A Nearly Relativistic Rocket

or

Why Your Instructor Believes UFOs are Not of Extraterrestrial Origin

A rocket scientist decides to design a rocket capable of accelerating a 1.0-ton payload to a speed of 0.1 % of the speed of light, \( i.e., \, v_f = 0.001c = 3\times10^5 \text{ m/s} \). If available technology limits her to rocket engines having a maximum exhaust speed (with respect to the rocket) of \( u = 5 \text{ km/s} = 5000 \text{ m/s} \), how much fuel must she design the rocket to carry?

**D_1:** No figure is needed here.

**D_2:** Besides the parameters defined in the statement of the problem, let \( M_f = 2000 \text{ lbs/g} = 907 \text{ kg} \) be the rocket payload, \( M_i \) be the initial mass of the rocket (including fuel) and \( -\Delta M = M_f - M_i \) be the fuel load to be carried by the rocket.

**D_3:** Solve for \( \Delta M \). Assume that the total system mass consists only of fuel and payload.

**BE:** Use the rocket equation, \( v_f = v_i + u \ln(M_i/M_f) \).

**S:** Since \( v_i = 0 \), the rocket equation becomes \( v_f = u \ln(M_i/M_f) \). Solving for \( M_i \), \( M_i = M_f e^{v_f/u} \). Hence \( -\Delta M = M_i - M_f = M_f \left( e^{v_f/u} - 1 \right) \).

**N:** Plugging in numbers, \( -\Delta M = (907 \text{ kg}) \left( e^{(300000 \text{ m/s})/5000 \text{ m/s}} - 1 \right) \approx (907 \text{ kg}) \times 60 = 1.04 \times 10^{29} \text{ kg} \).

**U:** ✓

**B:** This is about 17300 times the mass of the earth or 1/19 the mass of the sun! Such a space ship would require 4240 years to reach the nearest star!