1. Prayer
2. While waiting, see how many of these blanks you can fill out:

**Centripetal Accel.:**
Causes change in **direction** but not **speed**
It points **inward** but not **__________**
Magnitude: \( a_c = \frac{v^2}{r} \)
How to use with N2: Always include on the right hand side

**Tangential Accel.:**
Direction: **tangent**
Causes speed to **change**
Causes angular speed to **change**
Therefore, causes: a**ngular acceleration**

Definitions: \( \theta = \text{angle} \) \( \omega = \frac{\Delta \theta}{\Delta t} \) \( \alpha = \frac{\Delta \omega}{\Delta t} \)

**Connecting eqns:** arc length \( s = \theta r \) \( v_{\text{tan}} = \omega r \) \( a_{\text{tan}} = \alpha r \)

**Angular Kinematic Equations:**

1. \( \omega_f = \omega_i + \alpha t \)
2. \( \theta_f = \theta_i + \omega_i t + \frac{1}{2} \alpha t^2 \)
3. \( \omega_f^2 = \omega_i^2 + 2 \alpha \Delta \theta \)

Angular acceleration is caused by **torque**.
“Which of the problems from last night's HW assignment would you most like me to discuss in class today?”
Torque

Definition: \[ \tau = r \perp F \]

Measure \( r \) from a "pivot point" (or potential pivot point)
From warmup

In order to apply the most torque to a bolt, you should:

a. use a wrench with a long handle
b. use a wrench with a short handle
c. there would be no difference
Demo: T-handle torque
Torque tug-of-war
“Lever Arm”

some force $F$

pivot point

Perpendicular distance $r_\perp = \text{"lever arm" of the force}$

$F_\perp$

$F_\parallel$

$F$

$r$

$F_\perp$

$F_\parallel$

$\tau = r_\perp F = rF_\perp = rF \sin \theta$
Torque Summary

\[ \tau = r \perp F = rF \perp = rF \sin \theta \]

→ but be careful about which angle you call \( \theta \)!

**Note:** If you are familiar with vector cross products, you can write it like this: \( \mathbf{\tau} = \mathbf{r} \times \mathbf{F} \)
Two people sit on a seesaw. They sit in positions such that the seesaw is balanced in a horizontal position. The two people must weigh the same amount.

a. true
b. false
Where should the large elephant stand in order to balance the seesaw? (big elephant mass = 4 \times\) the little elephant mass)

- a. \(d\)
- b. \(d/2\)
- c. \(d/4\)  \(\text{Correct Answer}\)
- d. \(d/6\)
- e. \(d/8\)
Clicker quiz

When the see-saw is balanced, what is the upwards force from the pivot point? (Or, equivalently, the downward force on the pivot point.)

- a. $mg$
- b. $4mg$
- c. $5mg$
- d. $6mg$
- e. $8mg$

$$4F_y = 0$$
$$N - mg - 4mg = 0$$
$$N = 5mg$$
Center of mass

Where is the center of mass of the elephants?
Demos: Center of mass (balanced objects)
Equilibrium

What concepts are involved?

1. If an object is not moving ("translational equilibrium"), then…

\[ \sum F_x = 0 \]
\[ \sum F_y = 0 \]

2. If an object is not *rotating* ("rotational equilibrium"), then…

\[ \sum \tau = 0 \]

A new blueprint equation!
From warmup

If an object is in equilibrium:

a. the net force on it must be zero
b. the net torque on it must be zero
c. both of the above
d. neither of the above
Blueprint advice

\[ \sum \tau = 0 \]

if it has no ang. accel.

Think carefully about the pivot point and the sign of the torques.

Colton - Lecture 14 - pg 15
Worked problem
A 1500 N man is standing on a board supported by a wall and a rope. He is 1 meter from the right end. The board weighs 800 N and is 4 meters long. What is the tension in the rope?

1. Draw all of the forces present.
   Note: gravity acts at the center of mass

1b. Divide forces into components

2. Use $\sum \mathbf{F}$ blueprint equation(s)

   $\Sigma F_x = 0$

   $N - T\sin\theta = 0$

   $\Sigma F_y = 0$

   $(F_f) + (T\sin\theta) - W_b - W_m = 0$

3. Use $\sum \tau$ blueprint equation

   $\rightarrow$ which point to use as the "pivot point"?

   $\Sigma \tau = 0$

   $-W_b(2x) - W_m(3x) + (T\sin\theta)(4x) = 0$
4. Use the filled-in blueprints to solve for what you’re looking for.

\[-w_b \cdot 2 - w_m \cdot 3 + T \sin \alpha \cdot 4 = 0\]

\[T \sin \alpha \cdot 4 = 2w_b + 3w_m\]

\[4 \, s = 0\]

\[T = \frac{2(800 \text{ N}) + 3(1500 \text{ N})}{4 \, s = 30^\circ}\]

\[= 3050 \text{ N}\]

Answer: \( T = 3050 \text{ N} \)
**Additional question**

What are the horizontal and vertical forces of the wall on the board?

\[ N - T \cos \theta = 0 \]

\[ N = T \cos \theta \]

\[ = 3050 \text{ N} \cdot \cos 30^\circ \]

\[ = 2641 \text{ N} \]

\[ F_f = T \sin \theta - W_b - W_m = 0 \]

\[ F_f = 800 \text{ N} + 1500 \text{ N} - 3050 \text{ N} \cdot \sin 30^\circ \]

\[ = 775 \text{ N} \]

Answers: \( F_x = 2641 \text{ N} \) to right, \( F_y = 775 \text{ N} \) up
From warmup
Ralph noticed that both torque and work are obtained by multiplying a force times a distance. He wants to know: how are they different? Do they have the same units? What can you tell Ralph to help him out?

“Think-pair-share”
- Think about it for a bit
- Talk to your neighbor, find out if he/she thinks the same as you
- Be prepared to share your answer with the class if called on

Clicker: I am now ready to share my answer if randomly selected.
  a. Yes

Note: you are allowed to "pass" if you would really not answer.
Problem:
(Like HW 15-2)

A ladder leans against a frictionless wall. The ground has static coefficient of friction $\mu$. What’s the smallest angle $\theta$ such that the ladder doesn’t slip? Length of ladder is $d$, mass of ladder is $m$.

Draw a FBD of ladder:

Clicker quiz: I have done so
a. yes
The ground’s frictional force is ________ compared to the wall’s normal force.

a. more than  

b. less than  

c. the same  

d. can’t tell
The ground’s normal force pushing upward is ________ compared to the weight.

- a. more than
- b. less than
- c. the same
- d. can’t tell

\[ \mathbf{f} = \mu \mathbf{N}_2 \]

\[ 3f_g = 0 \]

\[ \mathbf{N}_2 - mg = 0 \]

\[ \mathbf{N}_2 = mg \]
Clicker quiz

To solve the problem, we need to use $\sum \tau = 0$... but about which point should we compute the torques?

a. A
b. B
(c) C
Solved problem

\[ \sum \tau = 0 \]
\[ mg(\frac{L}{2} \cos \theta) - N_1 \left( L \sin \theta \right) = 0 \]
\[ \frac{mg}{2} \cos \theta = N_1 \frac{L \sin \theta}{\cos \theta} \]
\[ \tan \theta = \frac{mg}{2 \cdot N_1} \]
\[ \tan \theta = \frac{mg}{2 \left( \frac{L}{2} \sin \theta \right)} = \frac{1}{2 \mu} \]
\[ \theta = \tan^{-1} \left( \frac{1}{2 \mu} \right) \]

Numerical answers: if \( \mu = 0.5 \rightarrow \theta = 45^\circ; \quad \mu = 0.7 \rightarrow \theta = 35.5^\circ; \quad \mu = 0.9 \rightarrow \theta = 29.1^\circ \)

Answer: \( \theta = \tan^{-1} \left( \frac{1}{2 \mu} \right) \)
Modification

Suppose the wall also has friction, $\mu$. What’s the angle $\theta$ now? (Think: bigger or smaller?)

New FBD:

Equations:

\[ \Sigma F_x = 0 \]

\[ N_1 = \mu N_2 \]

\[ \Sigma F_y = 0 \]

\[ M N_1 + N_2 = mg \]

\[ \Sigma \tau_1 = 0 \]

\[ mg \frac{L}{2} \cos \theta - N_1 L \sin \theta - \mu N_1 L \cos \theta = 0 \]
Solved problem

If $\mu = 0.5 \rightarrow \theta = 36.9^\circ$; $\mu = 0.7 \rightarrow \theta = 20.0^\circ$; $\mu = 0.9 \rightarrow \theta = 6.0^\circ$

Answer: $\theta = \tan^{-1}\left(\frac{1}{2\mu} - \frac{\mu}{2}\right)$
A 0.4 kg meterstick is suspended from pulleys and support pillars (not shown) via two strings at $\theta_1 = 41.4^\circ$ and $\theta_2 = 60^\circ$, with tensions of 2 N and 3 N, respectively. The strings are attached at 10 cm and 5 cm from the two ends of the meterstick. The stick is not in equilibrium until an additional mass is hung from a point in the middle. Find the unknown $x$ and $m$. 
Answers: 0.171 kg, 38.2 cm