Work done by a gas

1 m³ of an ideal gas at 300 K supports a weight in a piston such that the pressure in the gas is 200,000 Pa (about 2 atm). The gas is heated up. It expands to 3 m³. How much work did the gas do as it expanded?

\[ W = F \cdot \Delta y \quad (P = \frac{F}{A}) \]

\[ W = (PA) \Delta y \]

\[ W = P \Delta V \]

\[ = (2 \times 10^5 \text{ Pa}) (2 \text{ m}^3) \]

\[ \frac{\text{N} \cdot \text{m}}{\text{m}^2} = \text{Nm} = J \]

Result: \[ W_{\text{by gas}} = P \Delta V \]

(for constant P)

\[ \text{5th edition} \]

\[ \text{5th, 7th, 8th editions} \]

Work done on a gas

\[ W_{\text{on gas}} = -P \Delta V \]

(for constant P)

\[ W_{\text{on gas}} > 0 \text{ when... gas is expanding} \]

\[ W_{\text{on gas}} > 0 \text{ when... gas is contracting} \]
Internal energy of an ideal gas: $U$

Return to Equipartition Theorem:

The total kinetic energy of a system is shared equally among all of its independent parts, on the average, once the system has reached thermal equilibrium.

Each "degree of freedom", of each molecule, has an energy of: $\frac{1}{2} k_B T$

independent parts: larger for molecules that can
- rotate
- vibrate

(requires more than one atom)

→ such molecules have more "internal energy"

Monatomic ideal gas: only kinetic energy possible (3 directions)
average KE/molecule = $3/2 \ k_B T$
total KE = $N \times (3/2 \ k_B T)$

\[ U = \frac{3}{2} N k_B T = \frac{3}{2} nRT \]  (monoatomic)

Other substances: $U$ is more complicated, depends on temperature
Diatomic, around 300K: $U = \frac{5}{2} nRT$
(2 rotational directions that take energy)
P-V diagrams

State postulate: any two (independent) variables determine the state: P, V, T, U, etc.

\[ PV = nRT \]
\[ T = \frac{PV}{nR} \]

Work done: area under curve (but careful with sign)

How to tell at a glance if the temperature has increased or decreased: Isothermal curves, contours of constant T

\[ PV = \text{const} \]
\[ P = \frac{\text{const}}{V} \]
\[ y = \frac{A}{x} \]

\[ \Delta U \text{ for an isothermal process is } 0 \text{ because... } T_f = T_i \]

\[ U = \frac{3}{2}nRT \rightarrow \Delta U = \frac{3}{2}nR \Delta T \]

What is \( \Delta U \) for the constant P process at top of page?
\[ \Delta U > 0 \]
1\textsuperscript{st} Law of Thermodynamics

\[ \Delta U = Q_{\text{added}} + W_{\text{on system}} \]

(note: 5\textsuperscript{th} edition uses \(-W_{\text{by system}}\))

System: the object you are studying.
Environment: what it interacts with

*Typically "where heat comes from”*

**What does it mean??** Use 5\textsuperscript{th} edition version:

\[ \Delta U = Q_{\text{added}} - W_{\text{by system}} \rightarrow Q_{\text{added}} = \Delta U + W_{\text{by system}} \]

**Meaning of 1\textsuperscript{st} Law:**

Heat added can go either towards

- increasing internal energy (temperature), or
- doing work by the gas

**Final warning:** Be careful with all the signs!!!

\( \Delta U \) is positive if: \textit{temp increases}

\( Q_{\text{added}} \) is positive if: \textit{heat flows into the system}

\( W_{\text{on system}} \) is positive if: \textit{volume decreases}
P-V diagram examples

Isothermal process

\[ \Delta U = 0 \]
\[ Q = Q_{\text{added}} + W_{\text{on gas}} \]
\[ W_{\text{on gas}} \text{ negative} \]
\[ Q \text{ positive} \]

Another process

\[ Q_{\text{added}} = \Delta U - W_{\text{on}} \]
\[ \Delta U \text{ positive} \]
\[ W_{\text{on gas}} \text{ negative} \]
\[ Q \text{ positive} \]
A gas in a piston expands from point A to point B on the P-V plot, via either path 1 or path 2. Path 2 is a “combo path,” going down first then over.

**Clicker quiz 1:** The gas does the most work in:
- a. path 1
- b. path 2
- c. neither; it’s the same

**Clicker quiz 2:** In process 1, the work done:
- a. puts energy into the system $W_{\text{on}} > 0$
- b. takes energy out of the system $W_{\text{on}} < 0$
- c. has no effect on the energy of the system

**Clicker quiz 3:** The process in which $\Delta U$ is the greatest (magnitude) is:
- a. path 1
- b. path 2
- c. neither; it’s the same

How much work is done in first half of path 2? What is this path physically?
Adiabatic expansion or compression

Adiabatic: no heat added, either because...
- system is insulated, or
- ΔV is fast, so no time for much heat to go in/out of gas

\[ Q = 0 \]
\[ W \]

\[ \Delta U < 0 \] negative
[expanding]
\[ \Delta U > 0 \] positive
[compressing]

Adiabatic curves are steeper than isothermal curves

→ "No heat added" does not mean "no temperature change"

Demos: adiabatic compression and cotton freezing by expansion

Ralph question: how does isothermal compression work?
Two situations...

**Clicker quiz:** You compress air very quickly in an engine cylinder. Determine the signs of $Q$, $W$, and $\Delta U$.

- a. $Q_{\text{added}} = +$  $W_{\text{on gas}} = +$  $\Delta U = +$
- b. $Q_{\text{added}} = 0$  $W_{\text{on gas}} = +$  $\Delta U = +$
- c. $Q_{\text{added}} = +$  $W_{\text{on gas}} = -$  $\Delta U = +$
- d. $Q_{\text{added}} = +$  $W_{\text{on gas}} = 0$  $\Delta U = +$
- e. $Q_{\text{added}} = -$  $W_{\text{on gas}} = +$  $\Delta U = 0$

**Clicker quiz:** You heat a spray can in a fire, and volume stays about the same (it doesn't explode). System = gas in the can. Determine the signs of $Q$, $W$, and $\Delta U$.

- a. $Q_{\text{added}} = +$  $W_{\text{on gas}} = +$  $\Delta U = +$
- b. $Q_{\text{added}} = 0$  $W_{\text{on gas}} = +$  $\Delta U = +$
- c. $Q_{\text{added}} = +$  $W_{\text{on gas}} = -$  $\Delta U = +$
- d. $Q_{\text{added}} = +$  $W_{\text{on gas}} = 0$  $\Delta U = +$
- e. $Q_{\text{added}} = -$  $W_{\text{on gas}} = +$  $\Delta U = 0$
Cyclical Processes

\[ \Delta U = Q_{\text{added}} + W_{\text{gas}} \]
\[ \text{net } Q_{\text{added}} = -W_{\text{gas}} \]
\[ \text{area enclosed positive} \]

\[ Q = \text{work} \]

Engines
The basic idea: energy transformation

\( (\text{fuel burning}) \)
heat \( Q_h \)
\( T_h \)

Notation: \( Q_h, Q_e, T_h, T_c, |W_{\text{net}}| \)

Efficiency: how good is your engine at converting heat to work?

Definition:
\[ e = \frac{|W_{\text{net}}|}{Q_h} \]

Engine Power: work per time (as usual)
\[ = \frac{|W_{\text{net}}|}{\text{time for one cycle}} \]
Demo: Thermoelectric converter engine

Worked Problem: An engine produces power of 5000 W, at 20 cycles/second. Its efficiency is 20%. What are \( |W_{net}|, Q_h, \) and \( Q_e \) per cycle?

\[
P = \frac{|W_{net}|}{\text{time per cycle}}
\]

\[
|W_{net}| = \frac{5000 \text{J}}{20 \text{cycles}} = 250 \text{J/cycle}
\]

\[
\eta = \frac{|W_{net}|}{Q_h}
\]

\[
Q_h = \frac{|W_{net}|}{\eta} = \frac{250 \text{J}}{0.2} = 1250 \text{J}
\]

\[
1250 = 250 + Q_e
\]

\[
Q_e = 1000 \text{J}
\]

What do those quantities represent?

Answers: 250 J, 1250 J, 1000 J
Carnot’s Theorem:
You can’t even convert *most* of the heat into work

\[ e_{\text{max}} = "e_c" = 1 - \frac{T_c}{T_h} \]

T in Kelvin
C for Carnot

(Organized) Energy lost by “irreversibilities”
Irreversibilities occur when heat is added during a temperature change

**Most efficient engine possible:** Carnot engine
→ all heat added during constant temperature processes

How much power? Isothermal = slow, typically