**Practice Test Question 5**

A particle with $S_1 = 2$ is combined with a particle with spin $S_2 = 1$.

a) How many possible values can the spin $S$ of the composite system take, and what are they?

Equation 4.184 states:

$$s = (s_1 + s_2), (s_1 + s_2 - 1), (s_1 + s_2 - 2), \ldots, |s_1 - s_2|.$$

We have $s_1 + s_2 = 3$ and $s_1 - s_2 = 1$ so the are 3 possible vaules for s. $s = 3, 2, 1$.

b)Using the Clebsch-Gordan table 4.8 provided to write down the expression for the composite state $|sm\rangle = |31\rangle$ as a linear combination of the individual states $s_1 m_1\rangle|s_2 m_2\rangle$.

$|sm\rangle = \sqrt{\frac{1}{15}}|22\rangle|1-1\rangle + \sqrt{\frac{8}{15}}|21\rangle|10\rangle + \sqrt{\frac{6}{15}}|20\rangle|11\rangle$

c)When measuring the spin $S_z^{(2)}$ of the second particle in the composite state $|sm\rangle = |31\rangle$, whac is the probability of finding $+\hbar$ (meaning $m_2 = +1$)?

The last term above is the only one where the m of the second ket equals 1. So we take the coefficient in front of that term and square. $P = \frac{6}{15}$

d)Apply the lowering operator $S_- = S_-^{(1)} + S_-^{(2)}$ to $|sm\rangle = |31\rangle$ and, using table 4.8, check that the result is indeed a scalar multiple of $|sm\rangle = |30\rangle$.

$$
\begin{aligned}
S_-|sm\rangle &= (S_-^{(1)} + S_-^{(2)})(\sqrt{\frac{1}{15}}|22\rangle|1-1\rangle + \sqrt{\frac{8}{15}}|21\rangle|10\rangle + \sqrt{\frac{6}{15}}|20\rangle|11\rangle) \\
&= \sqrt{\frac{1}{15}}S_-^{(1)}|22\rangle|1-1\rangle + \sqrt{\frac{8}{15}}S_-^{(1)}|21\rangle|10\rangle + \sqrt{\frac{6}{15}}S_-^{(1)}|20\rangle|11\rangle \\
&+ \sqrt{\frac{1}{15}}|22\rangle S_-^{(2)}|1-1\rangle + \sqrt{\frac{8}{15}}|21\rangle S_-^{(2)}|10\rangle + \sqrt{\frac{6}{15}}|20\rangle S_-^{(2)}|11\rangle \\
\\
&= 2\hbar\sqrt{\frac{1}{15}}|21\rangle|1-1\rangle + \hbar\sqrt{\frac{8*6}{15}}|20\rangle|10\rangle + \hbar\sqrt{\frac{6*6}{15}}|2-1\rangle|11\rangle \\
&+ \hbar\sqrt{\frac{1*0}{15}}|22\rangle|1-2\rangle + \hbar\sqrt{\frac{8*2}{15}}|21\rangle|1-1\rangle + \hbar\sqrt{\frac{6*2}{15}}|20\rangle|10\rangle \\
\\
&= \frac{6\hbar}{\sqrt{3}}(\sqrt{\frac{1}{5}}|21\rangle|1-1\rangle + \sqrt{\frac{3}{5}}|20\rangle|10\rangle + \sqrt{\frac{1}{5}}|2-1\rangle|11\rangle
\end{aligned}
$$