4b) Write equ. of motion for spin $\chi(t)$

$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \qquad H = -\frac{\hbar^2}{2m}\nabla^2 + V$$

$$i\hbar \frac{\partial \chi(t)}{\partial t} = H\chi(t) \Rightarrow \boxed{i\hbar \frac{\partial \chi}{\partial t} = -\gamma B_0 S_z\, \chi(t)}$$

c) If initial state is $\chi_0 = a\chi_+^z + b\chi_-^z$, what is state $\chi(t)$ @ later?

$\chi_+^z = \begin{pmatrix}1\\0\end{pmatrix}$ $\chi_-^z = \begin{pmatrix}0\\1\end{pmatrix}$ for time dep, taek on $e^{-itE_n/\hbar}$ but for E_+, E_-

So $\chi(t) = a\chi_+^z e^{-itE_+/\hbar} + b\chi_-^z e^{-itE_-/\hbar} =$

$$= a\begin{pmatrix}1\\0\end{pmatrix}e^{-itE_+/\hbar} + b\begin{pmatrix}0\\1\end{pmatrix}e^{-itE_-/\hbar} = \boxed{\begin{pmatrix}a e^{-itE_+/\hbar}\\ b e^{-itE_-/\hbar}\end{pmatrix} = \chi(t)}$$

$E_+ = -\gamma B_0 \hbar/2 \Rightarrow e^{itB_0\gamma/2}$

$E_- = \gamma B_0 \hbar/2 \Rightarrow e^{-it\gamma B_0/2}$

$$\boxed{\begin{pmatrix}a e^{itB_0\gamma/2}\\ b e^{-itB_0\gamma/2}\end{pmatrix} = \chi(t)}$$

d) Since have χ_+^z & χ_-^z, have $|a|^2 + |b|^2 = 1$, so let

$a = \sin(\alpha/2)$

$b = \cos(\alpha/2)$

- $\langle S_x\rangle = \langle \chi_0 | S_x | \chi_0\rangle = [a e^{-itB_0\gamma/2} \quad b e^{+itB_0\gamma/2}]\frac{\hbar}{2}\begin{bmatrix}0&1\\1&0\end{bmatrix}\begin{bmatrix}a e^{itB_0\gamma/2}\\ b e^{-itB_0\gamma/2}\end{bmatrix}$

$$= \frac{\hbar}{2}[b e^{itB_0\gamma/2} \quad a e^{-itB_0\gamma/2}]\begin{bmatrix}a e^{itB_0\gamma/2}\\ b e^{-itB_0\gamma/2}\end{bmatrix} = \frac{\hbar}{2}ab\left(e^{itB_0\gamma} + e^{-itB_0\gamma}\right)$$

Note: $\cos x = \frac{e^{ix} + e^{-ix}}{2} \Rightarrow \hbar ab\cos(tB_0\gamma) = \frac{2\hbar}{2}\cos(\alpha/2)\sin(\alpha/2)\cos(tB_0\gamma)$

$2\cos(\alpha/2)\sin(\alpha/2) = \sin\alpha \quad \Rightarrow \boxed{\frac{\hbar}{2}\sin\alpha\cos(B_0\gamma t) = \langle S_x\rangle}$

- $\langle S_y\rangle = \langle \chi_0 | S_y | \chi_0\rangle = [a e^{-itB_0\gamma/2} \quad b e^{itB_0\gamma/2}]\frac{\hbar}{2}\begin{bmatrix}0&-i\\i&0\end{bmatrix}\begin{bmatrix}a e^{itB_0\gamma/2}\\ b e^{-itB_0\gamma/2}\end{bmatrix}$

$$= \frac{\hbar}{2}[ib e^{itB_0\gamma/2} \quad -ia e^{-itB_0\gamma/2}]\begin{bmatrix}a e^{itB_0\gamma/2}\\ b e^{-itB_0\gamma/2}\end{bmatrix}$$

$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$

$$= \frac{\hbar}{2}iba\left(e^{itB_0\gamma} - e^{-itB_0\gamma}\right) = \frac{\hbar}{2}\frac{i^2}{i}ba\left(e^{itB_0\gamma} - e^{-itB_0\gamma}\right) = -\hbar ba\cos(B_0 t\gamma)$$

$$= -\frac{\hbar 2}{2}\cos(\alpha/2)\sin(\alpha/2)\cos(B_0\gamma t) = \boxed{-\frac{\hbar}{2}\sin(\alpha)\sin(B_0\gamma t) = \langle S_y\rangle}$$