Terminal Velocity - An Illustration

A body falling at its terminal velocity is not accelerating. By Newton's 2nd Law, it therefore has no net force acting upon it, i.e., the sum of the forces acting upon it must be zero. This does not imply that there are no forces acting upon it. Any falling body in the earth's atmosphere will experience at least two* forces, its weight, \( W \), which acts downward and a force of air resistance, \( F_A \) which acts upward (in the absence of crosswinds). The gravitational force acting on the body (its weight) is proportional to its mass which is proportional to its volume. The air resistance force acting on the body is proportional to its surface area (upon which the air acts) and also proportional to its speed with respect to the air squared, so as a falling body speeds up, the air resistance force acting upon it increases from zero (when it is at rest) until it matches its weight. Thereafter, since the net force has become zero, there is no further acceleration and the body continues to fall at a constant speed (its terminal velocity).

To illustrate further, consider the simple case of a falling cube which has reached its terminal velocity, \( v_T \). The size of the cube pictured at the left is 2 units \( \times \) 2 units \( \times \) 2 units. Hence its volume is 8 units\(^3\) and its surface area is \( 6 \times 4 \text{ units}^2 = 24 \text{ units}^2 \). The weight of the object is therefore proportional to 8 units\(^3\) and its air resistance force is proportional to \( v_T^2 \times 24 \text{ units}^2 \).

Now suppose the cube is cut along the dotted lines into eight equal pieces of size 1 unit \( \times \) 1 unit \( \times \) 1 unit. The volume of each resulting piece is 1 unit\(^3\) and its surface area is \( 6 \times 1 \text{ units}^2 = 6 \text{ units}^2 \). Note then that the volume and therefore the mass and weight is only \( \frac{1}{8} \) that of the larger cube, but the surface area has only dropped to \( \frac{1}{4} \) of its value on the larger cube. It follows that \( v_T^2 \) of the smaller cube must only be \( \frac{1}{8} \) of the value of \( v_T^2 \) for the larger cube in order for the two forces to cancel so that the net force is zero. Therefore \( v_T^2 \text{(small)} = \frac{v_T^2 \text{(large)}}{2} \) or \( v_T \text{(small)} \equiv 0.7 \ v_T \text{(large)}, \ i.e., \ the \ smaller \ the \ body, \ the \ smaller \ the \ associated \ terminal \ velocity. \ The \ shape \ of \ the \ body \ is \ immaterial. \ The \ same \ result \ would \ hold \ for \ bodies \ of \ other \ shapes \ as \ well, \ including \ nearly \ spherical \ water \ droplets. (Actually the decrease in terminal velocity is more sensitive to particle size than is suggested by this oversimplified example. See the table in the next image for quantitative, experimentally verified results.)

*Actually there is a third force, the buoyant force, which affects all bodies immersed in any fluid, including the earth's atmosphere. The buoyant force, which acts upward, is due to the air pressure at the base of a body being greater than the pressure at the top of a body. Hence air pressure exerts a net force. Since an air parcel at rest experiences no air resistance, the sum of the two remaining forces acting on such a parcel, its weight and the buoyant force, must cancel each other and therefore must be equal in magnitude and opposite in direction. From this fact we can infer Archimedes Principle, namely that the buoyant force on any body, immersed in any fluid, must be equal to the weight of the fluid it displaces. (Hence the buoyant force on a helium balloon = the weight of the air it displaces, is greater than the weight of the helium-filled balloon, and the net force is upward.) If an immersed body is considerably denser than air, e.g., a water droplet, then the buoyant force is so much smaller than its weight that it can, for many purposes, be ignored.