3 hour time limit. One 3″×5″ handwritten note card permitted (both sides). Calculators permitted. No books.

Constants/some formulas:
- $g = 9.80 \text{ m/s}^2$
- $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
- Mass of Sun = 1.991 $\times 10^{30}$ kg
- Mass of Earth = 5.98 $\times 10^{24}$ kg
- Radius of Earth = 6.38 $\times 10^6$ m
- Radius of Earth’s orbit = 1.496 $\times 10^{11}$ m

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Keep four significant digits throughout your calculations; do not round up to less than four. When data is given, assume it has at least four significant digits. For example “15 meters” means 15.00 meters.

You are strongly encouraged to write your work on the exam pages and circle the correct answer. Of course also record your final answers on the bubble sheet.

→Write your CID above upper right corner. Did you do this ________? You won’t get your exam back without writing your CID.

Problem 1. A 1500 kg car must travel at what speed in order to have as much momentum as a 5500 kg truck moving at 30 m/s?

- a. Less than 74 m/s
- b. 74 - 84
- c. 84 – 94
- d. 94 – 104
- e. Faster than 104 m/s

1. $p = mv$
   
   \[
   m_{\text{car}}v_{\text{car}} = m_{\text{truck}}v_{\text{truck}}
   \]

   \[
   v_{\text{car}} = \left(\frac{m_{\text{truck}}}{m_{\text{car}}}\right)^{1/2}v_{\text{truck}} = 110 \text{ m/s}
   \]

   Choice E

Problem 2. In a collision between two objects with no outside forces, the total momentum of the system is conserved:

- a. Always
- b. Only when the collision is elastic

2. Momentum is always conserved if there are no external forces. Choice A

Problem 3. In class, a student in the rotating chair was rotating with his arms extended, holding weights. Then, he brought his arms closer towards his body and rotated faster. Why did this happen?

- a. the weights exerted a torque on his body
- b. his moment of inertia was reduced
- c. angular momentum was conserved
- d. a and b
- e. a and c
- f. b and c
- g. a, b, and c

3. Angular momentum was conserved (c is true), and therefore when I was reduced, $\omega$ increased (b is true). I guess the weights did exert a torque, but not in the clockwise/counterclockwise sense so choice a is not a good answer. Choice F
Problem 4. Did the student do work on the rotating system during this process?
   a. yes
   b. no

4. KE increased, so work was done. Choice A

Problem 5. Suppose Katie is floating in outer space with no forces acting on her. She is at rest, so her momentum is zero. Suddenly, she takes a ball out of her pocket and throws it. The ball goes one way, and she goes the other way. Was the momentum of the “Katie & the ball” system conserved?
   a. yes
   b. no

5. Momentum is always conserved if there are no external forces. Also, remember momentum is a vector. Choice A

Problem 6. A large cylinder is initially spinning at 24 rad/s. After it rotates through an angle of 16 radians while slowing down steadily, it is at rest. The angular acceleration magnitude was:
   a. less than 13 rad/s
   b. 13 - 16
   c. 16 – 19
   d. 19 – 22
   e. more than 22 rad/s

6. \( \omega_f^2 = \omega_i^2 + 2\alpha \Delta \theta \)

   \( 0 = 24^2 + 2\alpha(16) \)

   \( \alpha = 24^2/32 = 18 \text{ rad/s}^2 \)  
   Choice C

Problem 7. Which has greater linear speed (m/s), a horse near the outside rail of a merry-go-round or a horse near the inside rail?
   a. outside horse
   b. inside horse
   c. both the same

7. Outside horse is going faster. Choice A

Problem 8. Which has greater angular speed (rad/s), a horse near the outside rail of a merry-go-round or a horse near the inside rail?
   a. outside horse
   b. inside horse
   c. both the same

8. Both travel the same angle per second. Choice C

Problem 9. You swing a ball (mass \( m \)) in a vertical circle (radius \( r \)) with a string; it is traveling at a speed \( v \) as it passes through the lowest point. What is the tension there?
   a. \( T = mg - \frac{mv^2}{r} \)
   b. \( T = mg - \frac{mv^2}{2r} \)
   c. \( T = mg \)
   d. \( T = mg + \frac{mv^2}{2r} \)
   e. \( T = mg + \frac{mv^2}{r} \)

9. \( \sum F = ma_c = (mv^2)/r \)

   \( T - mg = (mv^2)/r \)

   \( T = mg + (mv^2)/r \)

   Choice E
Problem 10. A ball at the end of a pendulum string speeds up as it approaches the lowest point. See figure. What general direction is the acceleration of the ball at the point shown?

a.  

b.  

c.  

d.  

e.  

f.  

10. This is just like a car speeding up as it goes around a corner. The total acceleration is the vector sum of centripetal acceleration and tangential acceleration. Choice C

Problem 11. Assuming both objects have the same mass and are drawn to scale, which one will have the largest moment of inertia when rotating about the vertical axes shown?

a. object A  
b. object B  
c. same

11. The object with more mass far from the axis of rotation will have a greater moment of inertia. Choice B

Problem 12. Compare a ball rolling down a ramp (without slipping) with a block sliding down a similar ramp (with no friction). Which one will reach the bottom first?

a. the ball  
b. the block  
c. same

12. The block doesn’t need to put any potential energy into rotational energy, so more goes into translational (regular) kinetic energy. Choice B

Problem 13. A train car of mass 6400 kg moving at 3 m/s to the right attaches to another car (7100 kg) that is moving at 2 m/s to the left. How fast do they move together after the collision? (positive direction = to the right)

a. Less than -0.7 m/s  
b. -0.7 to -0.5  
c. -0.5 to -0.3  
d. -0.3 to -0.1  
e. -0.1 to +0.1  
f. +0.1 to +0.3  
g. +0.3 to +0.5  
h. +0.5 to +0.7  
i. More than +0.7 m/s

13. \[ \sum p_{\text{before}} = \sum p_{\text{after}} \]
\[ (6400)(3) - (7100)(2) = (13500)v_f \]
\[ v_f = 0.3704 \text{ (to the right) } \]

Choice G
Problem 14. How much kinetic energy was lost in the train collision?

a. Less than 41600 J
b. 41600 – 41700
c. 41700 – 41800
d. 41800 – 41900
e. 41900 – 42000
f. 42000 – 42100
g. More than 42100 J

14. \( \Delta KE = KE_f - KE_i \)

\[ KE_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (6400)(3)^2 + \frac{1}{2} (7100)(2)^2 = 43000 \text{ J} \]

\[ KE_f = \frac{1}{2}(m_1 + m_2)(v_1^2) = \frac{1}{2}(13500)(0.3704)^2 = 926 \text{ J} \]

\[ \Delta KE = 42074 \text{ J} \]

Choice F

Problem 15. A 700 kg car enters a flat highway via an unbanked turn of radius 20 m. The coefficient of static friction between the road and tires is 0.85. What is the maximum speed the car can have on this curve without slipping off the road?

a. Less than 11.8 m/s
b. 11.8 – 12.8
c. 12.8 – 13.8
d. 13.8 – 14.8
e. 14.8 – 15.8
f. More than 15.8 m/s

15. \( \sum F = ma = \frac{mv^2}{r} \)

\( Nmg = \frac{mv^2}{r} \)

\[ v = \sqrt{\frac{mg}{\mu g}} = 12.91 \text{ m/s} \]

Choice C

Problem 16. A satellite is in a circular orbit 5000 km above the earth’s surface (i.e. 11380 km from the earth’s center). How fast must it be moving?

a. Less than 5.5 km/s
b. 5.5 – 5.6
c. 5.6 – 5.7
d. 5.7 – 5.8
e. 5.8 – 5.9
f. Faster than 5.9 km/s

16. \( \sum F = ma = \frac{mv^2}{r} \)

\[ GmM/r^2 = \frac{mv^2}{r} \]

\[ v = \sqrt{\frac{GM}{r}} = \sqrt{(6.67 \times 10^{-11})(5.98 \times 10^{24})/11380 \times 10^3} = 5920 \text{ m/s} \]

Choice F

Problem 17. Suppose an alien astronaut in a circular orbit around the alien homeworld, 11380 km from the planet’s center, has an orbital speed of 7 km/s. How long would it take the alien to make one complete orbit?

a. Less than 10,000 sec
b. 10,000 – 11,000
c. 11,000 – 12,000
d. 12,000 – 13,000
e. 13,000 – 14,000
f. More than 14,000 sec

17. \( v = \frac{2\pi}{T} \)

\[ T = \frac{2\pi}{v} \]

\[ T = \frac{2\pi(11380 \times 10^3)/7 \times 10^3}{10215 \text{ m/s}} \]

Choice B

Problem 18. Titan is the largest moon of the planet Saturn; scientists believe it is composed in part of rocky material. Its mass is 1.345 \times 10^{23} \text{ kg} (0.0225 \text{ M_{earth}}) and its radius is 2,576 km (0.404 \text{ R_{earth}}). What is its surface gravity? (aka gravitational acceleration)

a. Less than 1.1 m/s^2
b. 1.1 – 1.2
c. 1.2 – 1.3
d. 1.3 – 1.4
e. 1.4 – 1.5
f. 1.5 – 1.6
g. More than 1.6 m/s^2

18. \( F = GmM/r^2 \) also = \( mg \) at surface

\[ GmM/r^2 = mg \]

\[ g = GmM/(2576 \times 10^3)^2/(2576 \times 10^3)^2 = 1.35 \text{ m/s}^2 \]

Choice D
Problem 19. How does escape velocity on Titan compare to escape velocity on Earth? That is, \( v_{\text{escape, Titan}} \) is what fraction of \( v_{\text{escape, Earth}} \)?

- a. \( v_{\text{escape, Titan}} = \) less than \( 0.21 \times v_{\text{escape, Earth}} \)
- b. \( 0.21 – 0.22 \)
- c. \( 0.22 – 0.23 \)
- d. \( 0.23 – 0.24 \)
- e. \( 0.24 – 0.25 \)
- f. \( v_{\text{escape, Titan}} = \) more than \( 0.25 \times v_{\text{escape, Earth}} \)

19. Escape velocity: \( \text{PE}_i + \text{KE}_i = \text{PE}_f + \text{KE}_f \)

\[-GMm/r + \frac{1}{2}mv^2 = 0 \]

\[\frac{1}{2} mv^2 = GMm/r \rightarrow v_{\text{escape}} = \sqrt{2GM/r} \]

\[v_{\text{escape, Titan}} / v_{\text{escape, Earth}} = \sqrt{(2GM_{\text{Titan}}/r_{\text{Titan}}) / (2GM_{\text{Earth}}/r_{\text{Earth}})} = \sqrt{(M_{\text{Titan}}/M_{\text{Earth}})(r_{\text{Titan}}/r_{\text{Earth}})} \]

\[= \sqrt{0.0225/0.404} = 0.236 \]

Choice D

Problem 20. On a frictionless airtrack: a cart moving to the right at 4 m/s bounces elastically off of a second cart of twice the mass which is moving at 3 m/s in the same direction. What velocity does the heavier cart now have?

- a. Less than 2.4 m/s
- b. 2.4 – 3.4
- c. 3.4 – 4.4
- d. 4.4 – 5.4
- e. 5.4 – 6.4
- f. 6.4 – 7.4
- g. 7.4 – 8.4
- h. More than 8.4 m/s

![Diagram](image)

20. conservation of momentum: \( 4m + 6m = v_1m + v_22m \)

\[v_1 + 2v_2 = 10 \]

“velocity reversal” since elastic \((v_1 - v_2)_{\text{before}} = -(v_2 - v_1)_{\text{after}}\)

\[4 - 3 = v_2 - v_1 \]

\[v_2 - v_1 = 1 \]

add together: \( 3v_2 = 11 \)

\[v_2 = 3.67 \text{ m/s} \] Choice C

Problem 21. A 100 g mousetrap with a 20 g piece of cheese is sliding across frictionless ice, going due north at 1 m/s. The mousetrap goes off, and as a result the cheese gets shot horizontally due east at 2 m/s. How fast must the mousetrap be going now?

- a. Less than 0.8 m/s
- b. 0.8 – 0.9
- c. 0.9 – 1.0
- d. 1.0 – 1.1
- e. 1.1 – 1.2
- f. 1.2 – 1.3
- g. 1.3 – 1.4
- h. More than 1.4 m/s

![Diagram](image)

21. (using units of grams; mass units cancel so it’s okay)

\[\sum P_{\text{before}} = \sum P_{\text{after}} \]

\[0 = 40 + (-100)v_{fx}\]

\[v_{fx} = .4 \text{ m/s west} \]

\[\sum P_{y\text{before}} = \sum P_{y\text{after}} \]

\[120 = 100v_{fy} \]

\[v_{fy} = 1.2 \]

\[v_f = \sqrt{v_{fx}^2 + v_{fy}^2} = 1.265 \text{ m/s} \] Choice F

Problem 22. As a result of the mousetrap “explosion”, the velocity of the center of mass of mousetrap & cheese system:

- a. is unchanged
- b. has changed direction
- c. has changed magnitude
- d. has changed direction and magnitude
- e. is zero

22. No outside forces, so center of mass can’t accelerate. Choice A.
Problem 23. A bullet (0.01 kg) imbeds itself into a small block of wood (0.25 kg) hanging from a string. The wood (with bullet inside) swings up to a height of 0.23 m above its original position. How fast was the bullet going initially?

a. Less than 54 m/s
b. 54 – 55
c. 55 – 56
d. 56 – 57
e. 57 – 58
f. 58 – 59
g. 59 – 60
h. More than 60 m/s

23. Start with 2→3: \( \frac{1}{2} m_{\text{tot}}v^2 = m_{\text{tot}}gh \)

\[ v = \sqrt{2gh} = 2.123 \text{ m/s} \]

Now do 1→2: \((.01)(v_0)+0=(.26)(2.123)\)

\[ v_0 = 55.2 \text{ m/s} \]

Choice C

Problem 24. A string attached to a block (mass \( M \)) is wound over a large (not zero-mass!) pulley. The pulley has moment of inertia \( I \) and radius \( R \). The pulley turns as the block is let fall. No energy is lost to friction. How fast does the block accelerate? (Note: you must provide free body diagrams for this problem, in problem 30.)

a. \( \frac{I}{Mg - I/R} \)
b. \( \frac{Ig}{MR^2 + I} \)
c. \( \frac{Mg}{I/R} \)
d. \( \frac{Mg}{I/R^2} \)
e. \( \frac{Mg}{M - I/R} \)
f. \( \frac{Mg}{M + I/R^2} \)

24. \( \sum F = Ma \)

\( Mg - T = Ma \)

\( \sum \tau = I \alpha \)

\( TR = I(a/R) \)

\( T = Ia/R^2 \)

Choice F

Problem 25. A 0.5 kg ball tied to the end of a 0.8 m string is spun in a horizontal plane with the cord making a 35° angle with the vertical (see figure). How fast must the ball be traveling? (Note: you must provide a free body diagram for this problem, in problem 30.)

a. Less than 1.8 m/s
b. 1.8 – 1.9
c. 1.9 – 2.0
d. 2.0 – 2.1
e. 2.1 – 2.2
f. 2.2 – 2.3
g. 2.3 – 2.4
h. More than 2.4 m/s

25. \( \sum F = 0 \rightarrow T \cos \theta = mg \)

\( T = mg/\cos \theta \)

\( \sum F = ma = mv^2/r \)

\( T \sin \theta = mv^2/L \sin \theta \)

\( (mg/\cos \theta) \sin \theta = mv^2/L \sin \theta \)

\( v = \sqrt{(gL \sin \theta^2/\cos \theta)} = 1.774 \text{ m/s} \)

Choice A
Problem 26. A marble rolls without slipping down a ramp (30° angle, 0.8 m long) from a vertical height of 0.4 m. What is the marble’s acceleration down the ramp? *(Hint: there are a couple of ways to solve this, but it’s probably easiest to first find the speed at the bottom using conservation of energy. Then use a kinematics formula to get the acceleration.)*

a. Less than 2.4 m/s²
b. 2.4 – 2.7
c. 2.7 – 3.0
d. 3.0 – 3.3
e. 3.3 – 3.6
f. 3.6 – 3.9
g. 3.9 – 4.2
h. 4.2 – 4.5
i. 4.5 – 4.8
j. Larger than 4.8 m/s²

26. \[ E_{\text{bef}} = E_{\text{aft}} \]
\[ mgh = \frac{1}{2} I \omega^2 + \frac{1}{2} m v^2 \]
\[ = \frac{1}{2} \left( \frac{2}{5} m R^2 \right) \left( \frac{v}{R} \right)^2 + \frac{1}{2} m v^2 \]
\[ mgh = \frac{1}{5} m v^2 + \frac{1}{2} m v^2 \]
\[ gh = \frac{1}{5} + \frac{1}{2} v^2 = .7 \ v^2 \]
\[ v = \sqrt{gh / .7} = 2.366 \text{ m/s} \]

Kinematics: \[ v_f^2 = v_0^2 + 2a \Delta x \]
\[ 2.366^2 = v_0^2 + 2a(.8) \]
a = 3.5 m/s²
Choice E

Problem 27. A 100 kg bear walks out on a beam in an attempt to retrieve some goodies hanging at the end (see figure). The 20 kg beam is 5 m long; the goodies have a mass of 9 kg. The support wire at a 60° angle as shown can withstand a maximum tension of 1100 N. How far out can the bear walk (distance \( x \)) before the wire breaks?

a. Less than 3.6 m
b. 3.6 – 3.7
c. 3.7 – 3.8
d. 3.8 – 3.9
e. 3.9 – 4.0
f. More than 4.0 m

27. \[ \sum \tau_p = 0 \]

\[ (T \sin \theta) L = (m_{\text{goodies}} g) L - m_{\text{beam}} g \left( \frac{L}{2} \right) - (m_{\text{bear}} g) x = 0 \]
\[ x = \frac{(T \sin \theta L - m_{\text{goodies}} g L - m_{\text{beam}} g (L/2))}{(m_{\text{bear}} g)} \]
\[ x = 1100 \sin 60° \cdot 5.9 \cdot 9.8 \cdot 5. \]
\[ 20 \cdot 9.8 \cdot 2.5 / (100 \cdot 9.8) = 3.91 \text{ m} \]
Choice E
Problem 28. Two children (masses $m_1$ and $m_2$) are balanced on the ends of a lopsided seesaw (length $L$, mass $M$) as shown. (a) Draw in and label all forces on the seesaw. (b) Write down the “sum of torques” equilibrium blueprint equation, and fill in the equation with the given quantities using the point marked “p” as your pivot point.

\[
\sum \tau_p = 0
\]

\[m_1g(L/4) - Mg(L/4) - m_2g(3L/4) = 0\]

Problem 29. Provide free body diagrams for the following. Include only the real forces acting on the objects. Label each force.

(a) The pulley and block from problem 24. (Be sure to include the pulley’s weight and the support holding it up.)

(b) The ball spinning in a circle from problem 25 (in the position pictured).