$W = \text{absorption limit (for gasses, i.e. UV)}$

Real part of $\tilde{\nu}$

$\nu = \frac{\hbar}{\sqrt{1 + \hbar^2 / 2m_0}}$

$\hbar = 1$

$W$

except for this limit, "anomalous dispersion"

index always rises w/frequency

regular "normal dispersion"

visible $\rightarrow \nu$

$\nu$

$\nu$

$\nu$

Ions - can also act like charge separations. Frequencies are much lower

$N_{\text{ion}} = 4/3$ atoms/unit cell / 3D

$\chi(\omega)$

$\text{optical phonons} \sim 2 \text{TO}$

Assume degeneracy

$\omega = \frac{1}{2} \omega_{\text{TO}}$

$\epsilon(\omega) = 1 + \frac{\omega^2}{\omega_{\text{TO}}^2}$

$\epsilon_{\infty}$

Hilbert's notation, p3 911

$\epsilon = \epsilon_{\infty} + \frac{n_{\text{ion}}^2}{m_0 c_0^2} \left( \frac{1}{\omega^2} - \frac{1}{\omega_{\text{TO}}^2} \right)$

Impurities

$\mu_{\text{imp}}$

E

$\epsilon(\omega)$

"high freq, dielectric constant"

$i\omega > m_0 \omega_{\text{TO}}$

freq: much higher than vibrational freqs, but below excitonic excitation energies

$0 \leq \text{volume of ions}$
Chuuy of variable.

Note that when \( \omega = \omega_0 - \frac{\omega^2}{\omega_0} \times \frac{1}{\mu_0 \epsilon_0} \)

\[
\epsilon = \epsilon_\infty + \frac{\epsilon_0 - \epsilon_\infty}{1 - \frac{\omega^2}{\omega_0^2}}
\]

\( \frac{\epsilon_0}{\epsilon} \)

\( \rightarrow \) can calculate \( \eta \)

\( \rightarrow \) can calculate \( R \)

\( 100\% \)

\( \rightarrow \) which \( \epsilon = 0 \)

call it \( \omega_L \)

Turns out...

\( \omega_L \): frequency of TH planes

\( \omega_T \): LO planes

\( \omega_L \rightarrow \) regular case worn

\( \omega_T \rightarrow \) strange case

\( E_\infty = 15 \)

\( \epsilon_\infty = 12 \)

\( \frac{\omega}{\omega_L} = 0.02 \) (maybe)

Mag/\( \vec{D} \) = 0 \( \rightarrow \) \( \nabla \cdot (\vec{E} \vec{B}) = 0 \)

\( \vec{E} \rightarrow \frac{\epsilon_0}{\epsilon} \vec{B} \)

\( \epsilon = 0 \)
The response of the ion to the driving force of the electric field will be greatest when their motion is one of the normal lattice waves in the crystal. It is not difficult to identify the frequency at which this occurs. First of all, since the motion of the positive and negative ions is in opposite directions, the lattice wave belongs to the optical branch of the dispersion curve. Sound, since the displacement of the ions is 90° to the direction of the wave vector, the lattice wave is transverse.

Also, wavelengths must match \( \rightarrow k_{\text{light}} = k_{\text{lattice}} \)

\[
\begin{align*}
\text{essentially 0,} & \\
\text{as discussed before.} & \\
\text{Therefore zone center} & \\
\text{So, } \frac{\lambda}{2} = \text{fgr of } T_0 \text{ at zone center!} & \\
(\text{That's IR light -} & \\
\text{for KBr } \sim 90 \mu m) &
\end{align*}
\]

Also has explanation of LD on p. 187

\[
\begin{align*}
\nabla \cdot D &= 0 \\
\rightarrow & \varepsilon_0 \kappa \cdot \varepsilon = 0 \\
\text{can be satisfied if } & \kappa \cdot \varepsilon = 0 \text{ (usual !)} \\
\text{however} & \\
\varepsilon &= 0 \\
\text{can then a longitudinal wave can be supported}
\end{align*}
\]
\[ \varepsilon = \varepsilon_\infty \left\{ 1 - \frac{\varepsilon_0 - \varepsilon_\infty}{\varepsilon_0} \right\} \]

when \( \varepsilon = 0 \), \( \mu = \mu_0 \)

\[ \varepsilon = \varepsilon_0 + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\mu_0^2}{\mu^2}} \]

\[ \varepsilon = A + \frac{B - A}{1 - x} \]

\[ x = \frac{B}{A} \]

\[ \left( \frac{\mu_0}{\mu} \right)^2 = \frac{\varepsilon_0}{\varepsilon_\infty} \]
Polarizing - causing of EM wave w/ something

Phonon polarizability (the only type mentioned in lecture)

\[ \text{phonon polarizability} \]

\[ \text{optical phonons couple to EM wave.} \]

\[ W = \sqrt{k} = \left( \frac{\omega}{n} \right) k \]

In regions where frequencies are degenerate (near),

really need degenerate pair theory (like we did for broad structure, when two electron modes had same frequency).

Result - two coupled modes, "upper branch." 

\[ \text{physically branches, gap between them.} \]

All frequencies further away, lacks prior region.

Instead of VM degenerate pair theory, use this method which gives essentially correct result.

\[ \text{Problem Figs. 14.11 + 14.12} \]

\[ \text{pg 411, 412} \]

\[ \text{relevant area around} \ W_0 = \left( \frac{\omega}{n} \right) k \]

Force dielectric constants to be the same

Light: \[ \frac{\omega}{k} = \frac{C}{n} = C k \]

\[ \Rightarrow \ \epsilon = \left( \frac{C k}{n} \right)^2 \]

To phonons: \[ \epsilon = \epsilon_{\infty} + \frac{\epsilon_0 - \epsilon_{\infty}}{1 - \frac{v^2}{c^2}} \]

\[ \text{(no damping) for simplicit.} \]
\[
\frac{\varepsilon^2 k^2}{\omega^2} = \varepsilon_\infty + \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}}
\]

\[\text{poloctor dispersion solution}\]

\[\text{can in theory solve } \omega \text{ vs } k\]

\[\varepsilon^2 k^2 = \varepsilon_\infty \omega^2 + \omega^2 \left( \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}} \right) - \frac{\omega^2}{\omega_T^2} \]

Hard to solve for \( k \), easier for \( \omega \).

For a given \( \omega \), I can tell you \( k \)!

\[ k = \frac{1}{c} \sqrt{\varepsilon_\infty \omega^2 + \omega^2 \left( \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}} \right)} \]

\[ k = \frac{1}{c} \sqrt{\varepsilon_0} \]

Consider lower branch, small \( k / \omega \)

\[ k = \frac{1}{c} \sqrt{\varepsilon_\infty} \]

Consider upper branch, large \( k / \omega \)

\[ k = \frac{1}{c} \sqrt{\varepsilon_\infty \omega^2 + \omega^2 \left( \frac{\varepsilon_0 - \varepsilon_\infty}{1 - \frac{\omega^2}{\omega_T^2}} \right)} \]

\[ k = \frac{1}{c} \sqrt{\varepsilon_0 \omega^2 + \frac{\omega^2 \left( \varepsilon_0 - \varepsilon_\infty \right)}{\omega_T^2}} \]

\[ k = \frac{1}{c} \sqrt{\varepsilon_0} \]

No (real) solution for \( k \) in here.

\( k \) is purely imaginary for these \( \omega \)s.

That's why it's very refractive for these freqs.