5) $\sigma_A^2 \sigma_B^2 \geq \left(\frac{\langle [A,B] \rangle}{2i}\right)^2$ Generalized Uncertainty Principle (provided)

a- $\sigma_x \sigma_p \geq \frac{\langle [x,p] \rangle}{2i}$

dummy function

$[x,p] = [x, -i\hbar \frac{d}{dx}] f = \cancel{-x i\hbar \frac{df}{dx}} + \cancel{i\hbar x \frac{df}{dx}} + i\hbar f \rightarrow i\hbar$

$\sigma_x \sigma_p \geq \frac{\langle i\hbar \rangle}{2i} = \boxed{\frac{\hbar}{2}}$ ✓

dummy function

b- $[x,H] = [x, \frac{p^2}{2m} + V] = [x, \frac{p^2}{2m}] + \cancel{[x,V]}^{0} = \left(\frac{xp^2}{2m} - \frac{p^2 x}{2m}\right) f$

$= x\left(-\frac{\hbar^2}{2m}\frac{d^2 f}{dx^2}\right) - \frac{p}{2m}\left(-i\hbar \frac{df}{dx} - i\hbar f\right)$

$= \cancel{-\frac{\hbar^2}{2m} x \frac{d^2 f}{dx^2}} + \frac{\hbar^2}{2m}\left(\cancel{x\frac{d^2 f}{dx^2}} + \frac{df}{dx} + \frac{df}{dx}\right) \Rightarrow \boxed{\frac{\hbar^2}{m}\frac{d}{dx}}$

$\sigma_x \sigma_H \geq \frac{\langle \frac{\hbar^2}{m}\frac{d}{dx} \rangle}{2i} = \boxed{\frac{\hbar |\langle p \rangle|}{2m}}$

c- $\frac{d\langle Q \rangle}{dt} = \frac{i}{\hbar}\langle [H,Q] \rangle + \left\langle \frac{dQ}{dt} \right\rangle$ Heisenberg equation of motion (provided)

$\frac{d\langle x \rangle}{dt} = \frac{i}{\hbar}\langle [H,x] \rangle + \cancel{\left\langle \frac{dx}{dt} \right\rangle}^{0} = \frac{1}{i\hbar}\langle [x,H] \rangle = \frac{1}{\cancel{i\hbar}} \frac{\cancel{i\hbar}\langle p \rangle}{m} = \frac{\langle p \rangle}{m}$

$\boxed{\langle p \rangle = m \frac{d\langle x \rangle}{dt}}$

$\frac{i}{\hbar}\langle [H,Q] \rangle = \frac{d\langle Q \rangle}{dt} - \left\langle \frac{dQ}{dt} \right\rangle \rightarrow \langle [H,Q] \rangle = i\hbar\left(\left\langle \frac{dQ}{dt} \right\rangle - \frac{d\langle Q \rangle}{dt}\right)$ since we really mean Δt, not Q

$\sigma_H^2 \sigma_Q^2 \geq \left(\frac{\langle [H,Q] \rangle}{2i}\right)^2 = \left(\frac{\cancel{i}\hbar\left(\cancel{\left\langle \frac{dQ}{dt} \right\rangle}^{0} - \frac{d\langle Q \rangle}{dt}\right)}{2\cancel{i}}\right)^2$ we can drop the minus sign because we are squaring the answer

$= \left(\frac{\hbar}{2}\frac{d\langle Q \rangle}{dt}\right)^2$

Here we define $\Delta t = \frac{\sigma_Q}{d\langle Q \rangle / dt}$

$\Delta H \Delta t \geq \boxed{\frac{\hbar}{2}}$

dummy function

d- $\frac{d\langle p \rangle}{dt} = \frac{i}{\hbar}\langle [H,p] \rangle + \cancel{\left\langle \frac{dp}{dt} \right\rangle}^{0} = \frac{i}{\hbar}\left\langle \left[\frac{p^2}{2m} + V, p\right] \right\rangle = \frac{i}{\hbar}\left\langle \cancel{\left[\frac{p^2}{2m}, p\right]} + [V,p] \right\rangle f$

$= \frac{i}{\hbar}\left\langle \cancel{-Vi\hbar\frac{df}{dx}} + \cancel{i\hbar V \frac{df}{dx}} + i\hbar \frac{dV}{dx} f \right\rangle = -\left\langle \frac{dV}{dx} \right\rangle$

$\frac{d\langle p \rangle}{dt} = -\left\langle \frac{dV}{dx} \right\rangle$ Ehrenfest's Theorem