Finish "hard way" for 110 (Example 4) to get results.

then an "easy way" for 110 longitudinal (Example 3)

Example 4: "Easy way" for (110) transverse in \( \frac{x-y}{\sqrt{2}} \) direction

\[
e^{i(k \cdot (x \hat{a} + y \hat{b} - u \hat{c}) / \sqrt{2})}
\]

\( u_y = -u_x \)

\[
r^2 \psi^2 u_x = C_{11} \left( \frac{k^2}{2} \right) u_x + C_{44} \left( \frac{k^2}{2} + 1 \right) u_x + (C_{11} + C_{12}) \left( \frac{-k^2}{2} u_x + 0 \right)
\]

\[
pw^2 = \frac{1}{2} \left[ C_{11} + C_{44} - C_{12} - C_{11} \right] k^2
\]

\[
\frac{k^2}{2} = \frac{1}{\sqrt{2} \rho} (C_{11} - C_{12})
\]

That's not the only transverse direction, though!

\( \psi = (0, 0, u_2) \) is also transverse to (110)

Make \( \frac{w^2}{\rho} = \sqrt{\frac{C_{44}}{\rho}} \) is result

The three eigen vectors are:

1. (1 1 0) long
2. (1 -1 0) transverse
3. (0 0 1) transverse

Note: 1) All other transverse direction (eg (5 1 5, 3) can be written as linear combo of the two "special" transverse ones.

2) You can't always guess at "special" directions, sometimes very long, is it a given value.
Wave information

1) General exponential dependence is 
\[ e^{i(kx - wt)} \]

2) Direction of travel tells you what \( \mathbf{k} \) components are present.

3) Direction of oscillation (i.e., longitudinal vs. transverse, and which specific transverse direction) tells you which \( \mathbf{u} \) components are present.

How to use Eqn 3.57 (a), (b), (c).

1) The Easy Way — If you know the specifics of \( u_x, u_y, u_z \), plug that in from the start.

Example 1 — (100) waves, longitudinal

Given \( \mathbf{k} = (k_x, 0, 0) \) and \( \mathbf{u} = u_x \hat{x} e^{i(kx - wt)} \)

Eqn 3.57 a becomes

\[ p w^2 / x = C_{11} k_x^2 / \rho \rightarrow \sqrt{\frac{p w^2}{k_x}} = \sqrt{\frac{C_{11}}{\rho}} \]

Example 2 — (101) waves, transverse in the (0,1,0) direction

Given \( \mathbf{k} = (k, 0, 0) \) and \( \mathbf{u} = (0, u_y, 0) \)

Eqn 3.57 a becomes

\[ p w^2 / x = C_{44} k_y^2 / \rho \rightarrow \sqrt{\frac{p w^2}{k_y}} = \sqrt{\frac{C_{44}}{\rho}} \]

Example 3 — (110) waves, longitudinal

Given \( \mathbf{k} = k_x \hat{x} + k_y \hat{y} \) and \( \mathbf{u} = u_x \hat{x} + u_y \hat{y} + u_z \hat{z} e^{i(kx + k_y y - wt)} \)

Eqn 3.57 a becomes

\[ p w^2 / x = C_{11} k_x^2 / \rho + C_{12} k_y^2 / \rho + C_{44} \left( \frac{k_y^2}{2} u_y + \frac{k_z^2}{2} u_z \right) \rightarrow \sqrt{\frac{p w^2}{k_x}} = \sqrt{\frac{C_{11}}{\rho}} \]
2) The Hard Way — if you don’t know all of the specifics, you’ll need to solve simultaneous eqns in an eigenvalue-type matrix eqn.

Example 9: (110) waves, unknown direction in X-Y plane

Given \( \kappa = k_0^2 \left( \frac{k_x}{k_0} \right) \) \( k_x = \frac{k^2}{k_0} \)

\[
\begin{bmatrix}
\omega^2 & 0 \\
0 & \omega^2
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y
\end{bmatrix}
= \frac{1}{2} k_0^2 \begin{bmatrix}
C_{11} + C_{44} & C_{12} + C_{44} \\
C_{12} + C_{44} & C_{11} + C_{44}
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y
\end{bmatrix}
\]

or

\[
\omega^2 \begin{bmatrix}
u_x \\
u_y
\end{bmatrix}
= \frac{1}{2} k_0^2 \begin{bmatrix}
C_{11} + C_{44} & C_{12} + C_{44} \\
C_{12} + C_{44} & C_{11} + C_{44}
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y
\end{bmatrix}
\]

Eqs. 3.57a.

Combine in matrix eqn:

\[
\begin{bmatrix}
\omega^2 - \frac{k_0^2}{2} (C_{11} + C_{44}) & C_{12} + C_{44} \\
C_{12} + C_{44} & \omega^2 - \frac{k_0^2}{2} (C_{11} + C_{44})
\end{bmatrix}
\begin{bmatrix}
u_x \\
u_y
\end{bmatrix}
= 0
\]

To have non-zero solns, det \((J) = 0 \)

\[
\left( \frac{k_0^2}{2} (C_{11} + C_{44}) - \omega^2 \right)^2 - \left( \frac{k_0^2}{2} (C_{11} + C_{44}) \right)^2 = 0
\]

\[
\frac{k_0^2}{2} (C_{11} + C_{44}) - \omega^2 = \pm \frac{k_0^2}{2} (C_{11} + C_{44})
\]

\[
\frac{\omega^2}{k_0^2} = \frac{1}{2p} \left[ (C_{11} + C_{44}) \mp (C_{12} + C_{44}) \right]
\]

Two solns: either

\[
\frac{\omega}{k} = \sqrt{\frac{1}{2p} \left( C_{11} + C_{12} + 2C_{44} \right)}
\]

or

\[
\frac{\omega}{k} = \sqrt{\frac{1}{2p} \left( C_{11} - C_{12} \right)}
\]

To determine direction of oscillation for these velocities, plug that \( \omega \)

value back into Eq. 3.57 and solve for \( \nu_y \) ratio. One turns out to be

\( \nu_y = \nu_x \) the other \( \nu_y = -\nu_x \).
Dispersion relation:
\[ v = \frac{\omega}{k} \]  
- Only true for long \( \lambda \) (small \( k \))

"Acoustic waves"

**Diagram:**
- \( k \) in \([100]\)
- \( k \) in \([110]\)
- \( k \) in \([111]\)

Done w/ Ch 3

Solved "Elastic Energy Density" p 77 - 78
and "Bulk Modulus & Compressibility" p 80
Chapter 4: Phonons

So far we've focused on long wavelength oscillations (small k).

There vs k is straight line, and slope is speed of elastic wave (p.402 problem 3.2).

Figure 11, pg 101

What about larger k?

Now we'll answer some of those questions.

Basic answers: leaving the continuum limit!

Consider \( \lambda = 10a \Rightarrow k = \frac{2\pi}{10a} = \frac{\pi}{5a} \)

But what if \( \lambda = a \) ? \( k = 2\pi a \)

Is there a wave?

\( \lambda = a/2 \Rightarrow k = 4\pi a \)

That's why there's a kink! \( \Rightarrow \) for \( \lambda < a \) doesn't make sense.

Solid curve looking no more info than dashed curve. Only \( k = 0 \) or \( \lambda = 0 \) are needed!