Lecture 18 Announcements

1. **No homework due Saturday**, light HW for next Wed

2. **Evening exam review session tonight**
   a. 7 – 9 pm, room C215 (right below the TA lab)

3. **Exam 3 starts today!**
   a. Exam begins Thurs Oct 30, 10:15 am
   i. Late fee if you pick up your exam after 5 pm
   b. Exam ends Wed Nov 5, end of day
      i. ...but cumulative, of course
   c. Covers Chapters 6, 7 & 8, Homeworks 9-14
      i. …but cumulative, of course
   d. 3 hour time limit like last time
      i. 0.5 pt/min deducted after 5 min grace period
   e. Last year’s exam
      i. Last year students took an average of 2.0 hours
      ii. Average score was 66%
      iii. I attempted to make this year’s exam 3 a little easier. Time will tell if I succeeded.
   f. Format
      i. In response to student suggestions, I have attempted to make problem numbers and answer choices a little more clear (see last year’s posted exam)
      ii. There will be at least one (maybe 2 or 3) problems again where you turn in work
   g. Note card allowed as usual
      i. 3×5 note card, front & back, handwritten only
      ii. I will not give you most formulas on the exam
      iii. I will give you moments of inertia for the common shapes
      iv. I will give constants like $g = 9.8 \text{ m/s}^2$, radius & mass of Earth, $g = 6.67 \times 10^{-11} \text{ N⋅m}^2/\text{kg}^2$, etc.
   h. Calculators allowed as usual
      i. Things to study as usual, very roughly in order of importance
         1. HW problems
         2. Last year’s exam
         3. Conceptual stuff, especially:
            1. clicker quizzes
            2. Warmup questions
         iv. Worked problems from class
         v. Textbook problems
         vi. Etc.

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José sits on frictionless ice, holding a spinning bicycle wheel. View from above it is going **clockwise** (CW). Neglect external friction.

**Clicker quiz 1:** If he grabs on to the wheel edge firmly and “stops” it he will then be
a. turning CW (viewed from the top)
   b. turning CCW
   c. not turning

Maria is on a spinning merry-go-round. What will happen to its **rotational speed** if she…

**Clicker quiz 2:** Walks towards the center?
a. it slows down
b. it stays same speed
c. it speeds up

**Clicker quiz 3:** Runs opposite to the spinning so she is at rest vs the ground? (same choices)

**Clicker quiz 4:** Slips off when she steps on a frictionless icy part? (same choices)

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**Formulas Review**

**Definitions and Fundamental Laws**

Final exam: you will be expected to know these

- Definition of momentum: $\vec{p} = m \vec{v}$
- Conservation of momentum: $\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$ (if …)

**Angular formulas from old formulas:**
- $x \rightarrow \theta$
- $v \rightarrow \omega$ e.g. definitions/three kinematic equations
- $a \rightarrow \alpha$
- $m \rightarrow I$ e.g. rotational kinetic energy, angular momentum
- $F \rightarrow \tau$ e.g. Newton’s 2nd Law for torques
- $p \rightarrow L$

**Definition of torque:** $\tau = r_1 F = r F_\perp = r F \sin \theta$
- Conservation of $L$: $\sum L_{\text{before}} = \sum L_{\text{after}}$ (if …)

**New stuff, but not quite as basic**

Final exam: I will give you these
- arc length: $s = r \theta$
- tangential $v = r \omega$
- tangential $a = r \alpha$
- $a_c = v^2/r$
- Universal gravity: $F = \frac{G M m}{r^2}$, $P E_g = -\frac{G M m}{r}$
Moments of inertia:

- \( I = mR^2 \) (point mass going in circle)
- \( I = \frac{2}{5} mR^2 \) (sphere rotating about center)
- \( I = mR^2 \) (hoop rotating about its axis)
- \( I = \frac{1}{2} mR^2 \) (disk or cylinder about its main axis)
- \( I = mR^2 \) (rod about its center)
- \( I = \frac{1}{2} mL^2 \) (disk or cylinder about its main axis)
- \( I = \frac{1}{3} mL^2 \) (rod about its end)

Angular momentum, definition 2:

Things which you might consider to be formulas (but I don’t really, so I won’t give them to you on exam)

- Relationship between speed and period: \( v = \frac{2\pi r}{T} \)
- Quick derivation of satellite orbital velocity:
  \[ \Sigma F = ma \rightarrow \frac{GMm}{r^2} = \frac{mv_{\text{orbit}}^2}{r} \]
- Quick derivation of escape velocity:
  \[ E_{\text{kin}} = E_{\text{pot}} \rightarrow \frac{GMm}{r} + \frac{1}{2}mv_{\text{escape}}^2 = 0 \]
  (be clear about which \( r \) to use…)

\( I_{\text{tot}} = I_1 + I_2 + \ldots \)

Exam 3 - Review of important concepts

1. Momentum
   a. Definition
   b. Conservation Law
   c. Collision problems
      i. 1D
      ii. 2D – think of momentum components in each direction
   d. Elastic vs. Inelastic
   e. Combination problems (e.g. bullet into block of wood)
   f. Center of mass motion

2. Rotational motion
   a. Angular quantities: \( \theta, \omega, \alpha \)
      i. How they relate to “regular” quantities
      ii. Radians – \( 2\pi \) radians in a circle
   b. Connection between linear and rotational motion:
      \[ v = \omega r, \text{ etc.} \]
   c. Kinematic equations for constant angular acceleration
      i. also constant tangential acceleration
   d. Period vs. velocity vs. \( \omega \)

3. Centripetal acceleration, \( a_c = \frac{v^2}{r} \)
   a. Difference between centripetal and tangential
      i. …and when the two are combined
   b. Newton’s Law of Gravity and orbits
      i. Force equation
      ii. Potential energy equation
      iii. Orbital velocity and/or escape velocity
   c. “Roller coaster” problems
      i. Normal force = 0 when you “fly out of your seat”

4. Torque
   a. Definition
      i. “lever arm” concept, \( r \perp \)
   b. Equilibrium problems: \( \Sigma F = 0, \Sigma \tau = 0 \)
   c. Moment of inertia
      i. Equation for \( I \) for various situations will be given
   d. Newton’s 2nd Law for rotations: \( \Sigma \tau = I\alpha \)
   e. Torques and rotation
      i. Combining Newton 2 with kinematics

5. Rotational kinetic energy & momentum
   a. Definitions
   b. Conservation laws
   c. The two expressions for \( L \):
      \[ L = r \perp \cdot p \quad \text{and} \quad L = I \omega \]

Some HW problems (missed by many):

HW 10 Problem 2. A 11.4-g object moving to the right at 21 cm/s makes an elastic head-on collision with a 14.3-g object moving in the opposite direction with some unknown velocity. After the collision, the second object is observed to be moving to the right at 14.6 cm/s. Find the initial velocity of the second object.

Answer: 36 m/s to the left
HW 10 Problem 4. An 8.29-kg mass moving east at 11.6 m/s on a frictionless horizontal surface collides with a 18.5-kg mass that is initially at rest. After the collision, the first mass moves south at 4.6 m/s. What is (a) the magnitude and (b) the direction of the velocity of the second mass after the collision? (c) What percentage of the initial kinetic energy is lost in the collision?

Answers: 5.59 m/s, 21.6 N of east, 32.4% lost

HW 11 Problem 6. A 0.537-kg ball that is tied to the end of a 1.83-m light cord is revolved in a horizontal plane with the cord making a 28.6° angle with the vertical. (a) Draw a free-body diagram of the ball. (b) Determine the ball’s speed. (c) If instead the ball is revolved so that its speed is 4.24 m/s, what angle does the cord make with the vertical? (d) If the cord can withstand a maximum tension of 8.76 N, what is the highest speed at which the ball can move?

Answers: 2.16 m/s, 51.9°, 4.37 m/s

HW 13 Problem 1 A hungry 728-N bear walks out on a beam in an attempt to retrieve some “goodies” hanging at the end (see figure). The beam is uniform, weighs 216 N, and is 5.17 m long; the goodies weigh 82 N. (a) When the bear is at x = 1.00 m, find the tension on the wire and the magnitude of the hinge force where the beam is connected to the wall. (b) If the wire can withstand a maximum tension of 900 N, what is the maximum distance the bear can walk before the wire breaks?

Answers: 382 N, 721 N; 4.19 m

HW 14 problem 5 A space station has a radius 118 m and a moment of inertia of 4.77E08 kg·m². 150 people are on the rim, and the station is rotating so that they experience an apparent acceleration of 1 g. The people add to the total angular momentum of the system. When 100 people move to the center of the station, the angular speed changes. What apparent acceleration do the 50 remaining people feel? Assume m = 65 kg for all the inhabitants.

Answer: 13.5 m/s²
Some conceptual quizzes

Clicker quiz: An elastic collision means:
   a. the objects deform when they collide
   b. each object keeps its kinetic energy when they collide
   c. the total kinetic energy of the objects stays the same
   d. both b and c

Clicker quiz: Newton’s second law (ΣF=ma) for rotational motion is:
   a. Στ = Ια
   b. Στ = Ιω
   c. Στ = Lα
   d. Στ = Lω

Clicker quiz: You go around a curve in your car at constant speed. The tangential acceleration of the car is zero.
   a. True
   b. False

Clicker quiz: The net horizontal force on the car is:
   a. Toward the center of the circle
   b. Away from the center of the circle
   c. Tangent to the circle, in the direction of travel
   d. Tangent to the circle, opposite the direction of travel

More worked problems

A satellite of mass 185 kg is launched from a site on the Earth’s equator into an orbit at 380 km above the surface of the Earth. (a) Assuming a circular orbit, what is the orbital period of this satellite? (b) What is the satellite’s speed in orbit? (c) What is the minimum energy necessary to place this satellite in orbit, assuming no air friction? Neglect any initial kinetic energy due to the rotation of the earth about its axis. (d) What is the actual initial kinetic energy if the equatorial velocity is 465 m/s?

More Conceptual Questions

Clicker quiz. A boy is at the stern (back) of a sailboat with a bunch of beanbags. The wind has stopped. If the boy throws the beanbags against the sail with sufficient velocity, he can get the boat to move forward.
   a. True
   b. False

Clicker quiz. Two snowballs are thrown at each other. One is 2 kg traveling to the right at 3 m/s. The second is 0.5 kg traveling to the left at 8 m/s. After they collide they stick together. In this collision, total kinetic energy was conserved.
   a. True
   b. False

Clicker quiz: Their velocity after they stick is:
   a. to the right
   b. to the left

Clicker quiz: A large solid steel sphere and a small steel hoop are rolled down an inclined plane. Which reaches the bottom first?
   a. Sphere
   b. Hoop
   c. Tie
   d. Need to know masses
   e. Need to know diameters

Answers: 92.2 min, 7.68 km/s, 6.11E9 J, 2.00E7 J

A light string is wrapped around a solid cylindrical spool of radius 0.565 m and mass 1.79 kg. A 1.04 kg mass is hung from the string, causing the spool to rotate and the string to unwind. If the system starts from rest and no slippage takes place, determine the time required for the mass to drop 4.19 m.

Answer: 1.26 s
**More worked problems**

A 0.5 kg ball is tied to a string of length 4 m, and the ball is whirled around in a horizontal circle. The tension in the string is 60 N. What is the speed of the ball as it goes around the circle?

Three masses of 2 kg each are arranged in an equilateral triangle of side 6 m. Treat them as point masses and find the moment of inertia of this system about the vertical axis as shown.

A satellite has a moment of inertia of 900 kg m$^2$ about the central axis. It is initially not spinning, but rockets 2 m from the center provide 40 N of force each to rotate the satellite. After a time of 20 seconds, the rockets turn off. What is its angular velocity then?

A 6 m beam weighs 800 N. The weight of the ball is 1200 N. What is the tension in the angled supporting cable?

A solid cylinder of mass 30 kg and radius 80 cm is rolling on the floor without slipping, and has 500 J of total kinetic energy. (a) What is the speed of the cylinder? (b) How far up an incline will it roll (vertical height)?

Answers: 21.9 m/s, 36 kg m$^2$

Answer: 3.56 rad/s

Answer: 11085 N

Answers: 4.71 m/s, 1.70 m