Practice Test Question 4

The stationary states of the electron in the hydrogen atom are described by the wave function $\Psi_{nlm}(r,\theta,\phi)$ in spherical coordinates.

a)What are the respective names of the quantum numbers n, l, and m?

n is the principal quantum number.

l is the azimuthal quantum number.

m is the magnetic quantum number.

b)$\Psi_{nlm}(r,\theta,\phi)$ is an eigenstate of three different operators. What are these operators and the corresponding eigenvalues, in terms of n, l, and m and the ground state energy E_1?

The Hamiltonian: H$\Psi_{nlm}(r,\theta,\phi)$=E$\Psi_{nlm}(r,\theta,\phi)$.

The orbital angular momenentum squared:

$L^2\Psi_{nlm}(r,\theta,\phi) = \hbar^2 l(l+1)\Psi_{nlm}(r,\theta,\phi)$.

The z component of the orbital angular momenentum:

$L_z\Psi_{nlm}(r,\theta,\phi) = \hbar m\Psi_{nlm}(r,\theta,\phi)$.

$E_1 = -13.6eV$.

c)Using table 4.3 and 4.7 write the wave function $\Psi_{421}(r,\theta,\phi)$.

$R(r) = \frac{a^{-3/2}}{64\sqrt{5}}(1-\frac{r}{12a})(\frac{r}{a})^2 e^{-r/4a}$.

$Y_2^1(\theta,\phi) = -(\frac{15}{8\pi})^{1/2} sin(\theta)cos(\theta)e^{\imath\phi}$.

So $\Psi_{421}(r,\theta,\phi) = RY_2^1 = -\frac{\sqrt{3}}{128\sqrt{2\pi}}a^{-3/2}(1-\frac{r}{12a})(\frac{r}{a})^2 sin(\theta)cos(\theta)e^{\imath\phi}$

d)What is the energy of the particle and what is its associated degeneracy? (explain).

$E_n = \frac{E_1}{n^2} = \frac{-13.6}{16} = -.85eV$.

The degeneracy is 16, because the energy only depends of n. So the degeneracy is n^2 because of the different values of l and m associated with each n.

e)If the electron would transit from the energy level E_4 to grond state E_1, what would be the wavelength of the light emitted? Where in the electromagnetic sepectrum would it fall?

$\delta E = E_4 - E_1 = E_1(\frac{)}{1}16 - 1) = -13.6 * -.75 = 10.2eV$.

$E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E} = 97nm$.

Which is Ultraviolet.

f)What do we get when applying L^2, L_z, L_+, L_-, L_x, and L_y on $\Psi_{421}(r, \theta, \phi)$? (Express your result in termls of $\Psi_{nlm}(r, \theta, \phi)$)

For L^2: $L^2\Psi_{421}(r, \theta, \phi) = \hbar^2 2(2 + 1)\Psi_{421}(r, \theta, \phi) = 6\hbar\Psi_{421}(r, \theta, \phi)$.

For L_z: $L_z\Psi_{421}(r, \theta, \phi) = \hbar m\Psi_{421}(r, \theta, \phi) = \hbar\Psi_{421}(r, \theta, \phi)$.

For L_+: $L_+\Psi_{421}(r, \theta, \phi) = \hbar\sqrt{2(2 + 1) - 1(1 + 1)}\Psi_{422}(r, \theta, \phi) = 2\hbar\Psi_{422}(r, \theta, \phi)$.

For L_-: $L_-\Psi_{421}(r, \theta, \phi) = \hbar\sqrt{2(2 + 1) - 1(1 - 1)}\Psi_{420}(r, \theta, \phi) = \sqrt{6}\hbar\Psi_{420}(r, \theta, \phi)$.

For L_x: $L_x\Psi_{421}(r, \theta, \phi) = \frac{1}{2}(L_+\Psi_{421}(r, \theta, \phi) + L_-\Psi_{421}(r, \theta, \phi)) = \frac{\hbar}{2}(2\Psi_{422}(r, \theta, \phi) + \sqrt{6}\Psi_{420}(r, \theta, \phi))$.

For L_y: $L_x\Psi_{421}(r, \theta, \phi) = \frac{1}{2\imath}(L_+\Psi_{421}(r, \theta, \phi) - L_-\Psi_{421}(r, \theta, \phi)) = \frac{\hbar}{2\imath}(2\Psi_{422}(r, \theta, \phi) - \sqrt{6}\Psi_{420}(r, \theta, \phi))$.