The Equation of Radiative Transfer

Let \( j_{\nu} \) [erg g\(^{-1}\) s\(^{-1}\) Hz\(^{-1}\)] be the mass emission coefficient. It represents the radiant energy emitted, per unit mass of emitting material, per second, per unit frequency interval. In most cases of interest, the emission is isotropic.

Let \( \kappa_{\nu} \) [cm\(^2\) g\(^{-1}\)] be the mass absorption coefficient. It represents the absorption cross section, per unit mass of absorbing material. In most cases of interest, \( \kappa_{\nu} \) is also isotropic.

For radiation flowing in the direction indicated, through the incremental pillbox of the figure, total energy must be conserved, i.e., energy out = energy in + energy gains in the beam − energy losses in the beam, or, stated in terms of our radiation field variables,

\[
(I_{\Omega} + dI_{\Omega}) dA d\Omega = I_{\Omega} dA d\Omega + \frac{j_{\nu}}{4\pi} \rho ds dA d\Omega - \kappa_{\nu} \rho I_{\nu} ds dA d\Omega.
\]

Simplifying,

\[
dI_{\nu} = \frac{j_{\nu}}{4\pi} \rho ds - \kappa_{\nu} \rho I_{\nu} ds,
\]

or

\[
\frac{1}{\rho} \frac{dI_{\nu}}{ds} = \frac{j_{\nu}}{4\pi} - \kappa_{\nu} I_{\nu}.
\]

This is known as the equation of radiative transfer.