Optics: what you should already know, by Dr. Colton

From Phys 123 section 2

Complex numbers:
- Euler’s identity: \( e^{ix} = \cos x + i \sin x \)
- Complex numbers as points in the complex plane; polar ↔ rectangular conversion

General wave properties:
- what all of these parameters mean: \( x, t, A, \lambda, f, v, k, \omega, \phi \)
- \( f = A \cos(kx - \omega t + \phi) \leftrightarrow A e^{i(kx - \omega t + \phi)} \)
  (and how to extend that to 3D for arbitrary wave direction and arbitrary oscillation direction)
- \( k = \frac{2\pi}{\lambda}; \quad \omega = \frac{2\pi}{T} \)
- \( v = \lambda f \)
- wave packets: \( v_{phase} = \omega k; \quad v_{group} = (d\omega/dk)|_{k_0} \)

Uncertainty relationships:
- \( \Delta x \Delta k \geq \frac{\hbar}{2} \)
- \( \Delta x \Delta p \geq \frac{\hbar}{2} \)
- \( \Delta t \Delta \omega \geq \frac{\hbar}{2} \)
- \( \Delta t \Delta E \geq \frac{\hbar}{2} \)

Reflection/transmission coefficients at normal incidence:
- \( r = \frac{n_2 - n_1}{n_2 + n_1}; \quad t = \frac{2n_2}{n_2 + n_1} \)
- \( R = |r|^2; \quad T = 1 - R \)

Fourier series:
- \( f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos \left( \frac{2\pi nx}{L} \right) + \sum_{n=1}^{\infty} b_n \sin \left( \frac{2\pi nx}{L} \right) \)  \((2\pi/L = k_0 = \text{fundamental [spatial] frequency})\)
  \[ a_0 = \frac{1}{L} \int_{0}^{L} f(x) \, dx \]
  \[ a_n = \frac{2}{L} \int_{0}^{L} f(x) \cos \left( \frac{2\pi nx}{L} \right) \, dx \]
  \[ b_n = \frac{2}{L} \int_{0}^{L} f(x) \sin \left( \frac{2\pi nx}{L} \right) \, dx \]

Fourier series in time: \( x \rightarrow t; \quad L \rightarrow T; \quad k_0 \rightarrow \omega_0 \)

Index of refraction, \( n \)
- speed of light = \( c/n \)
- \( \lambda_{\text{material}} = \lambda_{\text{vacuum}}/n \)

Laws of reflection/refraction:
- \( \theta_{\text{incident}} = \theta_{\text{reflected}} \)
- \( n_1 \sin \theta_1 = n_2 \sin \theta_2 \)  \((\theta \text{ measured from the perpendicular})\)

Total internal refraction: \( \theta_{\text{critical}} \) of high index material is when \( \theta_2 = 90^\circ \)

Polarization
- Difference between linear and circular polarization
- \( \theta_{\text{Brewster}} = \tan^{-1}(\theta_2/\theta_1) \)

Difference between s- and p-polarization

Lenses/mirrors
- Thin lens equation: \( 1/f = 1/p + 1/q \)
- Mirror: \( f = R/2 \)
- Lensmaker’s eqn: \( \frac{1}{f} = (n-1) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \)  \((R_1 = \text{pos}, R_2 = \text{neg if convex-convex})\)
- magnification: \( M = h/h_o = -q/p \)
f-number of a lens = \( f/D \)

**Diffraction through slits/apertures:**

- Phase difference due to path-length difference: \( \phi = 2\pi(\Delta PL/\lambda) \)
- Parallel ray approximation, if screen distance >> slit separation: \( \Delta PL = dsin\theta \) (\( d \) = distance to reference of phase)
- Field at location on screen is sum of fields from each slit: \( E = E_0 \left( e^{i\phi_1} + e^{i\phi_2} + \ldots \right) \) (integrate if needed)

Intensity \( I \sim |E|^2 \)

- 2 slit result: \( I = I_0 \cos^2 \left( \frac{2\pi d}{\lambda} \sin\theta \right) \); \( dsin\theta = m\lambda \) (maxima); \( = (m + \frac{1}{2})\lambda \) (minima)
- 1 wide slit result: \( I = I_0 \sin^2 \left( \frac{\pi a}{\lambda} \right) \); \( asin\theta = m\lambda \) (minima)
- 2 wide slits result: \( I = (2 \text{ slit result}) \times (1 \text{ wide slit result}) \)
- Arbitrary number/arrangement of slits: how to apply this technique to get \( I(\theta) \)

Small angle approximation sometimes applies: \( \theta \approx \sin\theta \approx \tan\theta = y/L \)

- Circular aperture result, Rayleigh criterion: \( \theta_{\text{min,resolve}} = 1.22\lambda/D \)

- Grating result: \( dsin\theta_{\text{height}} = m\lambda \)
- Spectrometer: \( R = \lambda_{\text{ave}}/\Delta\lambda = \# \text{slits} \times m \)

**Thin film interference:**

- \( OPL = PL \times n \)
- \( \Delta OPL + \text{other phase shifts} = m\lambda \) (constructive); \( \ldots = (m + \frac{1}{2})\lambda \) (destructive)

**Photons:** (possibly not learned until Phys 222)

- photon momentum \( p = h/\lambda \)
- photon energy \( E = pc = hc/\lambda \)

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**From Phys 220:**

**Coulomb’s Law:**

\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} = \frac{1}{4\pi\varepsilon_0} \frac{q\vec{r}}{r^2} \quad \text{(field from a point charge located at origin; } \frac{1}{4\pi\varepsilon_0} = k_c \text{ the Coulomb force constant)}
\]

\[
\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{q(\vec{r} - \vec{r}')}{(\vec{r} - \vec{r}')^3} \quad \text{(field from a point charge located at } \vec{r}' \text{ )}
\]

**Biot-Savart Law:**

\[
\vec{B} = \frac{\mu_0}{4\pi} \int \frac{I d\ell \times \hat{r}}{r^2} \quad \text{(field from a current-carrying wire, } \hat{r} \text{ should really be written } \frac{\vec{r} - \vec{r}'}{r'} \text{ and } d\vec{\ell} = d\vec{r}' \text{)}
\]

**Gauss’s Law (Maxwell #1):**

\[
\oint_{\text{closed surface}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{enclosed}}}{\varepsilon_0} \quad \text{(electric flux is proportional to } q_{\text{enclosed}})\]

**Gauss’s Law for magnetism (Maxwell #2):**

\[
\oint_{\text{closed surface}} \vec{B} \cdot d\vec{A} = 0 \quad \text{(no magnetic monopoles)}\]

**Faraday’s Law (Maxwell #3):**

\[
\oint_{\text{closed path}} \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt} \quad \text{(induced EMF is } -\text{d}(\text{flux})/\text{dt}; \text{ minus sign is Lenz’s Law)}
\]

**Ampere’s Law, with Maxwell correction (Maxwell #4):**

\[
\oint_{\text{closed path}} \vec{B} \cdot d\vec{\ell} = \mu_0 I_{\text{enclosed}} + \varepsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \text{(currents act as sources of magnetic fields; so do changing electric fields)}
\]
From Multivariable Calculus:

Scalar and vector functions:

\[ f = \text{a scalar function of } x, y, z. \text{ Example: } f(x, y, z) = x^2y + \sin z. \]

\[ \vec{F} = \text{a vector function of } x, y, z. \text{ Example: } \vec{F}(x, y, z) = (x^2y + \sin z, xyz, 4). \text{ I.e., } F_x = x^2y + \sin z; \ F_y = xyz; \ F_z = 4 \]

Gradient \( \nabla f = \left( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right) \) (which is a vector)

Divergence \( \nabla \cdot \vec{F} = \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \) (which is a scalar)

Curl \( \nabla \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix} \) (which is a vector)

Laplacian of scalar function \( \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \) (which is a scalar)

Laplacian of vector function \( \nabla^2 \vec{F} = \left( \nabla^2 F_x, \nabla^2 F_y, \nabla^2 F_z \right) \) (which is a vector)

Handy “curl of curl” formula: \( \nabla \times (\nabla \times \vec{F}) = \nabla (\nabla \cdot \vec{F}) - \nabla^2 \vec{F} \)

Gradient Theorem:

\[ f(b) - f(a) = \int_{\text{path from } a \text{ to } b} (\nabla f) \cdot d\vec{l} \]

Divergence Theorem:

\[ \oint_{\text{closed surface}} \vec{F} \cdot d\vec{A} = \int_{\text{volume bounded by the surface}} \nabla \cdot \vec{F} dV \]

Stokes’ Theorem:

\[ \oint_{\text{closed path}} \vec{F} \cdot d\vec{l} = \int_{\text{surface bounded by the path}} (\nabla \times \vec{F}) \cdot d\vec{A} \]