FT of shifted function:

\[
\text{FT} \left( \delta(x-x_0) \right) = \alpha \cdot \delta(x-x_0) \Rightarrow \text{FT} \left( \delta(x-x_0) \right) \ast \text{FT} \left( \begin{array}{c} \cos k_x \frac{a}{2} \\
\int_{-\infty}^{\infty} \delta(x-x_0) e^{-ikx} dx
\end{array} \right) = a \cdot \cos \frac{k_x a}{2} e^{ikx_0}
\]

2D:

\[
\text{FT} \text{(shifted optical)} = e^{-ik_x x_0} e^{-ik_y y_0} \ast \text{FT} \text{(regular aperture)}
\]

Array theory:

\[
\text{FT} \text{(array)} = \text{FT} \text{(aperture)} \cdot \sum_{\text{all apertures}} R(x_x,y_x) e^{-ik_x x_0} e^{-ik_y y_0}
\]

\[\text{Origin not worrying about factor, \text{FT} \text{(optical)}!}\]

\[\text{(x_0,y_0), center \text{ FT of } \sum (x_x,y_x)} \]

\[\text{Real even have to be in an array!}\]

Random vs. Non-random array:

\[\text{Exp fund solution, covers} \]

\[\text{Exp for real, given your solution!}\]

\[I = I(\text{one aperture})\]

\[I = I(\text{optical}) \text{ modulated by } I(\text{one aperture})\]

\[\text{like covering } = \int \text{ real for double slit example}
\]
Diffraction Grating

a) General function

\[
\frac{\lambda}{\sin \theta} = \frac{\lambda}{\sin \theta} \quad (\text{ignore y-direction})
\]

\[
\frac{\lambda}{\sin \theta} \quad \text{max} \quad \frac{\lambda}{\sin \theta} \\
0 \quad \text{min} \quad \frac{\lambda}{\sin \theta} \\
\lambda \quad \text{pass} \quad \frac{\lambda}{\sin \theta}
\]

"Stop function, remember?"

2N+1 total slits (odd)

\[
\text{FT} = \frac{1}{\sqrt{2\pi}} \sum \frac{\sin(\omega x + \phi_0)}{\sin \omega x} \quad \text{from handbuch}
\]

Let N now = total slits = odd

\[
= \frac{1}{\sqrt{2\pi}} \frac{\sin(Nk\Delta x)}{\sin k_x \Delta x}
\]

\[
= \frac{1}{2\pi} \int \text{exp}(k \cdot x) \frac{\sin^2 k_x \Delta x}{2 \Delta x} \frac{\sin(Nk\Delta x)}{\sin k_x \Delta x} \text{d}x
\]

\[
I = I_0 \frac{\sin^2 k \cdot x}{2 \Delta x} \frac{\sin^2 Nk \cdot x}{2 \Delta x}
\]

\[
= \frac{N^2 \sin^2 k \cdot x}{2 \Delta x} \frac{\sin^2 Nk \cdot x}{2 \Delta x}
\]

Recall \( \frac{\sin N\pi x}{\pi x} \) plotted in delta function, broad for \( \lambda \to 0 \)

Now squared

Multiplied by \( \sin^2 = \frac{\sin^2 k \cdot x}{N^2} \)

Recall \( \frac{\sin N\pi x}{\pi x} \) plotted in delta function, broad for \( \lambda \to 0 \)

Now squared

Multiplied by \( \sin^2 = \frac{\sin^2 k \cdot x}{N^2} \)

\[\text{Fraunhofer 'shadow'}\]

Multiples

Annular

"first order peak" (no other peaks)

\[\sin^2 \] on wavelength (via k)

Analyze for visibility
Spectra

We use a (first order) diffraction spot to give our wavelength spacing.

Let grating spacing = \( h \) now.

A) A grating perfect set by comb function

B) A grating perfect set by sine function

In plate case

\[ \frac{k x}{h} = m \pi \]

\[ \frac{2\pi x}{h} = m \pi / \lambda \]

\[ \lambda = \frac{x h}{m \pi} \]

Again from grating function formula

Distance (in k-space) to first max = \( \frac{2\pi}{N h} \)

\[ \Delta k_x = \frac{2\pi}{N h} \]

\[ \frac{x \Delta k}{k} = \frac{2\pi}{N h} \]

\[ \frac{x 2 \pi \lambda}{\lambda^2} = \frac{2\pi}{N h} \]

\[ \lambda = \frac{\lambda}{x N h} \]

\[ x^2 = m \lambda \]

\[ (\Delta \lambda) = \frac{\lambda}{m N} \]

Close to \( \Delta \lambda \) max

\[ R^2 = \frac{\lambda}{\Delta \lambda} \]

\[ R^2 = m N \]

in lower \#p indistinct \# (not illuminated light)

\( \lambda \) or \( m \) (but this decreases intensity)