what is the total kinetic energy before the collision? (It’s just the sum of the two particle’s kinetic energies.) (b) In the frame of reference of either particle, what is the total kinetic energy before the collision? (In Galilean relativity, the answer to part (b) would be only twice as large as the answer to part (a).)

41-5. A $^{57}$Fe nucleus at rest emits a $[06] \quad $-keV photon. Use conservation of energy and momentum to deduce the kinetic energy of the recoiling nucleus. (Use $Mc^2 = 8.60 \times 10^{-9}$ J for the final state of the $^{57}$Fe nucleus.)

41-6. (Paper only.) A photon collides with an electron that is at rest. The photon imparts momentum to the electron, and a new photon recoils in the opposite direction (exactly backward relative to the incoming photon). Show that the momentum of the recoiling photon $p'_{\text{photon}}$ is related to the momentum of the incident photon $p_{\text{photon}}$ through

$$\frac{1}{p'_{\text{photon}}} - \frac{1}{p_{\text{photon}}} = \frac{2}{m_ec}.$$ 

This shift in photon momentum (and hence wavelength, as you will learn) is called the Compton shift.

**Extra problems I recommend you work (not to be turned in):**

- (a) According to Newtonian physics (kinetic energy $= \frac{1}{2}mv^2$), how much work is required to accelerate an electron from rest to $0.99c$? (b) If we do that much work on an electron, what will its final speed actually be? (Answers: $4.012 \times 10^{-14}$ J, $0.7414c$.)

- In an accelerator, an electron experiences a constant electric field of $E = 1.00$ MV/m. What is its speed (a number times $c$) after 11.1 ns, assuming it starts from rest? Hint: The force on the electron is $F = qE$, where $q = 1.602 \times 10^{-19}$ C is the electron charge. Note that a megavolt per meter is the same as $10^6$ N/C. Eq. (39.20) is trivially integrated since $F$ is constant. (Answer: 0.9884$c$.)
Consider an electron which has been accelerated to a total energy of 30 GeV. (a) What fraction of the electron’s energy is due to its rest mass (i.e., the ratio of rest energy to total energy)? (b) What is the momentum in kg·m/s and (c) in GeV/c? (d) The speed of the electron can be written in the form \( v = (1 - \delta)c \). Find the value of \( \delta \).

Hint: \( \sqrt{1 + x} \approx 1 + \frac{1}{2}x \) when \( x \) is small. (Answers: \( 1.703 \times 10^{-5} \), \( 1.60 \times 10^{-17} \) kg·m/s, 30 GeV/c, \( 1.451 \times 10^{-10} \).)

A 1 kg block of copper is heated from a temperature of 25°C to a temperature of 150°C. How much does the mass of the copper change? (Answer: \( 5.38 \times 10^{-13} \) kg.)
Answers to Homework Problems, Physics 123, Fall Semester, 2010
Section 2, John Colton

1-1a. $1.00 \times 10^5$, $3.00 \times 10^5$ Pa
1-1b. $2.00 \times 10^5$, $4.00 \times 10^5$ Pa
1-2. 90, 140 cm$^2$
1-3. 150, 400 tons
1-4. 1.0, 20.0 lb
1-5. 10.0, 30.0 cm
1-6. 25.0, 50.0 kN
2-1. 40, 100 kg
2-2. 1.40, 2.30 cm
2-3b. 0.170, 0.320 m
2-4a. 500.0, 1100.0 N
2-4b. 500.0, 1100.0 N
2-4c. 5.0, 12.0 N
2-4d. 80.0, 100.0 N
2-5. 3.50, 6.00 g/cm$^3$
2-6. −1.00, −2.00 g
3-1. 10.0, 25.0 min
3-2a. 1000, 1900 ± 10 Pa
3-2b. 100, 300 lb
3-3. $1.00 \times 10^5$, $2.20 \times 10^5$ Pa
3-4a. 1.20, 2.50 cm
3-4b. 10.0, 20.0 m/s
4-1. −200.0, −400.0 °B
4-2b. 1.60010, 1.6050 cm
4-3. 500, 1100 gal
4-4a. −0.20, −1.50 mm
4-4c. 10.0, 54.0 s
4-5. 9.9920, 9.9970 cm
4-6. 100, 900 balloons
4-70. $1.5 \times 10^{12}$, $3.5 \times 10^{12}$
4-8. 100, 600 atm
5-1. $1.50 \times 10^9$, $3.00 \times 10^9$ molecules
5-2a. 0.60, 0.90 N
5-2b. 1.00, 1.40 Pa
5-3a. 60.00, 65.00 mi/h
5-3b. 60.00, 65.00 mi/h
5-4a. $2.50 \times 10^{23}$, $4.00 \times 10^{23}$
5-4b. $5.50 \times 10^{-21}$, $7.50 \times 10^{-21}$ J
5-4c. 1200, 1500 ± 10 m/s
5-4d. 1200, 1500 ± 10 m/s
5-5a. $1.00 \times 10^6$, $2.00 \times 10^6$ m
5-5b. 0.50, 0.99 h
6-1. 40, 70°C
6-2. 2.00, 5.00 h
6-4. 10.0, 20.0°C
6-5. 10.0, 20.0 g
7-1. 0.0600, 0.1100 J/s-m°C
7-2. 10.0, 25.0 J/s
7-3. 30, 100°C
8-1a. $1.00 \times 10^5$, $1.30 \times 10^5$ Pa
8-1b. −17.0, −21.0 J
8-2a. −600, −850 J
8-2b. −400, −650 J
8-2c. −200, −450 J
8-4a. −100, −140 J
8-4b. 0 J
8-4c. 100, 140 J
9-1a. 3000, 7000 ± 20 J
9-1b. 3000, 7000 ± 20 J
9-2b. 7.50, 9.00 L
9-2c. 750, 900 K
9-2d. 200, 350 J
9-3a. $3.00 \times 10^4$, $6.00 \times 10^4$ Pa
9-3b. 1000, 2000 ± 10 J
9-3c. 1000, 2000 ± 10 J
9-4a. 100, 500°C
9-4b. 10, 90 J
9-4c. 10, 90 J
9-5a. 5.0, 12.0 kJ
9-5b. $3.0 \times 10^6$, $9.5 \times 10^6$ Pa
9-5c. 700, 1200 ± 10 K
9-5d. 5.0, 12.0 kJ
10-1a. 600, 950 J
10-1b. 400, 650 J
10-2. 0.20, 0.50°C
10-3a. 4.0, 8.0 kJ
10-3b. 10.0, 30.0 kJ
10-3c. −10.0, −20.0 kJ
10-3d. 20.0, 30.0%
10-4a. $1.20 \times 10^5$, $1.500 \times 10^5$ J/s
10-4b. 25.0, 30.0 mi/gal
11-1a. 15.0, 30.0 J
11-1b. 100, 200 J
11-2. 1900, 2900 ± 10 J
11-3b. 1000, 2000 ± 10 J
11-4b. 1000, 2000 ± 10 J
11-5d. 5
11-3. 2000, 3000 ± 10 W
11-4a. 7.00, 9.99
11-4b. 30.0, 50.0 W
11-4c. 20.0, 40.0 dollars
12-1. 1.5, 3.5 J/K
12-2a. 3.00, 7.00 J/K
12-2b. 3.00, 7.00 J/K
12-3a. 6.0, 11.0 J/K
12-3b. 6.0, 11.0 J/K
12-3c. 6.0, 11.0 J/K
13-1a. 1.5 × 10^{27}, 3.0 × 10^{27}
13-1b. 4.5 × 10^4, 9.5 × 10^4 J/K
14-1a. 210, 340 m
14-1b. 2.80, 3.60 m
14-2a. 10, 30 cm
14-2b. 1.0, 3.0 s^{-1}
14-2c. 0.010, 0.025 cm^{-1}
14-2d. 3.0, 4.0 rad
14-3a. 0.80, 1.30 Hz
14-3b. 5.00, 8.00 rad/s
14-3c. 1.00, 2.00 m
14-3d. 3.00, 7.00 rad/m
14-3e. 0.50, 1.50 cm
14-3f. 0.50, 3.00 m/s
15-1. 100, 300 N
15-2. 25.0, 55.0 J
17-1. 1.00, 9.99 %
17-2a. 0.60, 0.90
17-2b. 1.00, 1.50
17-2c. 0.950, 0.999
17-3. 0.300, 0.600
18-1. 10, 50
18-2a. 1.50 × 10^{-4}, 1.90 × 10^{-4} W/m^2
18-2b. 82.0, 84.0 dB
18-3a. 5.0, 70.0 mW/m^2
18-3b. 95.0, 110.0 dB
18-3c. 0.50, 3.00 m
18-4. 0.0100, 0.0600 J
18-5. 1.3, 3.0
18-6. 1.20, 2.00 s
19-1. 42.20, 42.70 kHz
19-2a. 320, 380 Hz
19-2b. 460, 520 Hz
19-3. 20.0, 35.0 m/s
19-4b. 1.00 × 10^8, 2.00 × 10^8 m/s
19-5a. 0.10, 0.60 m
19-5b. 0.50, 1.50 m
19-6a. 8000, 9000 ± 10 rad/s
19-6b. 20.0, 30.0 rad/m
19-6c. 0.0100, 0.0300 Pa
19-6d. 80, 180 degrees
19-6e. 1000, 1500 ± 10 Hz
20-1. 2.0, 7.0 g/m
20-2a. 100, 200 lb
20-2b. 10.0, 20.0 tons
20-3a. 0.400, 0.600 m
20-3b. 300, 450 Hz
20-4a. 23.00, 25.00 cm
20-4b. 0.30, 0.70 cm
20-4c. 365.0, 371.0 Hz
21-1. 9.0, 17.0 Hz
21-2a. 525.0, 529.0 Hz
21-2b. −1.0, −2.0%
21-3. 2.0, 6.0 Hz
21-4. 1.0, 4.0 cm
21-5a. 10, 25
21-5b. 300, 350 Hz
21-5c. 1, 10
21-5d. 1450, 1490 Hz
21-5e. 15, 25
25-1a. 34.0, 38.0°
25-1b. 60.0, 70.0°
25-2a. 49.00, 50.00°
25-2b. 49.00, 51.00°
25-2c. 48.00, 50.00°
26-1a. 1.00, 2.00 m
26-2. 25.0, 85.0 degrees
27-1. 30, 50°
27-2a. 0.00, 6.00 W/m^2
27-2b. 0.00, 6.00 W/m^2
27-2c. 0.00, 6.00 W/m^2
27-2d. 0.00, 6.00 W/m^2
27-3b. 36.00, 38.00 degrees
28-1a. 3.70, 4.30 cm
| 28-1b. | -0.20, -0.50          | 32-1. | 160, 300 Hz |
| 28-1c. | -0.50, -0.90 cm       | 32-2. | 1.9, 3.7 mm |
| 28-2a. | -2.0, -9.9 cm         | 32-3. | 400, 800 ± 10 nm |
| 28-2b. | 1.0, 5.0              | 32-4. | 0.000, 0.100 W/cm² |
| 28-2c. | 1.0, 5.0 cm           | 33-1. | 480, 660 nm |
| 28-3a. | -1.50, -1.80 cm       | 33-2. | 200, 400 nm |
| 28-3b. | 0.140, 0.200          | 33-3b. | 6.0, 9.0 µm |
| 28-3c. | 0.50, 0.99 cm         | 33-4. | 1.000300, 1.000500 |
| 28-4a. | 25.0, 35.0 cm         | 34-1. | 400, 800 ± 10 nm |
| 28-4c. | -25.0, -35.0 cm       | 34-2. | 50.0, 99.9 µm |
| 29-1.  | 50, 80 cm             | 35-1. | 0.1, 1.0 m |
| 29-2a. | 5.50, 6.50 cm         | 35-2a. | 5.0, 25.0 cm |
| 29-3a. | 4.00, 9.50 cm         | 35-2b. | 10.0, 60.0 |
| 29-3b. | -1.00, -4.00          | 35-3a. | 5, 30° |
| 29-3c. | -1.00, -4.00 cm       | 35-3b. | 3.0 × 10⁻⁵, 5.0 × 10⁻⁵ degrees |
| 29-4a. | -3.0, -9.0 cm         | 35-4a. | 0.80, 1.30 D |
| 29-4b. | 1.5, 2.5              | 35-4b. | 1.00, 1.60 λ |
| 29-4c. | 1.5, 2.5 cm           | 35-5a. | 1.00, 1.50 m |
| 29-5a. | -2.50, -3.50 cm       | 35-5b. | 2.00, 4.00 m |
| 29-5b. | 0.30, 0.50            | 35-6a. | 0.040, 0.120° |
| 29-5c. | 1.50, 2.50 cm         | 35-6b. | 200, 600 ± 10 µm |
| 29-6a. | -20.0, -30.0 cm       | 35-6c. | 900, 1100 slits |
| 29-6b. | 2.0, 4.0              | 37-1a. | 50.0, 80.0 m/s |
| 29-7a. | 110, 220 cm           | 37-1b. | 10.0, 40.0 m/s |
| 29-7c. | 0.800, 0.900          | 37-1c. | 40.0, 60.0 m/s |
| 30-1.  | -40, -60 cm           | 38-1. | 6.0 × 10⁻¹², 9.9 × 10⁻¹² m |
| 30-2.  | 25.0, 35.0 cm         | 38-2. | 0.999880, 0.999950c |
| 30-3a. | 40.0, 70.0 cm         | 38-3. | 0.999850c, 0.999950c |
| 30-3b. | 0.10, 0.70 mm         | 38-4a. | 6.00, 9.50 y |
| 30-3c. | 0.040, 0.099 mm       | 38-4b. | 1.50, 3.50 y |
| 30-3d. | 0.010, 0.040 mm       | 38-4c. | 1.50, 3.50 ly |
| 30-4.  | 7.00, 9.00 cm         | 38-5. | 0.150, 0.300 c |
| 30-5.  | 10.0, 50.0 cm         | 38-6b. | 2.50 × 10⁸, 3.00 × 10⁸ m/s |
| 31-1a. | 0.100, 0.150°         | 38-6c. | 1.20 × 10¹⁰, 1.35 × 10¹⁰ light years |
| 31-1b. | 0.150, 0.200°         | 38-6d. | 1.20 × 10¹⁰, 1.35 × 10¹⁰ y |
| 31-1c. | 1.30, 1.60            | 39-1a. | 2.20 × 10⁸, 2.70 × 10⁸ m/s |
| 31-2a. | 2.00, 4.00            | 39-1b. | 4.70, 5.70 m |
| 31-2b. | 2.00, 4.00            | 39-1c. | -1.10 × 10⁻⁸, -1.60 × 10⁻⁸ s |
| 31-3a. | 0.150, 0.250 mm       | 39-2a. | 40.0, 70.0 years |
| 31-3b. | 1.00, 1.60°           | 39-2b. | 0.060, 0.099 years |
| 31-3c. | 30.0, 60.0 cm         | 39-2c. | 40.0, 70.0 years |
| 31-4b. | 0.2, 5.0 mm           | 39-3. | 0.65, 0.95c |
39-4. 0.950, 0.990c
40-1c. 14.0, 19.0 min
41-1b. 0.60, 0.99 c
41-2a. 2.00, 3.50 MeV
41-2b. 0.960, 0.990c
41-3a. 0.50, 0.99 keV
41-3b. 50, 99 keV
41-4a. $1.00 \times 10^{-12}$, $4.00 \times 10^{-12}$ J
41-4b. $1.00 \times 10^{-11}$, $9.00 \times 10^{-11}$ J
41-5. $1.50 \times 10^{-3}$, $4.00 \times 10^{-3}$ eV
In this lab you will measure the density of an unknown liquid. You do this by forcing the liquid up a tube using a known amount of pressure (see figure).

Pressurize the bottle of liquid by squeezing the hand pump repeatedly. The liquid should be forced up the tube. Be sure that the silver air release value is closed (twist it clockwise). Increase the pressure until the level of the liquid in the tube is almost 2 m above the floor. If you overshoot 2 m, you may lower the level of the liquid by opening the air release valve (twist it counter-clockwise).

Using the 2-meter stick, measure $h_1$ and $h_2$ (relative to the bottom of the bottle) and calculate $\Delta h = h_2 - h_1$. Record the results below. Record the pressure measured by the gauge. (Note that this is the pressure $P - P_0$ relative to the atmospheric pressure $P_0$. Also note that the units of pressure measured by the gauge is oz/in$^2$. 16 oz = 1 lb.) Using $P = P_0 + \rho gh$, calculate the density $\rho$ of the liquid and record the result below. Your result should be accurate to the nearest 0.01 g/cm$^3$. Please release the air pressure when you are finished.

$h_1 = \underline{\hspace{2cm}}$

$h_2 = \underline{\hspace{2cm}}$

$h = \underline{\hspace{2cm}}$

$P - P_0 = \underline{\hspace{2cm}}$

$\rho = \underline{\hspace{2cm}}$
In this lab, you will measure the specific heat of aluminum. A strap is wound around an aluminum cylinder of mass $m = 216$ g and radius $r = 1.00$ inch. One end of the strap is attached to a weight of mass $M = 1.00$ kg, and the other end is secured to a fixed support. As you turn the cylinder, the weight is lifted up slightly. The strap slips around the cylinder, and the weight is lifted due to a frictional force $Mg$ between the strap and cylinder. When you turn the cylinder one revolution, the work done by the friction is equal to $W = (Mg)(2\pi r)$. This work becomes heat which causes the temperature of the cylinder to rise.

The temperature of the cylinder is measured using a thermocouple wire which is connected to a digital meter. Insert the wire into the shallow hole at the center of the red circle drawn on the end of the aluminum cylinder. Hold it there for 30 seconds. If you hold it with your fingers, then be sure to keep your fingers at least two inches away from the end of the wire so that the heat from your fingers does not influence the reading of the temperature.

In order to minimize the effect of the heat flow between the cylinder and the surrounding air, we first cool down the cylinder to a few degrees below room temperature. This is done by pressing a piece of cold aluminum supplied with the apparatus against the rotating cylinder for about two seconds. If the temperature is still not below room temperature (perhaps because someone else had just finished the lab and left the cylinder hot), press the piece of cold aluminum against the cylinder for another two seconds or so. Do not lower the temperature below about 18°C.

Record the initial temperature $T_i$ below. Turn the crank on the cylinder 100 times. Note that every revolution of the crank produces 12 revolutions of the cylinder, so the cylinder has actually gone through 1200 revolutions. Record the final temperature $T_f$. Calculate the change in temperature $\Delta T$. Calculate the work $W$ per revolution done. Calculate the total work $W$ done. Calculate the specific heat $c$ of the cylinder.

$T_i = \underline{\hspace{2cm}}$

$T_f = \underline{\hspace{2cm}}$

$\Delta T = \underline{\hspace{2cm}}$

$W/\text{revolution} = \underline{\hspace{2cm}}$

$\text{total } W = \underline{\hspace{2cm}}$

$c = \underline{\hspace{2cm}}$
In this lab you will use a computer simulation to study how wave packets propagate in linear media. You will study both non-dispersive media in which sine-waves of all wavelengths travel at the same speed (like, for example, light traveling in a vacuum) as well as dispersive media (like light traveling through a piece of glass, electron quantum waves traveling through space, and just about every other real system).

The first step is to go to the class website and click the “Lab 3 - Dispersion” link. You can run the applet and get additional help there. Once the applet is running, you should see a screen with two graphs and some text. The next step is to click on the red “get help” button in the upper left-hand corner and read the instructions for the software. Before proceeding, you may want to play with the program for a bit to make sure that you understand how it works.

Uncertainty First let’s explore the uncertainty which is inherent in waves. To do this, first click on “Reset All.” In the upper graph you should see a depiction of a Gaussian wave packet (a little “burst” of a sine-wave with a Gaussian-shaped “envelope”). In the lower graph you can see the spectrum of the pulse (the amplitude of each of the sine waves which the computer added together to make the wave packet in the upper graph). On the far right-hand side of the program the computer displays $\Delta x$; (the standard deviation of the pulse in space), $\Delta k$ (the standard deviation of the pulse’s spectrum), and the product of the two.

We learned in class that in order to make pulses which were very narrow in space, we have to add a wide band of frequencies or wavenumbers together, making it difficult to state with certainty what the frequency of the pulse is. To make a wave packet with a very well defined frequency or wavenumber we have to let the packet extend over a large range in space such that it is difficult to assign a location to the packet with precision. Furthermore, we learned that if we defined uncertainty to be the RMS standard deviation, the uncertainties in $x$ and $k$ follow the uncertainty relation $\Delta x \Delta k \leq \frac{\hbar}{2}$.

Notice that our wave satisfies the above uncertainty relation. Now type in a different value for the pulse width ($w$). Notice that as the pulse shrinks, its spectrum widens. The uncertainty relation should still hold. Now change the central wavenumber ($k$) and see what happens.

Now click “Reset All,” enter 150 for $k$, and enter $\text{squarepulse}(x/w)$ for the “Envelope.” Now try different values for the pulse width and fill in the table below. Then answer the question below the table.
Note that the physical size of the pulse on the screen is about 4 times larger than $\Delta x$. This is just due to the fact that we have chosen to define uncertainty as the RMS standard deviation. This is the most commonly used but not always the most useful definition. So, you see, there is uncertainty in our definition of uncertainty! As a result, the uncertainty relation is often written in the less precise form: $\Delta x \Delta k \gtrsim 1$.

**Non-dispersive media.** In this part of the lab we will examine what happens when wave pulses travel in non-dispersive media. In non-dispersive media the angular frequency of a sine wave is simply proportional to the wavenumber of the wave: $\omega(k) = vk$, where $v$ is the velocity that waves travel through the medium. Wait a minute... is that the phase or group velocity? Think about this for one minute, and then answer the following two questions in the space provided.

- The dispersion relation of light traveling through a vacuum is just $\omega(k) = ck$, where $c$ is equal to $2.9979 \times 10^8 \text{ m/s}$. What is the phase velocity for a pulse of light whose central wavelength is 657 nm?

- What is the group velocity for such a light pulse?

Now let’s use the computer simulation to see what happens to a Gaussian-shaped pulse as it propagates through a non-dispersive medium. First click on the “Reset All” button. There should now be a pretty pulse displayed in the upper graph, with a nice spectrum centered around a wavenumber of 75 m$^{-1}$ in the lower graph. Now click on the “Go!” button to let time run and see what happens. The dispersion relation, shown just below the “Reset All” button, is $\omega(k) = 0.1 \text{ m/s} \cdot k$. Use this dispersion relation to answer the following question.
• What is the group velocity for a pulse in this medium centered at 75 m\(^{-1}\)?

Now click on the “Stop” button to stop the simulation if it hasn’t already stopped, and click on the “Reset t=0” button to set time back to zero. Now plug the group velocity you calculated above into the “x-Axis Velocity” box to make our “view window” move with the pulse. Click on “Go!” If you did your calculation correctly, the pulse should stand still in the window.

Based on what you have seen, answer the following question.

• What happens to the spatial size of a pulse and the spread of frequencies or wavenumbers in a pulse as it travels in a non-dispersive medium?

**Dispersive Media.** Now let’s pick a dispersion relation which is a little more interesting. Click on “Reset All,” and then enter the dispersion relation 0.001\(k^2\). Before you do anything else, use this dispersion relation to calculate the group and phase velocities for a pulse centered around \(k = 75\text{m}^{-1}\).

• Group Velocity

• Phase Velocity

Now click on “Go!” and see what happens. Now stop the simulation, set time to \(t = -10\), and set the “x-Axis Velocity” equal to the group velocity you calculated above. Press “Go!” again and watch what happens. Now stop the simulation, set time to \(t = -2.5\), and set the “x-Axis Velocity” equal to the phase velocity calculated above. Press “Go!” and see what happens (hint: this is the part of the lab where the vertical blue line in the center of the graph is useful). Finally, based on what you saw and in your own words explain what phase and group velocity represent:

• Group velocity is…
• Phase velocity is…

Now, based on what you have seen, answer the following question.
• What happens to the spatial “size” of a pulse when it travels through a dispersive medium?

• What happens to the spectrum of a pulse when it travels through a dispersive medium?

That’s the end of the lab, but I recommend that you take some additional time to play around with this simulation. If you can develop a solid understanding of dispersion, uncertainty, and group and phase velocities, you will be able to better understand many more concepts that you will learn in future courses in physics, chemistry, engineering, etc. After all, quantum mechanics tells us that everything is a wave, and that even a vacuum is dispersive for waves that represent matter!
In this lab, you will produce standing waves in a wire. This is done by placing the wire through the poles of a magnet and passing an alternating current (60.00 Hz) through the wire. The resulting force of the magnetic field on the current drives the wire into a vertical oscillation at 60.00 Hz. The tension in the wire is equal to the weight hanging at the end. At certain tensions, the wire will resonate and produce visible standing waves.

Produce a standing wave by adjusting the amount of water in the container and thus changing the tension in the wire. (Don't add any additional weight beside water. You may break the wire.) Adjust the tension until the amplitude of the antinodes is as large as possible (even though the nodes may not be as well defined). Using a meter stick, measure the wavelength $\lambda$ of the standing wave. Calculate the velocity $v$ of the waves in the wire. Weigh the container of water to obtain its mass $m$. Calculate the tension $F$ in the wire. From $F$ and $v$, calculate the linear mass density $\mu$ of the wire. Repeat this for a different standing wave.

<table>
<thead>
<tr>
<th>1st Standing Wave</th>
<th>2nd Standing Wave</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda$ =</td>
<td></td>
</tr>
<tr>
<td>$v$ =</td>
<td></td>
</tr>
<tr>
<td>$m$ =</td>
<td></td>
</tr>
<tr>
<td>$F$ =</td>
<td></td>
</tr>
<tr>
<td>$\mu$ =</td>
<td></td>
</tr>
</tbody>
</table>
In this lab, you will produce standing waves in a pipe. This is done by placing a speaker at an open end of the pipe and driving the speaker with an oscillator as shown below:

A piston is inserted into the other end of the pipe. At certain positions of the piston, the speaker will cause the pipe to resonate, thus producing standing waves.

Set the frequency $f$ of the oscillator at approximately 700 Hz. Read the frequency shown on the counter and record it below. Starting with the piston at the end of the pipe, push it in slowly. You will notice that at certain positions, the sound of the speaker is enhanced. This is caused by standing waves in the pipe. Use the sound meter to accurately determine the position of the piston where the enhanced sound is loudest. Measure the distance $l$ between the piston and the open end of the pipe at all positions of the piston for which this occurs and record it below. You ought to find 5 of them.

For each standing wave, the piston is at a position of a displacement node. From the data, you can thus obtain the distance between nodes and consequently the wavelength $\lambda$. Using the wavelength and frequency, calculate the velocity of sound to the nearest m/s (three significant figures). Record these results below.

\[
f = \underline{\quad} \\
\lambda = \underline{\quad} \\
v = f \lambda = \underline{\quad}
\]
Physics 123

Lab #6

Fourier Transforms

In this lab you will study the relationship between time dependent signals and their frequency spectrum (i.e., their Fourier transform). You will do this using a computer program which can generate or record waveforms or read-in pre-recorded waveforms. This program will display the waveform along with its Fourier transform.

The first step is to go to the class website and click the “Lab 6 - Fourier transforms” link. You can run the applet and get additional help there. The next thing to do is to play with the program and make sure that you understand how to use it. In particular, make sure you understand how to zoom in and out on the graphs, and how to find the exact value of a point by right-clicking on it.

**Musical Octaves.** Click on “RESET ALL”. This will set up the program to work with a “user defined” waveform and set the waveform equal to \( \sin(2\pi \times 440 \times t) \). This will generate a sine wave at 440 Hz (the A above middle C). Now, adjust the frequency (the 440) until you hear a tone which is one octave higher. Note the frequency below. Adjust the frequency again until the tone is another octave higher. Note the frequency below. Now think to yourself— does this agree with what we studied in class?

\[
\begin{align*}
\text{f_{One Octave Up}} &= \\
\text{f_{Two Octaves Up}} &=
\end{align*}
\]

**Generating a Square Wave.** Now enter \( \text{squarewave}(2\pi \times 440 \times t) \) as the user defined waveform to generate a 440 Hz square wave. Zoom in on the wave until you can see that it is, indeed, a square wave. Play the wave and hear what it sounds like. Now, using the “Its spectrum” graph, find the frequency and amplitude of the four lowest-frequency Fourier components and record them in the table below. Also record the frequency divided by the fundamental frequency (440 Hz). (Hint, \( f/440\text{Hz} \) should be an integer for all of the components, and should equal 1 for the lowest frequency component.)

<table>
<thead>
<tr>
<th></th>
<th>( f )</th>
<th>( A )</th>
<th>( f/440 \text{ Hz} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
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<tr>
<td>2.</td>
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<td>3.</td>
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<td>4.</td>
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</table>
Now let’s see what happens when we add together four sine waves with the above frequencies and amplitudes. Type

\[ A\sin(2\pi f_a t) + B\sin(2\pi f_b t) + C\sin(2\pi f_c t) + D\sin(2\pi f_d t) \]

in as the user defined waveform, where \( A, B, C \) and \( D \) are the amplitudes you measured above, and \( f_a, f_b, f_c, \) and \( f_d \) are the frequencies which go with each amplitude. Click on “Recalc/Record” and then zoom in on the graph of the wave to see if it looks like a square wave. Sketch what you see below:

For kicks, you might want to see what the wave looks like as you add more and more sine terms together. You can get a pretty decent looking square wave!

**Uncertainty Relations.** Now let’s make a short pulse of sound and explore the topic of “wave uncertainty”. Enter \( \sin(2\pi 440 t) \exp(-10000(t-0.5)^2) \) as the user defined waveform and click on “Recalc/Record.” Click on “Zoom to fit,” and take a look at the wave and its spectrum. Then play the wave. Now zoom in on the wave and on its spectrum and estimate \( \Delta t \) and \( \Delta f \). Now calculate \( \Delta \omega \) from \( \Delta f \), and calculate the uncertainty product \( \Delta \omega \Delta t \) and record everything below.

\( \Delta t: \)

\( \Delta f: \)

\( \Delta \omega: \)

\( \Delta \omega \Delta t: \)

Now make the pulse shorter and longer in time by changing the 10000 in the waveform to other numbers. Change it by at least a factor of 20 in both directions (smaller and larger). Describe below what happens to the width of the spectrum when you change the duration of the pulse in time. Why does this happen?
Describe below what happens to the tone of the note as you change the duration of the pulse in time. Why does this happen?

Playing Around. You have now finished the lab. But for your own learning experience I recommend that you play around with the program. In particular, you should do the following things.

(1) Record the sound of your hands clapping (or use the pre-recorded sound of my hands clapping, available under the “Waveform” drop-down box) and see if the uncertainty product \( \Delta \omega \Delta t \) makes sense.

(2) Listen to the various pre-recorded waveforms and note their spectral properties. Notice that most of the instruments have a spectrum which looks like a harmonic series and ask yourself why that is the case. Also notice that the percussive instruments do not have a spectrum which looks like a harmonic series. Not even the timpani which seems to generate a specific tone! Ask yourself why a timpani’s waveform does not consist of a harmonic series of frequencies.

(3) Try to generate different waveforms by adding sine waves together. You might want to actually calculate the Fourier transform of some waveform, and then plug the results in and see what you get.
In this lab, you will measure the Brewster angle for two different materials. From these measurements, you will then calculate the index of refraction for each material.

As shown in the figure below, a laser beam is directed towards the surface of a sample. The sample is mounted on a platform which can be rotated. The pointer attached to the platform points in a direction perpendicular to the surface of the sample. The incident angle $\theta$ of the beam can be read from a scale on the apparatus.

The reflected beam passes through a sheet of Polaroid and hits a white screen. The transmission axis of the Polaroid is horizontal. When the angle of the incident beam is equal to the Brewster angle, the reflected beam is polarized vertically and thus will not pass through the Polaroid. At this angle, the illuminated spot on the screen will disappear. (Actually, since the sample and the Polaroid are not ideal, the spot will not disappear completely, but will have a minimum intensity.)

There are two samples. One is ordinary glass, and the other is zirconium oxide (ZrO$_2$). First insert the glass into the sample holder. Rotate the sample platform and find the orientation where the reflected beam has a minimum intensity. Be sure that the Polaroid sheet is in place so that the reflected beam passes through it. Read the incident angle from the scale and record it below. This is the Brewster's angle $\theta_p$. Determine the index of refraction from $n = \tan \theta_p$ and record it below. Repeat this for the ZrO$_2$ sample.

**Warning:** Do not touch the sample surfaces. Fingerprints on the samples will affect your measurements. Wipe off any fingerprints with the tissues provided.

Glass sample: $\theta_p =$ _____________  $n =$ _____________

ZrO2 sample: $\theta_p =$ _____________  $n =$ _____________

When you are finished, remove the Polaroid sheet and notice how intense the reflected beam is. Then place a small circular Polaroid sheet in the path of the reflected beam and observe how its intensity changes as you rotate the sheet.
In this lab, you will construct a simple telescope using two lenses. Mount the source (illuminated arrow) and the screen on the optical bench, and mount one of the lenses between them. Adjust their positions until a real image of the arrow is focused on the screen. For best results, adjust the positions so that the lens is about half-way between the object and the image. Measure \( p \) and \( q \). Calculate \( f \) from the thin lens equation,

\[
\frac{1}{f} = \frac{1}{p} + \frac{1}{q}.
\]

Repeat for the other lens. Record your results below.

Construct a telescope by mounting the two lenses a distance \( f_1 + f_2 \) apart. Use the lens with the smaller focal length for the eyepiece. View the large scale mounted on the wall across the room. The distance between the two lenses may be adjusted to bring the image into better focus.

Measure the angular magnification \( m \) of the telescope by viewing the scale through the telescope with one eye and looking directly at the scale with the other eye. In this way, you ought to be able to see both the magnified and unmagnified scale superimposed on each other.

Finally, calculate \( m \) from the measured focal lengths.

<table>
<thead>
<tr>
<th>lens 1</th>
<th>lens 2</th>
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<tbody>
<tr>
<td>( p = )</td>
<td>( p = )</td>
</tr>
<tr>
<td>( q = )</td>
<td>( q = )</td>
</tr>
<tr>
<td>( f = )</td>
<td>( f = )</td>
</tr>
<tr>
<td>( m = ) measured</td>
<td>( m = ) calculated</td>
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In this lab, you will use a Michelson interferometer to measure the index of refraction of a gas. A chamber which can be evacuated is placed in one arm of the interferometer. All of the air is first evacuated from the chamber. As the gas to be studied is slowly allowed to enter the chamber, the number of fringes passing by the center of the screen is counted.

The index of refraction $n$ of the gas is given by

$$n = 1 + \frac{N \lambda}{2L},$$

where $N$ is the number of fringes counted, $\lambda$ is the wavelength of the laser in vacuum, and $L$ is the length of the chamber. See the Supplement on the next page for the derivation.

Turn on the vacuum pump and evacuate the chamber. Pump for at least a couple of minutes to obtain a good vacuum. Valve off the vacuum pump and \textit{slowly} open the chamber to air and count the fringes. (You will probably open the valve too fast the first time you try, and the fringes will go by too quickly to count. If this happens, evacuate the chamber again and start over.) Repeat using helium gas instead of aid. Measure $L$ and calculate $n$ for each gas.

<table>
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<tr>
<th></th>
<th>Air</th>
<th>Helium</th>
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<tbody>
<tr>
<td>$L =$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N =$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$n =$</td>
<td></td>
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Supplement to Michelson Interferometer

When the chamber is evacuated, the number of wavelengths along its length $L$ is given by

$$N_{\text{vac}} = \frac{L}{\lambda_{\text{vac}}},$$

where $\lambda_{\text{vac}}$ is the wavelength of the laser light in vacuum. When the chamber is filled with some gas, the number of wavelengths along its length is now given by

$$N_{\text{gas}} = \frac{L}{\lambda_{\text{gas}}},$$

where $\lambda_{\text{gas}}$ is the wavelength of the laser light in the gas.

Each time one arm of the interferometer gets behind (or ahead) by one wavelength, one fringe passes by the screens. As we fill the chamber with gas, that arm of the interferometer will get behind by $N = 2(N_{\text{gas}} - N_{\text{vac}})$ wavelengths. (The factor 2 is included since the light passes through the chamber twice, once going and once coming back.) From the two above equations, we thus obtain

$$N = 2 \left( \frac{L}{\lambda_{\text{gas}}} - \frac{L}{\lambda_{\text{vac}}} \right).$$

We also know that

$$\lambda_{\text{gas}} = \frac{\lambda_{\text{vac}}}{n},$$

where $n$ is the index of refraction of the gas. Using this to solve for $n$, we obtain

$$n = 1 + \frac{N\lambda_{\text{vac}}}{2L}.$$
In this lab, you will observe the interference pattern produced by shining a laser beam through a diffraction grating. From the distance between peaks in the pattern, you will determine the distance between the slits in the grating.

The He-Ne laser used in this lab produces red light of wavelength 633 nm. Turn on the laser. Its beam should pass through the diffraction grating. You should observe the interference pattern on the wall.

Use a meter stick to measure the distance $\Delta x$ between peaks in the interference pattern. Average this distance over several adjacent peaks so that your measurement will be as accurate as possible. Record your result below.

Use the tape measure to determine the distance $L$ between the diffraction grating and the interference pattern on the wall and record your result below.

Calculate the angle $\theta$ between adjacent bright spots in the interference pattern and record your result below.

Using $d \sin \theta = \lambda = 633$ nm, calculate the distance $d$ between the slits in the grating and record your result below.

$\Delta x = \underline{\hspace{2cm}}$

$L = \underline{\hspace{2cm}}$

$\theta = \underline{\hspace{2cm}}$

$d = \underline{\hspace{2cm}}$