\[ a. \quad 1 = \int_{-a}^{a} A^2 \, dx = A^2 x \bigg|_{-a}^{a} = 2A^2a = 1 \quad \Rightarrow A = \frac{1}{\sqrt{2a}} \]

\[ b. \quad \Phi(k) = \frac{1}{\sqrt{2\pi}} \int_{-a}^{a} e^{-ikx} \, dx = \frac{1}{2\sqrt{\pi a}} e^{ikx} \bigg|_{-a}^{a} = \frac{1}{2ik\sqrt{\pi a}} (e^{ika} - e^{-ika}) \]

\[ = \left( e^{ikx} - e^{-ikx} \right) \frac{1}{k\sqrt{\pi a}} = \frac{1}{k\sqrt{\pi a}} \sin(ka) \]

\[ c. \quad \Phi(x,t) = \frac{1}{\sqrt{2\pi a}} \int_{-\infty}^{\infty} \sin(ka) e^{ikx - k^2\frac{t}{2m}} \, dk \]

\[ d. \quad \text{for the free particle, } E = \frac{\hbar^2 k^2}{2m} \quad \text{(the coefficient of the time dependent phase)} \]

\[ e. \quad V_p = \frac{\hbar^2 k^2}{2m}, \quad V_G = \frac{\delta W}{\delta k} \quad \omega = \frac{\hbar k^2}{2m}, \quad \frac{\delta W}{\delta k} = \frac{\hbar k}{m} = V_G \]

\[ \text{The speed of individual phases} \quad \uparrow \quad \text{The speed of the wave packet} \]

\[ \rightarrow V_p \quad \rightarrow V_G \]

\[ \underline{Notes} \]

\[ \text{The integral to find } \Phi(k) \text{ only goes from } -a \to a \text{ in this case} \]

\[ \text{The integral to find } \Phi(x,t) \text{ is not given, its bounds remain } -\infty \to \infty \]