Announcements – 6 Oct 2009

0. 2nd projector back - yay!

1. Exam 2 still going on
   a. ends tomorrow, late fee after 1 pm

2. Next HW due Sat night (HW 9)

3. Don’t forget Oct 24 - Deadline to get extra point on extra credits
   a. You automatically get +1 if you turn in extra credit before Oct 24.

4) Warmup 10 (last lecture) will show up as
0 out of 0 (since there was no warmup) on your graderport

Don’t worry about it.
Which part of today's assignment was particularly hard or confusing?

It seemed like a big random jump from what we've been learning.

The part about the rocket propulsion didn't have to read

General comments:

What is the average on the test so far?

Answer: 77/92 = 84% ...but only 20% of class have taken exam, so probably not too meaningful

Is the test open all of Wednesday? Yes (lak fa after 1pm)

TRUCK vs. ROLLER SKATE!!! WOOO
Where are we now?

Topics

Kinematics (velocity, acceleration)
Vectors & 2D Motion
Forces & Newton's Laws
Work & Energy
Momentum
Rotations, Torque, and Angular Momentum
Pressure
Fluids & Solids
Temperature, Heat, and Heat Flow
Laws of Thermodynamics
Vibrations & Waves

Part Mechanics, Part Sound, Part Optics

“Mechanics”

“Thermodynamics”
Conserved quantities

Energy
→ When no non-conservative work done, \( E_{\text{bef}} = E_{\text{aft}} \)

Mass
→ If not converted to/from energy,
\[(\text{total mass})_{\text{bef}} = (\text{total mass})_{\text{aft}}\]

Charge
→ \((\text{total charge})_{\text{bef}} = (\text{total charge})_{\text{aft}}\)
I.e., if some positive charge flows out of a neutral object, it will leave the object with a negative charged

Often conserved (used to balance chemical reactions)
Number of each type of atom
Number of electrons

Etc.

A new conserved quantity… momentum
Define \( \vec{p} = m \vec{v} \) for each object, then
\[
\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \quad \text{(if no external forces)}
\]

Another blueprint equation!
Momentum: used for Collision Problems

Also Explosion

\[ \dot{v}_1 \text{ initial} \quad \dot{v}_2 \text{ initial} \]

\[ \Sigma F_1 = m_1 \dot{a}_1 \quad \Sigma F_2 = m_2 \dot{a}_2 \]

Derivation of conservation law:

Newton’s 3\textsuperscript{rd} Law: the forces in the collision are equal and opposite

If no other forces, then...

\[ F_{2-1} + F_{1-2} = m_1 \dot{a}_1 + m_2 \dot{a}_2 \]
\[ 0 = m_1 \Delta v_1/\Delta t + m_2 \Delta v_2/\Delta t \]

Multiply by \( \Delta t \) (which is the same for both)

\[ m_1 \Delta v_1 + m_2 \Delta v_2 = 0 \]
\[ m_1 (v_1 \text{ final} - v_1 \text{ initial}) + m_2 (v_2 \text{ final} - v_2 \text{ initial}) = 0 \]
\[ m_1 v_1 \text{ initial} + m_2 v_2 \text{ initial} = m_1 v_1 \text{ final} + m_2 v_2 \text{ final} \]

... and there you have it!

From warmup: The total momentum of an isolated system of objects is conserved

a. only if conservative forces act between the objects
b. regardless of the nature of the forces between the objects.
From warmup. A truck always has more mass than a roller skate. Does a truck always have more momentum than a roller skate?

a. yes  

b. no

\[ \text{if stationary, then } \vec{v} = 0 \]

\[ m \vec{v} \geq 0 \]

Why use conservation of momentum?

Make some problems much easier

Limitation: Like conservation of energy, conservation of momentum is a “before” and “after” law which doesn’t tell you about: the time it takes

If you want to know about \text{time}, you have to know \text{forces}

Another useful equation:

\[ \vec{F} \Delta t = \Delta \vec{p} \]

"Impulse equation" (if only one force)

\[ \Delta \vec{p} = m \vec{v}_f - m \vec{v}_i \]

Derivation: \[ \Sigma \vec{F} = ma = \frac{(m \Delta \vec{v})}{\Delta t}; \; \text{multiple both sides by } \Delta t \]
"\( p \) = momentum

**Demo Problem:** A cart moving at 4 m/s runs into a second cart of the same mass and sticks to it. What velocity do the two (stuck together) carts now have?

\[
\begin{align*}
\text{bef} & \quad \frac{\text{1 m}}{4 \text{ m/s}} \quad \text{ aft} \quad \frac{\text{2 m}}{\text{vf} \text{?}} \\
\end{align*}
\]

\[
\begin{align*}
\sum p_{\text{bef}} &= \sum p_{\text{aft}} \\
\text{m}(4 \text{ m/s}) &= \text{m}(2 \text{ m/s}) (vf) \\
\end{align*}
\]

\[
\begin{array}{c}
\text{vf} = 2 \text{ m/s}
\end{array}
\]

**Demo Problem:** A cart moving at 4 m/s runs into a second cart of with *twice* the mass and sticks to it. What velocity do the two (stuck together) carts now have?

\[
\begin{align*}
\text{bef} & \quad \frac{\text{2 m}}{4 \text{ m/s}} \quad \text{ aft} \quad \frac{\text{3 m}}{\text{vf} \text{?}} \\
\end{align*}
\]

\[
\begin{align*}
\sum p_{\text{bef}} &= \sum p_{\text{aft}} \\
\text{m}(4 \text{ m/s}) &= \text{m}(3 \text{ m/s}) (vf) \\
\end{align*}
\]

\[
\begin{array}{c}
\text{vf} = \frac{4}{3} \text{ m/s}
\end{array}
\]

**Demo Problem:** Two carts with the same mass spring apart. If one moves at 4 m/s to the right afterwards, what velocity does the second cart have?

\[
\begin{align*}
\text{bef} & \quad \frac{\text{m}}{4 \text{ m/s}} \quad \text{ aft} \quad \frac{\text{m}}{\text{vf} \text{?}} \\
\end{align*}
\]

\[
\begin{align*}
\sum p_{\text{bef}} &= \sum p_{\text{aft}} \\
0 &= \text{m}(4 \text{ m/s}) + \text{m} (vf) \\
\end{align*}
\]

\[
\begin{array}{c}
\text{vf} = -\frac{4}{3} \text{ m/s} \\
(4 \text{ m/s} \text{ going left})
\end{array}
\]

Colton - Lecture 11 - pg 6
Dr Colton’s Guide:
How to Solve Conservation of Momentum Problems

1. Draw initial and final pictures

2. Draw momentum or velocity vectors (arrows) in each picture

3. Use \( \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \) as “blueprint equation”

4. Divide into separate x- and y- equations if needed

5. Fill in both sides of blueprint equation(s) using initial and final pictures: one term in equation for each arrow in picture.

6. Reminder: be careful with signs! (Momentum is a vector)

Compare to previous two blueprint equations:

\[
\sum \vec{F} = m\vec{a}
\]

\[
E_{\text{before}} = E_{\text{after}} \quad \text{(if no non-conservative forces)}
\]

Similarities? Differences?
**Problem:** In the new sport of “ice football”, a 100 kg defensive end running north at 4 m/s tackles a 75 kg quarterback running east at 7 m/s. There’s no friction. What is their combined velocity right after the tackle?

\[
\begin{align*}
\Sigma F_{\text{net}x} &= \Sigma F_{\text{net}y} \\
(75 \text{ kg})(7 \text{ m/s}) &= (175 \text{ kg})v_x \\
v_x &= 3 \text{ m/s}
\end{align*}
\]

\[
\begin{align*}
\Sigma F_{\text{net}y} &= \Sigma F_{\text{net}y} \\
(100 \text{ kg})(4 \text{ m/s}) &= (175 \text{ kg})v_y \\
v_y &= 2.28 \text{ m/s}
\end{align*}
\]

\[
v_f = \sqrt{3^2 + 2.28^2} = 3.77 \text{ m/s}
\]

\[
\theta = \tan^{-1} \left( \frac{2.28}{3} \right) = 37.3^\circ \text{ north of east}
\]

**Answers:** \(v_x = 3 \text{ m/s}; v_y = 2.28 \text{ m/s}; v = 3.77 \text{ at 37.3}^\circ \text{ north of east} \)
Problem: An artillery shell of mass 20 kg is moving east at 100 m/s. It explodes into two pieces. One piece (mass 12 kg) is seen moving north at 50 m/s. What is the velocity (magnitude and direction) of the other piece?

\[ 2 \cdot \vec{p}_{\text{bef}} = \sum \vec{p}_{\text{aft}} \]
\[ (20 \text{ kg})(100 \text{ m/s}) = (8 \text{ kg}) \cdot v_x \]
\[ v_x = \frac{250 \text{ m}}{s} \]

\[ 0 = (12 \text{ kg})(50 \text{ m/s}) + (8 \text{ kg}) \cdot v_y \]
\[ v_y = -75 \text{ m/s} \]

\[ v_f = \sqrt{250^2 + 75^2} = 261 \text{ m/s} \]
\[ \theta = \tan^{-1} \left( \frac{75}{250} \right) = 16.7^\circ \text{ south of east} \]

Answers: \( v_x = 250 \text{ m/s} \); \( v_y = -75 \text{ m/s} \); \( v = 261 \text{ m/s} \) at 16.7° south of east
From warmup: Suppose Ralph is floating in outer space with no forces acting on him. He is at rest, so his momentum is zero. Now, he throws a ball. The ball goes one way, and he goes the other way. Before the collision, there was no momentum, and after the collision, there is plenty of momentum! Was momentum conserved?

Answer from the class: 741 ----------------
...it was still conserved because momentum is a vector quantity, so direction matters. The ball and Ralph were floating in opposite directions, but with the same momentum, making the net momentum zero, the same as when he was at rest.

From warmup, do as clicker quiz: A ping-pong ball moving forward with a momentum \( p \) strikes and bounces off backwards from a heavier tennis ball that is initially at rest and free to move. The tennis ball is set in motion with a momentum:

- a. greater than \( p \)
- b. less than \( p \)
- c. equal to \( p \)

\[
\text{a. greater than } p \quad \Sigma p_{\text{rel}} = \Sigma p_{\text{final}}
\]
\[
\text{b. less than } p \quad 1 = -5 + P_{\text{tennis}}
\]
\[
\text{c. equal to } p \quad P_{\text{tennis}} = 1.5 \text{ kg m/s}
\]

What about if ping-pong ball "thuds" and falls flat? \( P_{\text{tennis}} = 1 \text{ kg m/s} \)

Demo: Elastic and Inelastic Pendulum—which will cause the wood to be knocked over?

Question: Is energy conserved in collisions? All? Some? None?
Special Case: “Elastic” Collisions

In some special collisions, energy is also conserved!

*Elastic* collisions: no lost kinetic energy

→ they are “bouncy”

(but not all bouncy-looking collisions are elastic… don’t assume)

*Inelastic* collisions: some KE lost

Perfectly inelastic collisions:

most possible KE lost

two objects stick together
Dr. Colton's guide, cont.

#7. If it's an elastic collision ...

\[ \Sigma KE_{\text{before}} = \Sigma KE_{\text{after}} \]
\[ \rightarrow \text{This is in addition to } \sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}} \]

The two equations can be put together to give:

\[ (v_1 - v_2)_{\text{bef}} = (v_2 - v_1)_{\text{aft}} \]

"velocity reversal"

Careful with signs! “Right = positive, left = negative” still applies

**Derivation:**

**Cons. mom**

\[ m_1v_{li} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f} \]

\[ m_1(v_{li} - v_{1f}) = m_2(v_{2f} - v_{2i}) \]

**Cons. energy**

\[ \frac{1}{2} m_1v_{li}^2 + \frac{1}{2} m_2v_{2i}^2 = \frac{1}{2} m_1v_{1f}^2 + \frac{1}{2} m_2v_{2f}^2 \]

\[ m_1(v_{li}^2 - v_{1f}^2) = m_2(v_{2f}^2 - v_{2i}^2) \]

\[ m_1(v_{li} + v_{1f})(v_{li} - v_{1f}) = m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i}) \]

Divide the two equations.

\[ \frac{m_1(v_{li} + v_{1f})(v_{li} - v_{1f})}{m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i})} = \frac{m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i})}{m_2(v_{2f} + v_{2i})(v_{2f} - v_{2i})} \]

\[ v_{li} + v_{1f} = v_{2f} + v_{2i} \]

\[ v_{li} - v_{2i} = v_{2f} - v_{1f} \]
Demo Problem: A cart moving at 4 m/s bounces elastically off of a second cart of twice the mass which is moving at 2 m/s in the same direction. What velocity does each cart now have?

$$\begin{align*}
\text{Before:} & \quad m \quad v_1 \\
\text{After:} & \quad 2m \quad v_2
\end{align*}$$

$$\begin{align*}
(v_1 - v_2)_{\text{bef}} &= (v_2 - v_1)_{\text{aft}} \\
4 - 2 &= v_2 - v_1 \\
v_2 - v_1 &= 2
\end{align*}$$

$$v_1 = 1.33 \frac{m}{s} \quad v_2 = 3.33 \frac{m}{s}$$

Demo Problem: A cart moving at 4 m/s bounces elastically off of a second cart of the same mass which is stationary. What velocity does each cart now have? **Demo:** Newton’s cradle

$$\begin{align*}
\text{Before:} & \quad m \quad v_1 \\
\text{After:} & \quad m \quad v_2
\end{align*}$$

$$\begin{align*}
(v_1 - v_2)_{\text{bef}} &= (v_2 - v_1)_{\text{aft}} \\
4 - 0 &= v_2 - v_1 \\
v_2 - v_1 &= 4
\end{align*}$$

Solve:

$$v_1 = 0$$
$$v_2 = 4 \frac{m}{s}$$

Answer to first one: $v_1 = 1.33 \text{ m/s}; v_2 = 3.33 \text{ m/s}$