In this lab you will study the relationship between time dependent signals and their frequency spectrum (i.e., their Fourier transform). You will do this using a computer program which can generate or record waveforms or read-in pre-recorded waveforms. This program will display the waveform along with its Fourier transform.

The first step is to go to the class website and click the “Lab 6 - Fourier transforms” link. You can run the applet and get additional help there. The next thing to do is to play with the program and make sure that you understand how to use it. In particular, make sure you understand how to zoom in and out on the graphs, and how to find the exact value of a point by right-clicking on it.

**Musical Octaves.** Click on “RESET ALL”. This will set up the program to work with a “user defined” waveform and set the waveform equal to $\sin(2\pi 440 t)$. This will generate a sine wave at 440 Hz (the A above middle C). Now, adjust the frequency (the 440) until you hear a tone which is one octave higher. Note the frequency below. Adjust the frequency again until the tone is another octave higher. Note the frequency below. Now think to yourself— does this agree with what we studied in class?

$f_{\text{One Octave Up}} =

f_{\text{Two Octaves Up}} =

**Generating a Square Wave.** Now enter $\text{squarewave}(2\pi 440 t)$ as the user defined waveform to generate a 440 Hz square wave. Play the wave and hear what it sounds like. Now, using the “Its spectrum” graph, find the frequency and amplitude of the four lowest-frequency Fourier components and record them in the table below. Also record the frequency divided by the fundamental frequency (440 Hz). (Hint, $f/440$ Hz should be an integer for all of the components, and should equal 1 for the lowest frequency component.)

<table>
<thead>
<tr>
<th></th>
<th>$f$</th>
<th>$A$</th>
<th>$f/440$ Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td></td>
<td></td>
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<td>2.</td>
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<td>3.</td>
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<td>4.</td>
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Now let’s see what happens when we add together four sine waves with the above frequencies and amplitudes. Type

\[ A\sin(2\pi f_a t) + B\sin(2\pi f_b t) + C\sin(2\pi f_c t) + D\sin(2\pi f_d t) \]

in as the user defined waveform, where A, B, C and D are the amplitudes you measured above, and \( f_a, f_b, f_c, \) and \( f_d \) are the frequencies which go with each amplitude. Click on “Recalc/Record” and then zoom in on the graph of the wave to see if it looks like a square wave. Sketch what you see below:

For kicks, you might want to see what the wave looks like as you add more and more sine terms together. You can get a pretty decent looking square wave!

**Uncertainty Relations.** Now let’s make a short pulse of sound and explore the topic of “wave uncertainty”. Enter \( \sin(2\pi 440 t)\exp(-10000(t-0.5)^2) \) as the user defined waveform and click on “Recalc/Record.” Click on “Zoom to fit,” and take a look at the wave and its spectrum. Then play the wave. Now zoom in on the wave and on its spectrum and estimate \( \Delta t \) and \( \Delta f \). Now calculate \( \Delta \omega \) from \( \Delta f \), and calculate the uncertainty product \( \Delta \omega \Delta t \) and record everything below.

\( \Delta t: \)

\( \Delta f: \)

\( \Delta \omega: \)

\( \Delta \omega \Delta t: \)

Now make the pulse shorter and longer in time by changing the 10000 in the waveform to other numbers. Change it by at least a factor of 20 in both directions (smaller and larger). Describe below what happens to the width of the spectrum when you change the duration of the pulse in time. Why does this happen?
Describe below what happens to the tone of the note as you change the duration of the pulse in time. Why does this happen?

**Playing Around.** You have now finished the lab. But for your own learning experience I recommend that you play around with the program. In particular, you should do the following things.

1. Record the sound of your hands clapping (or use the pre-recorded sound of my hands clapping, available under the “Waveform” drop-down box) and see if the uncertainty product $\Delta \omega \Delta t$ makes sense.

2. Listen to the various pre-recorded waveforms and note their spectral properties. Notice that most of the instruments have a spectrum which looks like a harmonic series and ask yourself why that is the case. Also notice that the percussive instruments do not have a spectrum which looks like a harmonic series. Not even the timpani which seems to generate a specific tone! Ask yourself why a timpani’s waveform does not consist of a harmonic series of frequencies.

3. Try to generate different waveforms by adding sine waves together. You might want to actually calculate the Fourier transform of some waveform, and then plug the results in and see what you get.