Announcements – 8 Oct 2013

1. Exam 2 still going on
   a. ends tomorrow 2 pm, late fee after 2 pm today
Where are we now?

Topics

**Kinematics (velocity, acceleration)**
**Vectors & 2D Motion**
**Forces & Newton’s Laws**
**Work & Energy**
**Momentum**
**Rotations, Torque, and Angular Momentum**
**Pressure**
**Fluids & Solids**
**Temperature, Heat, and Heat Flow**
**Laws of Thermodynamics**
**Vibrations & Waves**

“Mechanics”

“Thermodynamics”

Part Mechanics, Part Sound, Part Optics
Conserved quantities

**Energy**
→ When no non-conservative work done, $E_{\text{bef}} = E_{\text{aft}}$

**Mass**
→ If not converted to/from energy ($E=mc^2$),
  
  $(\text{total mass})_{\text{bef}} = (\text{total mass})_{\text{aft}}$

**Charge**
→ $(\text{total charge})_{\text{bef}} = (\text{total charge})_{\text{aft}}$
  
  I.e., if some positive charge flows out of a neutral object, it will leave the object with negatively charged

Often conserved (used to balance chemical reactions)

- **Number of each type of atom**
- **Number of electrons**

Etc.
A new conserved quantity… **momentum**

Define \( \vec{p} = m\vec{v} \) for each object, then

\[
\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \quad \text{(if no external forces)}
\]

Another blueprint equation!
Momentum: used for Collision Problems

\[ \begin{align*}
    \text{\textbf{m}_1} & \quad \text{\textbf{v}_1 \text{\textit{i}}} \\
    \text{\textbf{m}_2} & \quad \text{\textbf{-m}_2 \text{\textit{v}_2 \text{\textit{i}}}} \\
    \text{\textbf{v}_1 \text{\textit{f}}} & \quad \text{\textbf{v}_2 \text{\textit{f}}} \\
\end{align*} \]

\[ \sum \text{\textit{P}} \text{\textit{bef}} = \sum \text{\textit{P}} \text{\textit{aft}} \]
Derivation of conservation law:

\[ \sum F_1 = m_1 a_1 \]
\[ \sum F_2 = m_2 a_2 \]

Newton’s 3\textsuperscript{rd} Law: the forces involved in the collision are \underline{equal} and \underline{opposite}.

If no other forces, then...

\[ \overrightarrow{F_{2-1}} + \overrightarrow{F_{1-2}} = m_1 a_1 + m_2 a_2 \]
\[ 0 = m_1 \Delta v_1/\Delta t + m_2 \Delta v_2/\Delta t \]

Multiply by \( \Delta t \) (which is the same for both)

\[ m_1 \Delta v_1 + m_2 \Delta v_2 = 0 \]
\[ m_1 (v_{1\text{ final}} - v_{1\text{ initial}}) + m_2 (v_{2\text{ final}} - v_{2\text{ initial}}) = 0 \]
\[ m_1 v_{1\text{ initial}} + m_2 v_{2\text{ initial}} = m_1 v_{1\text{ final}} + m_2 v_{2\text{ final}} \]

... and there you have it!
From warmup

The total momentum of an isolated system of objects is conserved
  a. only if conservative forces act between the objects
  b. regardless of the nature of the forces between the objects.
From warmup

A truck always has more mass than a roller skate. Does a truck always have more momentum than a roller skate?

a. yes

b. no
Why use conservation of momentum?

it makes some problems easier!

Limitation: Like conservation of energy, conservation of momentum is a “before” and “after” law which doesn’t tell you about:

_____________________

If you want to know about ________, you have to know ________
Another useful equation:

\[ \vec{F} \Delta t = \Delta \vec{p} \]

“Impulse equation” (focusing on one object)

**Derivation:** \( \Sigma F = ma = m\Delta v/\Delta t; \) multiple both sides by \( \Delta t \)

When to use?

\[
\begin{align*}
\vec{F} &= ma \\
\vec{F} &= m\frac{\Delta \vec{v}}{\Delta t} \\
\vec{F} \Delta t &= m \Delta \vec{v} \\
\Delta \vec{p} &= 0
\end{align*}
\]
**Demo Problem:** A cart moving at 1 m/s runs into a second cart (stationary) with the same mass and sticks to it. What velocity do the two stuck together carts now have?

\[
\text{before: \[ \text{2m \rightarrow 1m/s} \]}
\]

\[
\text{after: \[ \text{2m} \rightarrow \text{vf} \]}
\]

\[
3P_{	ext{before}} = 2P_{	ext{after}}
\]

\[
a(\text{1 m/s}) = (2m)v_f
\]

\[
v_f = \frac{1}{2} \text{ m/s}
\]

**Demo Problem:** A cart moving at 1 m/s runs into a second cart (stationary) with \textit{twice} the mass and sticks to it. What velocity do the two stuck together carts now have?

\[
\text{before: \[ \text{2m \rightarrow 1m/s} \]}
\]

\[
\text{after: \[ \text{3m} \rightarrow \text{vf} \]}
\]

\[
m(\text{1 m/s}) = (3m)v_f
\]

\[
v_f = \frac{1}{3} \text{ m/s}
\]
Demo Problem: A cart moving at 1 m/s runs into a second cart with twice the mass and sticks to it. The second cart is moving at 0.5 m/s towards the first one. What velocity do the two (stuck together) carts now have?
Dr Colton’s Guide: How to solve Conservation of Momentum problems

1. Draw initial and final pictures

2. Draw momentum or velocity vectors (arrows) in each picture

3. Use $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$ as “blueprint equation”

4. Divide into separate $x$- and $y$- equations if needed

5. Fill in both sides of blueprint equation(s) using initial and final pictures: one term in equation for each arrow in picture.

6. Reminder: be careful with signs! (Momentum is a vector)
The new blueprint

\[ \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \]

Compare to previous two blueprint equations:

\[ \sum \vec{F} = m \ddot{\vec{a}} \]

\[ E_{\text{before}} = E_{\text{after}} \quad \text{(if no non-conservative forces)} \]

Similarities? Differences?
From warmup
Suppose Ralph is floating in outer space with no forces acting on him. He is at rest, so his momentum is zero. Now, he throws a ball. The ball goes one way, and he goes the other way. Before the collision, there was no momentum, and after the collision, there is plenty of momentum! Was momentum conserved?

“Pair share”—I am now ready to share my neighbor’s answer if called on.
  a. Yes
**Worked Problem**

In the new sport of “ice football”, a 100 kg defensive end running north at 4 m/s tackles a 75 kg quarterback running east at 7 m/s. There’s no friction. What is their combined velocity right after the tackle?

\[ \begin{align*}
\Sigma P_{\text{bef}} &= \Sigma P_{\text{aftr}} \\
(175 \text{ kg}) V_x &= (175 \text{ kg}) V_x \\
V_x &= 3 \text{ m/s} \\
\end{align*} \]

\[ \begin{align*}
\Sigma P_{\text{bef}} &= \Sigma P_{\text{aftr}} \\
V_y &= \frac{2.28 \text{ m}}{3} \\
V_f &= \sqrt{3^2 + \left(\frac{2.28}{3}\right)^2} = 3.77 \text{ m/s} \\
\end{align*} \]

Answers: \( v_x = 3 \text{ m/s}; \ v_y = 2.28 \text{ m/s}; \ v = 3.77 \) at 37.3° north of east
Worked Problem

An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?

\[
\begin{align*}
\Sigma P_{x\text{bef}} &= \Sigma P_{x\text{aft}} \\
(20)(100) &= (0)(V_x \cos \theta) \\
2000 &= 0 \cdot V_x \cos \theta \\
V_x &= \frac{2000}{0} \\
&= 0 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\Sigma P_{y\text{bef}} &= \Sigma P_{y\text{aft}} \\
(12)(50) &= (8)(V_f \sin \theta) \\
600 &= 8 \cdot V_f \sin \theta \\
V_f \sin \theta &= 600 \\
V_f &= \frac{600}{8} \\
&= 75 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\cos \theta &= \frac{12}{20} \\
&= 0.6 \\
\theta &= \cos^{-1}(0.6) \\
&= 53.13^\circ \\
\end{align*}
\]

Answers: \(v_x = 250 \text{ m/s}\); \(v_y = -75 \text{ m/s}\); \(v = 261 \text{ m/s at } 16.7^\circ \text{ south of east}\)
From warmup, do as clicker quiz

A ping-pong ball moving forward with a momentum $p$ strikes and bounces off backwards from a heavier tennis ball that is initially at rest and free to move. The tennis ball is set in motion with a momentum:

- a. greater than $p$
- b. less than $p$
- c. equal to $p$

What about if ping-pong ball “thuds” and falls flat?

\[ p_{\text{new}} = -3p + p_t \]

\[ p_t = 1.3p \]

**Demo:** Elastic and Inelastic Pendulum—which will cause the wood to be knocked over?
Question

Is energy conserved in collisions? All? Some? None?
Special Case: “Elastic” Collisions

In some special collisions, energy is also conserved!

*Elastic* collisions: no lost kinetic energy

→ they are “bouncy”

(but not all bouncy-looking collisions are elastic… don’t assume)

*Inelastic* collisions:

\[ \text{KE not conserved} \]

*Perfectly inelastic* collisions:

\[ \text{stick together} \rightarrow \text{most lost KE} \]
#7. If it’s an elastic collision then…

\[ \sum KE_{\text{before}} = \sum KE_{\text{after}} \]

\[ \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \]

This is in addition to the two equations can be put together to give:

\[ (v_1 - v_2)_{\text{bef}} = (v_2 - v_1)_{\text{aft}} \]

used in addition to cons. of mom. for elastic collisions

"Velocity reversal"

Careful with signs! “Right = positive, left = negative” still applies
Derivation:
Cons. mom

\[ m_1 v_{i1} + m_2 v_{i2} = m_1 v_{f1} + m_2 v_{f2} \]

\[ m_1 (v_{i1} - v_{f1}) = m_2 (v_{f2} - v_{i2}) \]

Cons. energy

\[ \frac{1}{2} m_1 v_{i1}^2 + \frac{1}{2} m_2 v_{i2}^2 = \frac{1}{2} m_1 v_{f1}^2 + \frac{1}{2} m_2 v_{f2}^2 \]

\[ m_1 (v_{i1}^2 - v_{f1}^2) = m_2 (v_{f2}^2 - v_{i2}^2) \]

Divide the two equations.

\[ \frac{m_1 (v_{i1} + v_{f1}) (v_{i1} - v_{f1})}{m_1 (v_{i1} - v_{f1})} = \frac{m_2 (v_{f2} + v_{i2}) (v_{f2} - v_{i2})}{m_2 (v_{f2} - v_{i2})} \]

\[ v_{i1} + v_{f1} = v_{f2} + v_{i2} \]

\[ v_{i1} - v_{i2} = v_{f2} - v_{f1} \]
Demo Problem
A cart moving at 1 m/s bounces elastically off of a second cart of twice the mass which is moving at 0.5 m/s in the same direction. What velocity does each cart now have?

Answer: $v_1 = 0.33$ m/s; $v_2 = 0.83$ m/s
Demo Problem
A cart moving at 1 m/s bounces elastically off of a second cart of the same mass which is stationary. What velocity does each cart now have?

Demo: Newton’s cradle
Demo problem:
Elastic collision between big M and small m

Bowling ball and a marble! Marble is at rest.

\[ v_m \text{ going towards bowling ball, 15 g} \]

\[ v_b, 5 \text{ kg} \]

What are final speeds?

Simplification: \( v_{\text{bowling ball final}} \approx v_{\text{bowling ball initial}} \)

Demo: “Velocity amplifier”