Previous eqn: \[ R = \frac{n - 1}{n+1} \rightarrow \tilde{n} = \frac{n - 1}{n+1} \rightarrow \tilde{n}^{-1} = \frac{n+1}{n-1} \]

\[ n = \sqrt{\varepsilon_r} \rightarrow \varepsilon_r = n^2 \]

\[ \tilde{\varepsilon}_r = (\tilde{n})^2 \]

Kramers-Kronig Relations (ch 15 pg 430-433)

- **Theorem:** Whenever you have a complex response function, in order to guarantee the response lingers after the stimulus, it is necessary for the real and imaginary components of the function to be related to each other. Specifically, for dielectric constants:

\[ (\varepsilon_r(\omega))_{\text{real}} = \frac{2}{\pi} \Re \left[ \int_0^{\infty} \frac{\varepsilon_r(\omega') \omega' \omega}{\omega^2 - \omega'^2} d\omega' \right] \]

and

\[ (\varepsilon_i(\omega))_{\text{imag}} = -\frac{2}{\pi} \Im \left[ \int_0^{\infty} \frac{\varepsilon_r(\omega') \omega' \omega}{\omega^2 - \omega'^2} d\omega' \right] \]

"Cauchy principal value" related to contour integrals of complex numbers. That’s all I’ll say.

You measure eg absorption vs ω for large range of ω’s, and you can calculate reflectivity.
Locate model of dehnec -- to explain the UV feature of Figs 16.3 & 16.4 by YBY.

Main field theory

\[ e + E \rightarrow \theta_0 \]

"dehnec" action

Laid of like mass \( \Phi \) on a spring!

(Charged mass)

\[ \Phi \rightarrow -q \]

\[ \rightarrow \Phi \]

Let's assume there's a resonant field \( \Phi \) that describes this.

\[ E = ma \]

\[ \frac{m \dot{x}^2}{x^2} = -m \omega_0^2 \dot{x} + q \frac{E_0}{E_{\text{field}}} \]

\[ \dot{x} \text{ is def. from equilibrium.} \]

Would find a limit response when \( \omega = \omega_0 \)

... add damping

Simplest model, like air resistance

\[ \frac{\ddot{x}}{\dot{x}^2} = -\omega_0^2 \dot{x} + \frac{g E_0}{m}e^{-i(kx-\omega t)} \]

Guess solution \( x = x_0 e^{-i(kx-\omega t)} \)

\[ \frac{\ddot{x}}{\dot{x}^2} = -\omega_0^2 x_0 e^{-i(kx-\omega t)} + \frac{g E_0}{m} e^{-i(kx-\omega t)} \]

\[ (\omega_0^2 - \omega^2 - i\omega) x_0 = \frac{g E_0}{m} \]

\[ x_0 = \frac{g E_0}{m (\omega_0^2 - \omega^2 - i\omega)} \]

\[ \dot{x}_0 = \frac{q E_0}{m (\omega_0^2 - \omega^2 - i\omega)} e^{-i(kx-\omega t)} \]

\[ \dot{x} = \frac{q E_0}{m (\omega_0^2 - \omega^2 - i\omega)} e^{-i(kx-\omega t)} \]

Total \( \dot{x} \) at \( -Nq\Phi \)}
$$\rho = \frac{\text{dipole moment}}{\text{volume}} = \frac{q^2 n}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega \kappa} \rho_0 e^{i(\omega - \omega_0)}$$

$$\rho = e \chi G$$

This is $e \chi G$.

$$\varepsilon_r = 1 + e \chi$$

$$\left[ \varepsilon_r = 1 + \frac{q^2 n}{\varepsilon_0 m} \frac{1}{\omega_0^2 - \omega^2 - i\omega \kappa} \right]$$

We'll call it $$\omega_p = \sqrt{\frac{q^2 n}{\varepsilon_0 m}}$$

"plasma frequency"

It more than one resonant frequency simply sum! Weighted sum (eg. because of multiple mechanisms)


When written with $\lambda$ instead of $\omega$, (for empirical fitting) called the "Sellmeier Equation"

$$n^2 = 1 + \frac{B_1 \lambda^2}{\lambda^2 - C_1} + \frac{B_2 \lambda^2}{\lambda^2 - C_2} + \frac{B_3 \lambda^2}{\lambda^2 - C_3}$$

(used for glass)
Simplify eqn for a minute: no damping, just one force:

$$\xi = 1 + \frac{wp^2}{w_0^2 - w^2}$$

when $w$ small, $\xi = 1 + \frac{wp^2}{w_0^2}$ as $\xi \to 1$ at $w = w_0$.

when $w$ big, $\xi = 1 - \frac{wp^2}{w^2}$

Can measure visible index of refraction \((ignoring\ \text{dispersion})\)

Call it $n_{oq}$

$$n_{oq}^2 = 1 + \frac{wp^2}{w_0^2} = \frac{w_0^2 + wp^2}{w_0^2} \Rightarrow wp^2 = (n_{oq}^2 - 1) w_0^2$$

$$\xi = 1 + \left(\frac{n_{oq}^2 - 1}{w_0^2 - w^2}\right)$$

used in Stokes, eqn 16-43 pg 185

$$n = \sqrt{\xi}$$

$$n = \frac{n - 1}{n + 1}$$

pg 176, Stokes Fig 16-6 pg 185

Back to eqn w/ damping:

$$n = \sqrt{\xi} \rightarrow \text{plot real,imag parts of } n$$